Section-C
(Long Answer Type Questions)

Note- This section contains four questions from which **one** question is to be answered as long question. Each question carries 15 marks.

6. (a) Write the statement of Taylors theorem and expand the function $f(x) = \log(\sec x)$.

   (b) Evaluate indefinite integral $\int \frac{1}{1+\tan^2 x} dx$.

(Or)

7. (a) Find the unit vector perpendicular to both the vectors $\vec{a} = 3\hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = 2\hat{\imath} - 2\hat{\jmath} + \hat{k}$.

   (b) Evaluate integral by parts $\int x^2 \sin x \, dx$.

(Or)

8. (a) If $\cos y = x\cos(b + y)$ find $\frac{dy}{dx}$.

   (b) Evaluate integral $\int \frac{x^4}{(x-1)(x^2+1)} dx$ by using partial fraction method.

(Or)

9. (a) Find the work done by a force of magnitude 3 units acting in the direction $3i - 2j + 6k$ acting on a particle, which is displaced from the point $A(2, -3, -1)$ to point $B(4, 3, 1)$.

   (b) Verify vector triple product for any three vectors.

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K-026

Roll No. ....

Paper Code – BCA 1005

BCA 1st Year (Semester-I) EXAMINATION, 2023-24

MATHEMATICS-I

PAPER-V

Time : Two Hours] [Maximum Marks : 75

Note- This paper consists of three Section A, B and C. Carefully read the instructions of each Section in solving the question paper. Candidates have to write their answers in the given answer-copy only. No separate answer-copy (B Copy) will be provided.

Section-A
(Short Answer Type Questions)

Note- All questions are compulsory. Answer the following questions as short answer type questions. Each question carries 5 marks.

1. (A) Find the product of following matrices

$$[1 \quad 2 \quad 3]\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

   (B) State Lagrange's mean value theorem and find the value of c for $f(x) = x^4 + 5$ in interval $[1, 2]$.

(C) Evaluate the determinant of matrix $A=\begin{bmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{bmatrix}$.

(D) Evaluate $\lim_{x\to 3} \frac{x^2-5^2}{x-5}$.

(E) Find the angle between $\vec{a} = 2i + 2j - k$ and $\vec{b} = 3j + 2k$.

(F) If $y = (\log x)^3$ then find $\frac{dy}{dx}$.

(G) Evaluate indefinite integral $\int \frac{1}{\sqrt{9+x^2}} dx$.

(H) Prove that every square matrix can be written as sum of symmetric and skew symmetric matrices.

(I) Find the volume of parallelepiped with adjacent vertices vectors – $\vec{a} = 3i + 2j - k, b = i + j - k, c = 2i - k$.

Section-B

(Long Answer Type Questions)

Note- This section contains four questions from which **one** question is to be answered as long question. Each question carries 15 marks.

2. (a) Find the differential coefficient of $(x)^x$.

(b) Evaluate $\lim_{x\to 0} \frac{x^2 \sin\frac{1}{x}}{e^x}$.

(c) Discuss the continuity of $f(x) = \begin{Bmatrix} x^n \sin\frac{1}{x} & x \neq 0 \\ 0, & x = 0 \end{Bmatrix}$.

(Or)

3. (a) Using properties of determinants find $\begin{vmatrix} a+b+c & a & b \\ c & a+b+c & b \\ c & a & a+b+c \end{vmatrix}$

(b) Find Maclaurin's series expansion of function $\log(\sin x)$ about the point $x = 0$.

(c) Discuss the extrema of function $f(x) = x + \cos 2x, 0 < x < 2\pi$.

(Or)

4. (a) Write Statement of Caley Hamilton theorem and find characteristic equation of matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$.

(b) Verify statement of Caley Hamilton Theorem for matrix A given in part a.

(c) If matrix A is invertible, then find A^{-1}.

(Or)

5. (a) Find the second order derivative of $\frac{x^2}{(x-a)(x-b)}$.

(b) Find the inverse of matrix $A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 6 & 2 \\ -3 & 2 & -1 \end{bmatrix}$ using adjoint method.

(c) Evaluate the limit $\lim_{x\to\infty} \left(\frac{\log x}{x}\right)$.