



- Evaluate the left hand and right hand limits of the function defined by $f(x) = \begin{cases} 1+x^2, & 0 \leq x \leq 1 \\ 2-x, & x > 1 \end{cases}$ at $x=1$. Also, show that limit $f(x)$ does not exist. [2, 1]
- If $f(x) = \begin{cases} \frac{x-|x|}{x}, & x \neq 0 \\ 2x, & x = 0 \end{cases}$ show that $\lim_{x \rightarrow 0} f(x)$ does not exist.
- If $f(x) = \begin{cases} 5x-4, & 0 < x \leq 1 \\ 4x^3-3x, & 1 < x < 2 \end{cases}$, show that $\lim_{x \rightarrow 1} f(x)$ exist.
- Find the left hand and right hand limits of the greatest integer function $f(x) = [x]$ greatest integer less than or equal to x , at $x=k$, where k is an integer. Also show that $\lim_{x \rightarrow k} f(x)$ does not exist.
- Prove that $\lim_{x \rightarrow a^+} [x] = [a]$ for all $a \in \mathbb{R}$, $[.]$ denotes the greatest integer function.
- Show that $\lim_{x \rightarrow 0} \frac{e^x - 1}{\frac{1}{e^x + 1}}$ does not exist.
- Let $f(x) = \begin{cases} \cos x, & x \geq 0 \\ x+k, & x < 0 \end{cases}$. Find the value of constant k , given that $\lim_{x \rightarrow 0} f(x)$ exists. [$k=1$]
- If f is an odd function and if $\lim_{x \rightarrow 0} f(x)$ exists. Prove that this limit must be zero.
- If f is an even function, then prove that $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x)$.
- Let $f(x)$ be a function defined by $f(x) = \begin{cases} 4x-5, & x \leq 2 \\ x-\lambda, & x > 2 \end{cases}$ find λ , if $\lim_{x \rightarrow 2} f(x)$ exists. [$\lambda = -1$]
- If $f(x) = \begin{cases} mx^2+n, & x < 0 \\ nx+m, & 0 \leq x \leq 1 \\ nx^3+m, & x > 1 \end{cases}$ for what value of integers m, n does the limits $\lim_{x \rightarrow 0} f(x)$ and $\lim_{x \rightarrow 1} f(x)$ exist. [$m=n$]
- If $f(x) = \begin{cases} |x|+1, & x < 0 \\ 0, & x = 0 \\ |x|-1, & x > 0 \end{cases}$ for what value(s) of a does $\lim_{x \rightarrow a} f(x)$ exist? [for all $a \neq 0$]
- Suppose $f(x) = \begin{cases} a+bx, & x < 1 \\ 4, & x = 1 \\ b-ax, & x > 1 \end{cases}$ and if $\lim_{x \rightarrow 1} f(x)$. What are possible value of a and b ? [$a=0, b=4$]
- Evaluate $\lim_{x \rightarrow 2} \frac{x^2-4}{x+3}$ [0]
- Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} + \sqrt{1-x}}{1+x}$ [2]
- Evaluate $\lim_{x \rightarrow 0} \frac{2x^2+3x+4}{x^2+3x+2}$ [2]
- Evaluate $\lim_{x \rightarrow -1} \frac{x^3-3x+1}{x-1}$ [$-\frac{3}{2}$]
- Evaluate $\lim_{x \rightarrow 0} \frac{ax+b}{cx+d}, d \neq 0$ [$\frac{b}{d}$]
- Evaluate $\lim_{x \rightarrow 2} \frac{x^2-5x+6}{x^2-4}$ [$-\frac{1}{4}$]
- Evaluate $\lim_{x \rightarrow 2} \frac{x^3-3x^2+4}{x^4-8x^2+16}$ [3/16]
- Evaluate $\lim_{x \rightarrow 2} \frac{x^3-6x^2+11x-6}{x^2-6x+8}$ [1/2]
- Evaluate $\lim_{x \rightarrow 4} \frac{(x^2-x-12)^{18}}{(x^3-8x^2+16x)^9}$ [$7^{18}/4^9$]
- Evaluate $\lim_{x \rightarrow 3} \frac{x^3-7x^2+15x-9}{x^4-5x^3+27x-27}$ [2/9]



24. Evaluate $\lim_{x \rightarrow \sqrt{2}} \frac{x^9 - 3x^8 + x^6 - 9x^4 - 4x^2 - 16x + 84}{x^5 - 3x^4 - 4x + 12}$. $\left[\frac{8\sqrt{2}-31}{\sqrt{2}-3} \right]$
25. Evaluate $\lim_{x \rightarrow 1} \left(\frac{1}{x^2+x-2} - \frac{x}{x^3-1} \right)$. $[-1/9]$
26. Evaluate $\lim_{x \rightarrow 3} (x^2-9) \left[\frac{1}{x+3} + \frac{1}{x-3} \right]$. $[6]$
27. Evaluate $\lim_{x \rightarrow 1} \frac{\sqrt{x^2-1} + \sqrt{x-1}}{\sqrt{x^2-1}}, x > 1$. $\left[\frac{\sqrt{2}+1}{\sqrt{2}} \right]$
28. Evaluate $\lim_{x \rightarrow 0} \frac{x}{\sqrt{a+x} - \sqrt{a-x}}$. $[\sqrt{a}]$
29. Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{a^2+x^2} - \sqrt{a^2-x^2}}{x^2}$. $[1/a]$
30. Evaluate $\lim_{x \rightarrow 4} \frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}$. $\left[\frac{-1}{3} \right]$
31. Evaluate $\lim_{x \rightarrow 2} \frac{x^2-4}{\sqrt{3x-2} - \sqrt{x+2}}$. $[8]$
32. Evaluate $\lim_{x \rightarrow \sqrt{10}} \frac{\sqrt{7+2x} - (\sqrt{5} + \sqrt{2})}{x^2-10}$. $\left[\frac{\sqrt{5}+\sqrt{2}}{6\sqrt{10}} \right]$
33. Evaluate $\lim_{x \rightarrow 2} \frac{\sqrt{1+4x} - \sqrt{5+2x}}{x-2}$. $[1/3]$
34. Evaluate $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}, x \neq 0$. $[1/2\sqrt{x}]$
35. Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - \sqrt{1+x}}{\sqrt{1+x^3} - \sqrt{1+x}}$. $[1]$
36. Evaluate $\lim_{x \rightarrow 2} \frac{x^{10} - 1024}{x-2}$. $[5120]$
37. Evaluate $\lim_{x \rightarrow 9} \frac{x^{3/2} - 27}{x-9}$. $[9/2]$
38. Evaluate $\lim_{x \rightarrow a} \frac{x\sqrt{x-a} - a\sqrt{a}}{x-a}$. $\left[\frac{3}{2}\sqrt{a} \right]$
39. Evaluate $\lim_{x \rightarrow a} \frac{x^m - a^m}{x^n - a^n}$. $\left[\frac{m}{n} a^{m-n} \right]$
40. Evaluate $\lim_{x \rightarrow a} \frac{(x+2)^{5/3} - (a+2)^{5/3}}{x-a}$. $\left[\frac{5}{3}(a+2)^{2/3} \right]$
41. Evaluate $\lim_{x \rightarrow 2} \frac{x^n - 2^n}{x-2} = 80$, and $n \in \mathbb{N}$, find n . $[3(2)^{2/3}]$
42. Evaluate $\lim_{x \rightarrow 1} \frac{(x+x^2+x^3+\dots+x^n)-n}{x-1}$. $\left[\frac{n(n+1)}{2} \right]$
43. Evaluate $\lim_{x \rightarrow -a} \frac{x^9+a^9}{x+a} = 9$. Find the value of a . $[\pm 1]$
44. Evaluate $\lim_{x \rightarrow a} \frac{x^3-a^3}{x-a} = \lim_{x \rightarrow 1} \frac{x^4-1}{x-1}$, find all possible value of a . $\left[\pm \frac{2}{\sqrt{3}} \right]$
- Evaluate the following limits
45. $\lim_{x \rightarrow \infty} \frac{\sqrt{3x^2-1} + \sqrt{2x^2-1}}{4x+3}$ $\left[\frac{\sqrt{3}+\sqrt{2}}{4} \right]$
46. $\lim_{x \rightarrow \infty} \sqrt{x}(\sqrt{x+c} - \sqrt{x})$ $[c/2]$
47. $\lim_{x \rightarrow \infty} \sqrt{x^2+x+1} - \sqrt{x^2+1}$ $[1/2]$
48. $\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{n^2}$ $[1/2]$
49. let $f(x) = \frac{ax+b}{x+1}$, $\lim_{x \rightarrow 0} f(x) = 2$ and $\lim_{x \rightarrow \infty} f(x) = 1$
prove that $f(-2) = 0$
50. Show that $\lim_{x \rightarrow \infty} (\sqrt{x^2+x+1}) - x \neq \lim_{x \rightarrow \infty} (\sqrt{x^2+1} - x)$
51. $\lim_{x \rightarrow 0} \frac{\sin^2 ax}{\sin^2 bx}$ $[a^2/b^2]$
52. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$ $[1/2]$
53. $\lim_{x \rightarrow 0} \frac{1 - \cos 2mx}{1 - \cos 2nx}$ $[m^2/n^2]$
54. $\lim_{x \rightarrow 0} \frac{\sin 2x + \sin 6x}{\sin 5x - \sin 3x}$ $[4]$
55. $\lim_{y \rightarrow 0} \frac{(x+y) \sec(x+y) - x \sec x}{y}$ $[x \tan x \sec x + \sec x]$
56. $\lim_{x \rightarrow 0} \frac{\tan x + 4 \tan 2x - 3 \tan 3x}{x^2 \tan x}$ $[-16]$
57. $\lim_{x \rightarrow 0} \frac{\sin x - 2 \sin 3x + \sin 5x}{x}$ $[0]$
58. $\lim_{x \rightarrow 0} \frac{1 - \cos x \sqrt{\cos 2x}}{x^2}$ $[3/2]$
59. $\lim_{x \rightarrow 0} \frac{\sin(a+x) + \sin(a-x) - 2 \sin a}{x \sin x}$ $[-\sin a]$



60. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{\left(\frac{\pi}{2} - x\right)^2}$ $[1/2]$

61. $\lim_{x \rightarrow \frac{\pi}{6}} \frac{\sqrt{3} \sin x - \cos x}{x - \frac{\pi}{6}}$ $[2]$

62. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos 3x + 3 \cos x}{\left(\frac{\pi}{2} - x\right)^3}$ $[4]$

63. $\lim_{x \rightarrow \infty} 2^{x-1} \tan\left(\frac{a}{2^x}\right)$ $[a/2]$

64. $\lim_{x \rightarrow a} \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}}$ $[2\sqrt{a} \cos a]$

65. $\lim_{x \rightarrow \frac{\pi}{2}} (\sec x - \tan x)$ $[0]$

66. Prove that $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)} = -4$

67. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\sqrt{2} \cos^2 x}$ $[1/8]$

68. $\lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$ $[1/4\sqrt{2}]$

69. $\lim_{x \rightarrow \pi/4} \frac{\sec^2 x - 2}{\tan x - 1}$ $[2]$

70. $\lim_{x \rightarrow 3\pi/2} \frac{1 + \operatorname{cosec}^3 x}{\cot^2 x}$ $\left[\frac{-3}{2}\right]$