PDF

Vedantu
Learn LIVE Online

Eklavya 2021

ONE BOW GURU MANTRA GOAL

Get a Chance to Learn from **India's Best Teachers** & **Crack JEE ADVANCED 2021**

Vedantu's Eklavya 2021 is a Specialized Program for JEE ADVANCED, designed and headed by Co-Founder, IIT alumnus and Master Teacher Anand Prakash. Get mentored by **Top JEE Experts**, who have produced JEE Advanced **AIR 1, 2, 4, 7**, & many more.

Anand Prakash
Co-Founder, Vedantu
IIT- Roorkee

Vedantu
Learn LIVE Online

Our Previous EKLAVYANs & Their Achievements

Student Name	Main Rank	Advanced Rank
Chirag Jain	731	41
Prakhar Agrawal	445	177
Tanmay Gangwar	2133	227
Aditya Kukreja	1772	635
Eknoor Singh	3574	1243
Aditya Gupta	14275	1326

547 JEE Advanced Selections in 2020

JOIN NOW

Study Materials

- JEE Main & Advanced – Free Study Material
- NEET UG – Free Study Material
- NCERT Solutions for Class 1 to 12
- NCERT Books PDF for Class 1 to 12
- ICSE & ISC Free Study Material
- Free Study Material for Kids Learning (Grade 1 to 5)
- Olympiad Free Study Material
- Reference Books (RS Aggarwal, RD Sharma, HC Verma, Lakhmir Singh, Exemplar and More)
- Previous Year Question Paper CBSE & State Boards
- Sample Papers
- Access All Free Study Material Here

DIFFERENTIAL EQUATIONS

Single Type

1. The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where c is a positive parameter, is of

   (A) order 1 (B) order 2
   (C) degree 2 (D) degree 1

2. For any differentiable function y = f(x), the value of $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 \frac{d^2x}{dy^2}$ is

   (A) always zero (B) always non-zero
   (C) equal to $2y^2$ (D) equal to x^2

3. The function $f(\theta) = \frac{d}{d\theta}\int_0^\theta \frac{dx}{1-\cos\theta\cos x}$ satisfies the differential equation

   (A) $\frac{df}{d\theta} + 2f(\theta)\cot\theta = 0$ (B) $\frac{df}{d\theta} - 2f(\theta)\cot\theta = 0$
   (C) $\frac{df}{d\theta} + 2f(\theta) = 0$ (D) $\frac{df}{d\theta} - 2f(\theta) = 0$

4. If $y = e^{4x} + 2e^{-x}$ satisfies the relation $\frac{d^3y}{dx^3} + A\frac{dy}{dx} + By = 0$, then value of A and B respectively are

   (A) –13, 14 (B) –13, –12
   (C) –13, 12 (D) 12, –13

5. If f(x), g(x) be twice differentiable function on [0, 2] satisfying $f''(x) = g''(x)$, $f'(1) = 4$ and $g'(1) = 6$, f(2) = 3, g(2) = 9, then f(x) – g(x) at x = 4 equals

(A) 0 (B) –10

(C) 8 (D) 2

6. For any differential function y = f(x), the value of $\frac{d^2y}{dx^2}$ + $\left(\frac{dy}{dx}\right)^3 \frac{d^2x}{dy^2}$ is:

(A) Always zero (B) always non-zero

(C) Equal to $2y^2$ (D) equal to x^2

7. The degree and order of the differential equation of all parabolas whose axis is x-axis are

(A) 2, 1 (B) 1, 2

(C) 3, 2 (D) none of these

8. The differential equation of all ellipses centred at origin is:

(A) $y_2 + xy_1^2 - yy_1 = 0$ (B) $xyy_2 + xy_1^2 - yy_1 = 0$

(C) $yy_2 + xy_1^2 - xy_1 = 0$ (D) none of these

9. Particular solution of $y_1 + 3xy = x$ which passes through (0, 4) is :

(A) $3y = 1 + 11e^{-\frac{3x^2}{2}}$ (B) $y = \frac{1}{3} + 11e^{-x^2}$

(C) $y = 1 + \frac{11}{3}e^{-x^2}$ (D) $y = \frac{1}{3} + 11e^{\frac{3}{2}x^2}$

10. Solution of $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ is:

(A) $\sin\left(\frac{y}{x}\right) = kx$ (B) $\cos\frac{y}{x} = kx$

(C) $\tan\frac{y}{x} = kx$ (D) none of these

11. The degree and order of the differential equation of all tangent lines to the parabola $x^2 = 4y$ is:

(A) 2, 1 (B) 2, 2

(C) 1, 3 (D) 1, 4

12. Solution of $2y\sin x\frac{dy}{dx} = 2\sin x\cos x - y^2\cos x, x = \frac{p}{2}$, y = 1 is given by

(A) $y^2 = \sin x$ (B) $y = \sin^2 x$

(C) $y^2 = \cos x + 1$ (D) None of these

13. If $y = e^{-x}(A\cos x + B\sin x)$, then y satisfies

(A) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$ (B) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

(C) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$ (D) $\frac{d^2y}{dx^2} + 2y = 0$

14. If $y = e^{4x} + 2e^{-x}$ satisfies the relation $\frac{d^3y}{dx^3} + A\frac{dy}{dx} + By = 0$ then value of A and B respectively are:

(A) –13, 14 (B) –13, –12
(C) –13, 12 (D) 12, –13

15. If $x\frac{dy}{dx} = y$(logy – logx + 1) then the solution of the equation is:

(A) $\log\frac{x}{y} = cy$ (B) $\log\frac{y}{x} = cy$

(C) $\log\frac{x}{y} = cx$ (D) $\log\frac{y}{x} = cx$

16. The differential equation of all ellipses centerd at the origin and major and minor axes along coordinate axes is

(A) $y^2 + xy_1^2 - yy_1 = 0$ (B) $xyy_2 + xy_1^2 - yy_1 = 0$

(C) $yy_2 + xy_1^2 - xy_1 = 0$ (D) none of these

17. The solution of the differential equation dy/dx = cos(x – y) is

(A) $y + \cot\left(\frac{x-y}{2}\right) = C$ (B) $x + \cot\left(\frac{x-y}{2}\right) = C$

(C) $x + \tan\left(\frac{x-y}{2}\right) = C$ (D) none of these

18. $(x^2 + y^2)dy = xydx$. If $y(x_0) = e$, y(1) = 1, then value of x_0 =

(A) $\sqrt{3}e$ (B) $\sqrt{e^2 - \frac{1}{2}}$

(C) $\sqrt{\frac{e^2 - 1}{2}}$ (D) $\sqrt{\frac{e^2 + 1}{2}}$

19. The equation to the curve, which is such that portion of the axis of x cut off between the origin and the tangent at any point is proportional to the ordinate of that point, is

(A) $x = y\,(c - k \log y)$ (B) $\log x = ky^2 + c$

(C) $x^2 = y\,(c - k \log y)$ (D) none of these

(k is constant of proportionality)

20. The curve, which satisfies the differential equation $\frac{xdy - ydx}{xdy + ydx} = y^2 \sin(xy)$ and passes through (0, 1), is given by

(A) $y\,(1 - \cos xy) + x = 0$ (B) $\sin xy - x = 0$

(C) $\sin y + y = 0$ (D) $\cos xy - 2y = 0$

Integer Type

21. If $xdy = y(dx + ydy)$, $y > 0$ and $y(1) = 1$, then $y(-3)$ is equal to

22. The order of the differential equation of all tangent lines to the parabola $y = x^2$ is

23. The degree of the differential equation satisfying $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ is

24. The order of the differential equation of the family of circles with one diameter along the line $y = x$ is

25. The order of the differential equation of all tangent lines to the parabola $y = x^2$ is

26. The order of the differential equation whose general solution is given by
$y= (C_1 + C_2) \cos(X + C_3) – C_4 e^{X+C_5}$ where C_1, C_2, C_3, C_4, C_5 are arbitrary constant is

27. Form the differential equation having $y = (\sin^{-1}x)^2 + A \cos^{-1}x + B$, where A and B are arbitrary constants, as its general solution.

28. Find the differential equation of all the conic whose axes coincide with the co-ordinate axis.

29. Solve $(y \log x - 1) y dx = x dy$.

30. If the population of a country doubles in 50 years, in how many years will it triple under the assumption that the rate of increase is proportional to the number of inhabitants?

SOLUTIONS

Single Type

1. (A)

We have, $y^2 = 2c(x + \sqrt{c})$ …(i)

$\Rightarrow 2y\, y_1 = 2c \;\; \Rightarrow \;\; yy_1 = c$ …(ii)

Eliminating c from (i) and (ii), we get

$y^2 = 2yy_1(x + \sqrt{yy_1}) \;\; \Rightarrow \;\; y - xy_1 = \sqrt{y}{y_1}^{3/2} \;\; \Rightarrow \;\; (y - xy_1)^2 = yy_1^3$

Clearly, it is a differential equation of order one and degree 3.

Hence, (A) is the correct answer.

2. (A)

$\left(\frac{dy}{dx}\right) = \left(\frac{dx}{dy}\right)^{-1}$ for a differential coefficient

or $\frac{d^2y}{dx^2} = -1\left(\frac{dx}{dy}\right)^{-2} \frac{d}{dy}\left(\frac{dx}{dy}\right)\frac{dy}{dx} = -\left(\frac{dx}{dy}\right)^{-2}\frac{d^2x}{dy^2}\frac{dy}{dx}$

$= -\frac{d^2x}{dy^2}\left(\frac{dy}{dx}\right)^3$ or $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3\frac{d^2x}{dy^2} = 0$

Hence, (A) is the correct answer.

3. (A)

We have $f(\theta) = \frac{d}{d\theta}\int_0^{\theta}\frac{dx}{1 - \cos\theta\cos x} = \frac{1}{1-\cos^2\theta} = \operatorname{cosec}^2\theta$

Therefore $\frac{df(\theta)}{d\theta} = -2\operatorname{cosec}^2\theta\cot\theta$

Hence, (A) is the correct answer.

4. (B)

$\frac{dy}{dx} = 4e^{4x} - 2e^{-x} \Rightarrow \frac{d^2y}{dx^2} = 16e^{4x} + 2e^{-x} \quad \Rightarrow \quad \frac{d^3y}{dx^3} = 64e^{4x} - 2e^{-x}$

Putting these values in $\frac{d^3y}{dx^3} + A\frac{dy}{dx} + By = 0$, we have

$(64 + 4A + B)\, e^{4x} + (-2 - 2A + 2B)e^{-x} = 0$

Solving we get A = –13 and B = –12

Hence, (B) is the correct answer.

5. (B)

We have $f''(x) = g''(x)$

Integrating, we get $f'(x) = g'(x) + c$

Putting x = 1, we get $f'(1) = g'(1) + c \Rightarrow c = -2$

$\Rightarrow f'(x) = g'(x) - 2 \Rightarrow \quad f(x) = g(x) - 2x + C_1 \Rightarrow f(2) = g(2) - 4 + C_1 \Rightarrow C_1 = -2$

Thus we have $f(x) = g(x) - 2x - 2$

$\Rightarrow f(4) - g(4) = -10$

Hence, (B) is the correct answer.

6. (A)

$\left(\frac{dy}{dx}\right) = \left(\frac{dx}{dy}\right)^{-1}$ for a differential equation

or $\frac{d^2y}{dx^2} = -1\left(\frac{dx}{dy}\right)^{-2} \frac{d}{dy}\left(\frac{dx}{dy}\right)\frac{dy}{dx} = -\left(\frac{dx}{dy}\right)^{-2} \frac{d^2x}{dy^2}\frac{dy}{dx} = -\frac{d^2x}{dy^2}\left(\frac{dy}{dx}\right)^3$

or $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 \frac{d^2x}{dy^2} = 0.$

Hence, (A) is the correct answer.

7. (B)

Equation of required parabola is of the form $y^2 = 4a(x - h)$

Differentiating, we have $2y\frac{dy}{dx} = 4a \Rightarrow y\frac{dy}{dx} = 2a$

Required differential equation $\left(\frac{dy}{dx}\right)^2 + \frac{d^2y}{dx^2}y = 0$.

Degree of the equation is 1 and order is 2.

Hence, (B) is the correct answer.

8. (B)

Ellipse centred at origin are given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

……(1)

where a and b are unknown constants

$\frac{2x}{a^2} + \frac{2y}{b^2}y_1 = 0 \Rightarrow \frac{x}{a^2} + \frac{y}{b^2}y_1 = 0$ ……(2)

Differentiating again, we get

$\frac{1}{a^2} + \frac{1}{b^2}(y_1^2 + yy_2) = 0$ ……(3)

Multiplying (3) with x and then subtracting from (2) we get

$\frac{1}{b^2}(yy_1 - xy_1^2 - xyy_2) = 0 \Rightarrow xyy_2 + xy_1^2 - yy_1 = 0.$

Hence, (B) is the correct answer.

9. (A)

$\frac{dy}{dx} + (3x)y = x$

I.F = $e^{\int 3x\,dx} = e^{\frac{3}{2}x^2}$

$\therefore$ Solution of given equation is

$y e^{\frac{3}{2}x^2} = \int x.e^{\frac{3}{2}x^2} dx + c = \frac{1}{3}e^{\frac{3}{2}x^2} + c$

If curve passes through (0, 4), then

$4 - \frac{1}{3} = c \Rightarrow c = \frac{11}{3}$

$y = \frac{1}{3} + \frac{11}{3}e^{-\frac{3}{2}x^2} \Rightarrow 3y = 1 + 11e^{-\frac{3}{2}x^2}$.

Hence, (A) is the correct answer.

10. (A)

$\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$

put y = vx $\Rightarrow$ v + x $\frac{dv}{dx}$ = v + tan v

cot v dv = $\frac{dx}{x}$

Integrating, we get ln sin v = ln x + ln k $\Rightarrow$ sin $\frac{y}{x}$ = kx

Hence, (A) is the correct answer.

11. (A)

Equation of any tangent to $x^2 = 4y$ is $x = my + \frac{1}{m}$, where m is arbitrary constant.

$\Rightarrow 1 = m\frac{dy}{dx} \quad \Rightarrow m = \frac{1}{\frac{dy}{dx}}$

$\therefore$ Putting this value of m in $x = my + \frac{1}{m}$, we get

$x = \frac{y}{\frac{dy}{dx}} + \frac{dy}{dx} \quad \Rightarrow \left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$

Which is a differential equation of order 1 and degree 2.

Hence, (A) is the correct answer.

12. (A)

On dividing by sin x,

$$2y\frac{dy}{dx}+y^2\cot x=2\cos x$$

Put $y^2 = v \Rightarrow \frac{dv}{dx}+v\cot x=2\cos x$

I.F. = $e^{\int \cot x dx} = e^{\log \sin x} = \sin x$

$\therefore$ Solution is v. sin x ∫sinx . (2 cos x) dx + c

$\Rightarrow y^2 \sin x = \sin^2 x + c$

When $x=\frac{\pi}{2}$, y = 1, then c = 0

$\therefore y^2 = \sin x$.

Hence, (A) is the correct answer.

13. (C)

$y = e^{-x}(A\cos x + B\sin x)$ ....(1)

$\Rightarrow \frac{dy}{dx}=e^{-x}(-A\sin x + B\cos x) - e^{-x}(A\cos x + B\sin x)$

$\Rightarrow \frac{dy}{dx}=e^{-x}(-A\sin x + B\cos x) - y$ .....(2)

$\Rightarrow \frac{d^2y}{dx^2}=e^{-x}(-A\cos x - B\sin x) - e^{-x}(-A\sin x + B\cos x)$

$\left(-\frac{dy}{dx}\right)$

Using (1) and (2), we get

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}=-y-y-\frac{dy}{dx}$$

Hence, (C) is the correct answer.

14. (B)

On differentiating $y = e^{4x} + 2e^{-x}$, w.r.t x we get

$$\frac{dy}{dx} = 4e^{4x} - 2e^{-x} \Rightarrow \frac{d^2y}{dx^2} = 16e^{4x} + 2e^{-x} \Rightarrow \frac{d^3y}{dx^3} = 64e^{4x} - 2e^{-x}$$

Putting these values in $\frac{d^3y}{dx^3} + A\frac{dy}{dx} + By = 0$, we get

$(64 + 4A + B)e^{4x} + (-2 - 2A + 2B)e^{-x} = 0$.

On solving we get A= –13 and B = –12

Hence, (B) is the correct answer.

15. (D)

$$\frac{dy}{dx} = \frac{y}{x}\left(\log\frac{y}{x} + 1\right)$$

Put y = vx $\Rightarrow$ $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore \quad v + x\frac{dv}{dx} = v(\log v + 1)$

$\Rightarrow \quad \frac{dv}{v\log v} = \frac{dx}{x}$

$\Rightarrow$ log(log v) = logx + logc = logcx

$\Rightarrow$ log v = cx $\Rightarrow$ $\log\left(\frac{y}{x}\right) = cx$.

Hence, (D) is the correct answer.

16. (B)

The given family of curves is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (i)

where a and b are parameters

$\frac{2x}{a^2}+\frac{2y}{b^2}y_1 = 0 \Rightarrow \frac{x}{a^2}+\frac{y}{b^2}y_1 = 0$ (ii)

Differentiating again, we have

$\frac{1}{a^2}+\frac{1}{b^2}\left(y_1^2+yy_2\right) = 0$ (iii)

Multiplying (iii) with x and then subtracting from (ii), we have

$(1/b^2)\,(yy_1 - xy_1^2 - xyy_2) = 0 \Rightarrow xyy_2 + xy_1^2 - yy_1 = 0$

17. (B)

Putting u = x –y, we get du/dx = 1 –dy/dx. The given equation can be written as
1 – du/dx = cos u

$\Rightarrow(1 - \cos u) = du/dx$

$\Rightarrow \int \frac{du}{1-\cos u} = \int dx$

$\Rightarrow \frac{1}{2}\int \text{cosec}^2(u/2)du = \int dx$

$\Rightarrow x + \cot(u/2) = \text{const. or } x + \cot\frac{x-y}{2} = C.$

18. (A)

$\Rightarrow x^2dy + y^2dy = xydx$

$\Rightarrow x(xdy - ydx) = -y^2dy$

$\Rightarrow x\frac{(ydx - xdy)}{y^2} = dy$

$\Rightarrow \frac{x}{y}d\left(\frac{x}{y}\right) = \frac{dy}{y}$

Integrating

$$\Rightarrow \frac{x^2}{2y^2} = \log_e y + c$$

Given y(1) = 1 $\Rightarrow$ c = $\frac{1}{2}$

$$\Rightarrow \frac{x^2}{2y^2} = \log_e y + \frac{1}{2}$$

Now, $y(x_0) = e$

$$\Rightarrow \frac{x_0^2}{2e^2} - \log_e e - \frac{1}{2} = 0$$

$$\Rightarrow x_0^2 = 3e^2$$

$$\Rightarrow x_0 = \sqrt{3}e.$$

19. (A)

Let the curve be y = f (x). The equation of the tangent at any point (x, y) is given by Y – y = f′ (x) (X – x). The portion of the axis of X which is cut off between the origin and the tangent at any point is obtained by putting Y = 0. Therefore

$x - \frac{y}{f'(x)} = ky \Rightarrow x - y\frac{dx}{dy} = ky$

$\Rightarrow \frac{dx}{dy} - \frac{x}{y} = -k$

which is a linear equation in x, and its integrating factor is

$e^{-\int 1/y\,dy} = y^{-1}$.

Therefore, multiplying by y^{-1} we have

$\frac{d}{dy}(xy^{-1}) = -ky^{-1}$

$\Rightarrow xy^{-1} = -k \log y + c$

or $x = y(c - k \log y)$.

Hence, (A) is the correct answer.

20. (A)

The given differential equation can be written as

$$\left(\frac{xdy - ydx}{x^2}\right)\left(\frac{x^2}{y^2}\right) = (x\,dy + y\,dx)\sin xy$$

or $d\left(\frac{y}{x}\right) \left(\frac{x^2}{y^2}\right) = d\,(xy)\sin xy$

Integrating both the sides we get,

$-\frac{1}{y/x} = -\cos xy + c \Rightarrow \frac{x}{y} = \cos(xy) - c.$

For x = 0, y = 1, we get c = 1.

Hence, (A) is the correct answer.

Integer Type

21. (3)

$\Rightarrow xdy - ydx = y^2\,dy$

$\Rightarrow \frac{xdy - ydx}{y^2} = dy$

$\Rightarrow -d\left(\frac{x}{y}\right) = dy$

$\Rightarrow \frac{x}{y} + y = c$

$\because\ y(1) = 1 \Rightarrow c = 2$

$\Rightarrow \frac{x}{y} + y = 2$

for x = –3

$\Rightarrow y^2 - 2y - 3 = 0 \quad \Rightarrow y = -1 \text{ or } 3$

$\Rightarrow y = 3 \quad (\because y > 0)$

22. (1)

The parametric form of the given equation is x = t, $y = t^2$. The equation of any tangent at t is $2xt = y + t^2$. Differentiating, we get $2t = y_1$. Putting this value in the above equation, we have

$2xy_1/2 = y + (y_1/2)^2 \Rightarrow 4xy_1 = 4y + y_1^2$. The order of this equation is one.

23. (1)

Put $x = \sin\theta$, $y = \sin\phi$, so that given equation is reduced to

$$\cos\theta + \cos\phi = a(\sin\theta - \sin\phi)$$

$$\Rightarrow 2\cos\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2} = 2a\cos\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}$$

$\Rightarrow \cot\frac{\theta-\phi}{2} = a \Rightarrow \theta - \phi = 2\cot^{-1}a \Rightarrow \sin^{-1}x - \sin^{-1}y = 2\cot^{-1}a$.

Differentiating we get $\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$

So, the degree is one.

24. (2)

Centre of the circles can be taken as (a, a) and radius as r for some real numbers a and r. Thus, the family is a two parameter family.

Hence, order of the corresponding differential equation is 2.

25. (1)

The line x = my + $\frac{1}{4m}$ is tangent to the given parabola for all m. This line represents one parameter family of lines. Hence, the order of the corresponding differential equation is 1.

26. (3)

The given equation can be written as

$y = A\cos(x + C_3) - Be^x$ where A= C_1 + C_2 and B = $C_4 e^{C_5}$

Hence, there are three independent variables, (A, B, C_3).

Thus, the order of the differential equation will be 3.

27. (2)

y = $(\sin^{-1}x)^2$ + A $\cos^{-1}x$ +B = $(\sin^{-1}x)^2$ - A $\sin^{-1}x$ + $\frac{\pi A}{2} + B.$

Differentiating w.r.t. x, we have

$$\frac{dy}{dx} = \frac{2\sin^{-1}x}{\sqrt{1-x^2}} - \frac{A}{\sqrt{1-x^2}}$$

$$\Rightarrow \left(1\text{-}x^2\right)\left(\frac{dy}{dx}\right)^2 = 4\left(\sin^{-1}x\right)^2 - 4A\ \sin^{-1}x + A^2$$

= 4y - 4B + A^2 - $2\pi A$

Differentiating again w.r.t. x, we have

$$2\left(1-x^2\right)\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right) - 2x\left(\frac{dy}{dx}\right)^2 = 4\frac{dy}{dx}$$

$$\Rightarrow \left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$$

This is the required differential equation.

28. (0)

Any conic whose axes coincides with co-ordinate axis is,

$ax^2 + by^2 = 1$

$\Rightarrow 2ax + 2by.dy/dx=0$

$\Rightarrow$ ax + by dy/dx = 0 ……(i)

$\Rightarrow a + b\left(y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right) = 0$ ……(ii)

From (i) and (ii),

$a = \frac{-by}{x}.\left(\frac{dy}{dx}\right) = -b\left(y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right)$

$\Rightarrow \frac{y}{x}\left(\frac{dy}{dx}\right) = y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 \Rightarrow xy\left(\frac{d^2y}{dx^2}\right) + x\left(\frac{dy}{dx}\right)^2 - y\left(\frac{dy}{dx}\right) = 0$

29. (1)

The given differential equation can be written as

$x\frac{dy}{dx} + y = y^2 \log x$ . . . (1)

Divide by xy^2. Hence $\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{xy} = \frac{1}{x}\log x$

Let $\frac{1}{y} = v \Rightarrow -\frac{1}{y^2}\frac{dy}{dx} = \frac{dv}{dx}$

so that $\frac{dv}{dx} - \frac{1}{x}v = -\frac{1}{x}\log x$ (3)

(3) is the standard linear differential equation

with P = $-\frac{1}{x}, Q = -\frac{1}{x}\log x$

I.F = $e^{\int p dx} = e^{\int -1/x\, dx} = 1/x$

The solution is given by

$$v.\frac{1}{x} = \int \frac{1}{x}\left(-\frac{1}{x}\log x\right)dx = -\int \frac{\log x}{x^2}dx$$

$$= \frac{\log x}{x} - \int \frac{1}{x}.\frac{1}{x}dx = \frac{\log x}{x} + \frac{1}{x} + c$$

$\Rightarrow$ $v = 1 + \log x + cx = \log ex + cx$

or $\frac{1}{y} = \log ex + cx$ or $y(\log ex + cx) = 1$

30. (80)

Let x denote the population at a time t in years.

Then $\frac{dx}{dt} \propto x \Rightarrow \frac{dx}{dt} = kx,$

where k is a constant of proportionality.

Solving $\frac{dx}{dt} = kx,$ we get $\int \frac{dx}{x} = \int k\, dt$

$\Rightarrow \log x = kt + c \Rightarrow x = e^{kt+c} \Rightarrow x = x_o e^{kt},$

where x_0 is the population at time $t = 0$.

Since, it doubles in 50 years, at $t = 50$, we must have $x = 2x_0$.

Hence, $2x_0 = x_0 e^{50k} \Rightarrow 50k = \log 2$

$\Rightarrow k = \frac{\log 2}{50},$ so that $x = x_o e^{\frac{\log 2}{50}t}$

To find t, when it triples i.e. $x = 3x_o$

$\Rightarrow 3x_\circ = x_o e^{\frac{\log 2}{50}t} \Rightarrow 3 = e^{\frac{\log 2}{50}t}$

$\Rightarrow \frac{\log 2}{50}t = \log 3 \Rightarrow t = 50\frac{\log 3}{\log 2} = 80$ years