LIMITS

Approaching BUT NOT Achieving!!

MILKE BHI MIL NA SAKE!

Prepared by Neha Agrawal Mathematically Inclined

(I) DEFINITION

$\lim_{x\to a} f(x) = l$ As x approaches a, $f(x)$ approaches l

LEFT-HAND LIMIT (LHL)

$\lim_{x\to a^-} f(x) = l$ FINITE

Approaching x from the left side (very close to 'a' from the left).

NOTE: $x \to a^-$ does not mean NEGATIVE a

RIGHT-HAND LIMIT (RHL)

$\lim_{x\to a^+} f(x) = l$ FINITE

Approaching x from the right side (very close to 'a' from the right).

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = l \text{ (FINITE)}$$
$$\Rightarrow \lim_{x\to a} f(x) \text{ exists } \& = l$$

INDETERMINATE FORMS

$\frac{0}{0}, \frac{\infty}{\infty}, \infty-\infty, 0\cdot\infty$

$\infty^0, 0^0, 1^\infty$

(II) ALGEBRA OF LIMITS

let $\lim_{x\to a} f(x) = l$ and $\lim_{x\to a} g(x) = m$

1. $\lim_{x\to a} (f \pm g)(x) = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$
2. $\lim_{x\to a} (fg)(x) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$
3. $\lim_{x\to a} \left(\frac{f}{g}\right)(x) = \frac{l}{m}$; $m \neq 0$
4. $\lim_{x\to a} K f(x) = K \lim_{x\to a} f(x) = Kl$ (K → CONSTANT)

(III) METHODS OF SOLVING

① **DIRECT SUBSTITUTION (DS)** ☆ SABSE DUMDAAR

If by directly putting the value of $x = a$, we get $f(x)$ = finite no. (l)

$l \to$ LIMIT (END of THE STORY —x—)

② **FACTORISATION**

- $\lim_{x\to a} \frac{f(x)}{g(x)}$ By DS $x=a$, if we get $\frac{0}{0}, \frac{\infty}{\infty}, ..$ NOT FINITE
- Factorise $f(x)$ and $g(x)$

$(x-a)$ will be a factor, (Cancel IT!) → CULPRIT

- Now, go back to DS ① ☆

③ **RATIONALISATION**

$\frac{f(x)}{g(x)}$ one of them or both involve $\sqrt{\ }$

We Rationalize & go back to DS ① ☆

④ **TRIGONOMETRIC LIMITS**

(i) $\lim_{x\to 0} \frac{\sin x}{x} = 1$ (ii) $\lim_{x\to 0} \frac{\sin^{-1} x}{x} = 1$

(iii) $\lim_{x\to 0} \frac{\tan x}{x} = 1$ (iv) $\lim_{x\to 0} \frac{\tan^{-1} x}{x} = 1$

(v) $\lim_{x\to 0} \cos x = 1$

(i), (iii) x is in RADIANS

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⑦ L'HOSPITAL RULE (L'H-RULE)

- If $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0}, \frac{\infty}{\infty}$ AKA BEST FRIEND
- Both $f(x)$ & $g(x)$ are continuous & differentiable at $x=a$.
- $f'(x), g'(x)$ are continuous at $x=a$.

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

We can continue the process till we get a finite answer by DS ①

⑧ SQUEEZE/SANDWICH RULE

If $g(x) \le f(x) \le h(x)$ on an open interval containing 'a' and if

$$\lim_{x\to a} g(x) = \lim_{x\to a} h(x) = l$$

$$\Rightarrow \lim_{x\to a} f(x) = l$$

⑨ EXPANSIONS

- $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$
- $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$
- $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$
- $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$
- $\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \cdots$

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⑤ EXPONENTIAL and LOGARITHMIC LIMITS

(i) $\lim_{x\to 0}\frac{a^x - 1}{x} = \log_e a$

(ii) $\lim_{x\to 0}\frac{e^x - 1}{x} = 1$

(iii) $\lim_{x\to 0}\frac{\log(1+x)}{x} = 1$

(iv) $\lim_{x\to a}\left(\frac{x^n - a^n}{x-a}\right) = n \cdot a^{n-1}$ (a^n Ki Derivative wali Feeling)

(v) $\lim_{x\to a}(f(x))^{g(x)}$

WHEN $\lim_{x\to a} f(x) = 1$, $\lim_{x\to a} g(x) = \infty$

$= e^{\lim_{x\to a} g(x)(f(x)-1)}$

TREE-MONKEY TRICK

FOR U my Bandron!!

⑥ IMPORTANT RESULT TRICK for ∞

$$\lim_{x\to\infty}\frac{f(x)}{g(x)} = \frac{a_0 + a_1x + a_2x^2 + \cdots + a_mx^m}{b_0 + b_1x + b_2x^2 + \cdots + b_nx^n}$$

- If $m=n$: $\frac{a_m}{b_n}$
- If $m<n$: 0
- If $m>n$: ∞ $(a_m \cdot b_n > 0)$
- If $m>n$: $-\infty$ $(a_m \cdot b_n < 0)$

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