SPECTRUM CAREER INSTITUTE

CBSE -2020 MOCK (1)

MATEMATICS

(PAPER CODE M-2001)

Time : 3 hrs M.M. : 80

General Instructions

- All questions are compulsory.
- This question paper contains 36 questions divided into 4 sections A, B, C and D.
- Section A comprises of 20 questions of 1 mark each.
- Section B comprises of 6 questions of 2 marks each.
- Section C comprises of 6 questions of 4 marks each.
- Section D comprises of 4 questions of 6 marks each.
- There is no overall choice. However, internal choice has been provided in 3 questions of 1 mark each, 2 questions of 2 marks each, 2 questions of 4 marks each and 2 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculator is not permitted. You may ask for logarithmic tables, if required.

SECTION A (Objective Questions, 1 M)

Directions (Q. Nos. 1-10) *There are multiple choice type questions. Select the correct option.*

1. Let $f : R \to R$ be defined by $f(x) = x^2 + 1$. Then, pre-images of 17 and -3, respectively, are

(a) $\phi, \{4, -4\}$ (b) $\{3, -3\}, \phi$

(c) $\{4\}, \phi$ (d) $\{4, -4)\}, \{2, -2\}$

2. If the sides of an equilateral triangle are increasing at the rate of 2 cm/s, then the rate at which the area increases, when side is 10 cm, is

(a) $10\ cm^2/s$

(b) $\sqrt{3}\ cm^2/s$

(c) $10\sqrt{3}\ cm^2/s$

(d) $\frac{10}{3}\ cm^2/s$

3. Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let $z = px + qy$, where $p, q > 0$ conditions on p and q, so that the maximum of z occurs at (3, 0) and (1, 1) is

(a) $p = 2q$ (b) $2p = q$

(c) $p = 3q$ (d) $p = q$

4. If A and B are events such that $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cup B) = 0.5$, then $P(B' \cap A)$ equals to

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) $\frac{3}{10}$ (d) $\frac{1}{5}$

5. If $f(x) = x^2 \sin\frac{1}{x}$, where $x \neq 0$, then the value of the function f at $x = 0$, so that the function is continuous at $x = 0$, is

(a) 0 (b) −1

(c) 1 (d) None of these

6. $\int_{-1}^{1} \frac{dx}{1+x^2}$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

7. The equation of the normal to the curve $y = \sin x$ at (0, 0) is

(a) $x = 0$ (b) $y = 0$

(c) $x + y = 0$ (d) $x - y = 0$

8. $\int_{2}^{3} \frac{dx}{1-x^2}$ is equal to

(a) $\frac{1}{2}\log\frac{2}{3}$ (b) $\frac{1}{2}\log\frac{3}{2}$

(c) $\frac{2}{3}\log\frac{3}{2}$ (d) $\frac{1}{3}\log\frac{2}{3}$

9. If $f(x) = \cos^{-1}(\sin x)$, then $f'(x)$ is equal to

(a) 0 (b) 1

(c) −1 (d) None of these

10. $\int_{1}^{2} \frac{dx}{x\sqrt{x^2-1}}$ is equal to

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

Directions (Q. Nos. 11-15) *Fill in the blanks*

11. If A and B are symmetric matrices, then $AB - BA$ is a ..... .

Or

If the determinant of matrix A of order 3×3 is of value 4. Then, the value of $|3A|$ is equal to ..... .

12. Two or more vectors having same initial point are called ..... .

13. If A is an event associated with the sample space S of a random experiment, then $P\left(\frac{S}{A}\right) =$ ..... .

14. A function $f : A \to B$ is defined to invertible, then f must be ..... .

Or

If the set A contains 5 elements and the set B contains 6 elements, then number of one-one and onto mapping from A to B is equal to ..... .

15. Solution obtained by giving particular values to the arbitrary constants in the general solutions of a differential equation is called ..... .

Directions (Q. Nos. 16-20) *Answer the following questions.*

16. If $y = \sin^{-1}(\sin x)$, $x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$, then evaluate $\frac{dy}{dx}$.

Or

Verify Lagrange's mean value theorem for $f(x) = \log x$ in [1, 2].

17. Evaluate $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

18. Find all the vectors of magnitude $10\sqrt{3}$ that are perpendicular to the plane of $\hat{i} + 2\hat{j} + \hat{k}$ and $-\hat{i} + 3\hat{j} + 4\hat{k}$.

19. Evaluate $\int_{-\pi/6}^{\pi/6} x^3 \cos^2 x \, dx$.

20. Write the integrating factor of the differential equation

$$(1 + y^2) + (2xy - \cot y)\frac{dy}{dx} = 0$$

SECTION B (Short Answer Type Questions, 2 M)

21. Let f be an invertible function, then prove that $(f^{-1})^{-1} = f$.

Or

Let $A = \{a, b, c\}$ and the relation R be defined on A as follows

$$R = \{(a, a), (b, c), (a, b)\}.$$

Then, write minimum number of ordered pairs to be added in R to make R reflexive and transitive.

22. Find the position vector of a point A in space, such that OA is inclined at 60° to OX and at 45° to OY and $|OA| = 10$ units.

23. The total revenue (in ₹) received from the sale of x units of a product is given by $R(x) = 13x^2 + 26x + 15$. Find the marginal revenue when $x = 7$.

24. If the function $f(x) = \frac{1}{x+2}$, find the points of discontinuity of the composite function $y = f(f(x))$.

Or

If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ and $x \neq y$, prove that $\frac{dy}{dx} = -\frac{1}{(x+1)^2}$.

25. Using elementary transformations, find the inverse of the matrix $\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$, if it exists.

26. Find the probability of drawing a diamond card in each of the two consecutive draws from a well-shuffled pack of cards, if the card drawn is not replaced after the first draw.

SECTION C (Long Answer Type I Questions, 4 M)

27. If $x, y, z \in [-1, 1]$, such that $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{-3\pi}{2}$, find the value of $x^2 + y^2 + z^2$.

Or

Prove that

$$2\tan^{-1}\left(\frac{1}{5}\right) + \sec^{-1}\left(\frac{5\sqrt{2}}{7}\right) + 2\tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}.$$

28. Evaluate $\int (\sqrt{\tan x} + \sqrt{\cot x})\, dx$.

Or

Evaluate $\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$.

29. Solve $(x^3 - 3xy^2)\, dx = (y^3 - 3x^2 y)\, dy$.

30. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be non-zero, non-coplanar vectors. Prove that

$\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$ and $\vec{a} - 3\vec{b} + 5\vec{c}$ are coplanar vectors.

31. In a hockey match, both teams A and B scored same number of goals upto the end of the game, so to decide the winner, the referee asked both the captains to throw a die alternatively and decided that the team, whose captain gets a six first, will be declared the winner. If the captain of team A was asked to start, find their respective probabilities of winning the match.

32. David wants to invest atmost ₹ 12000 in bonds A and B. According to the rule, he has to invest atleast ₹ 2000 in bond A and atleast ₹ 4000 in bond B. If the rates of interest on bonds A and B, respectively are 8% and 10% per annum. Formulate the problem as linear programming problem and solve it graphically for maximum interest. Also, determine the maximum interest received in a year.

SECTION D (Long Answer Type II Questions, 6 M)

33. Find the area of the circle $x^2 + y^2 = 16$, which is exterior to the parabola $y^2 = 6x$, by using integration.

34. Find the image of line $\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$.

Or

Find the distance of the point (3, 4, 5) from the plane $x + y + z = 2$ measured parallel to the line $2x = y = z$.

35. Find the equations of tangent and normal to the curve $y = \frac{(x-7)}{(x-2)(x-3)}$ at the point, where it cut the X-axis.

Or

Show that the equation of normal at any point on the curve $x = 3\cos\theta - \cos^3\theta$, $y = 3\sin\theta - \sin^3\theta$ is $4(y\cos^3\theta - x\sin^3\theta) = 3\sin 4\theta$.

36. If $A = \begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that

$$I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}.$$