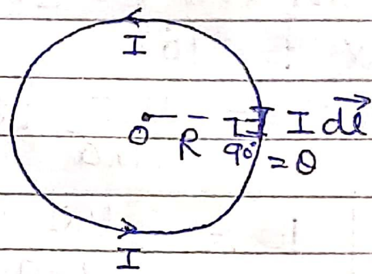
MAGNETIC FIELD- LECTURE-20

APPLICATIONS OF B-S LAW ---Continue.

(B) Magnetic field due current carrying loop or coil at its centre:-

Considering a circular loop of radius R and carrying current I.

Applying B-S Law.



$$dB = \frac{\mu_0 I}{4\pi}\frac{dl \sin 90^\circ}{R^2}$$

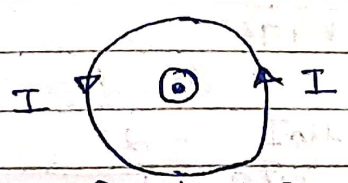
$$B = \frac{\mu_0 I}{4\pi R^2}\int_0^{2\pi R} dl = \frac{\mu_0 I}{4\pi R^2} \times 2\pi R$$

$\therefore B = \frac{\mu_0 I}{2R}$, If there are N turns then

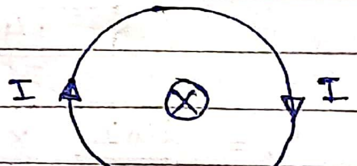
$$B = \frac{\mu_0 N I}{2R}$$

The direction of Magnetic field at the centre of the coil is given by Right hand Palm Rule.

"It state that if the fingers of Right hand are curved in such a way that these point in the direction of current in the circular loop, then thumb gives the direction of Magnetic field."



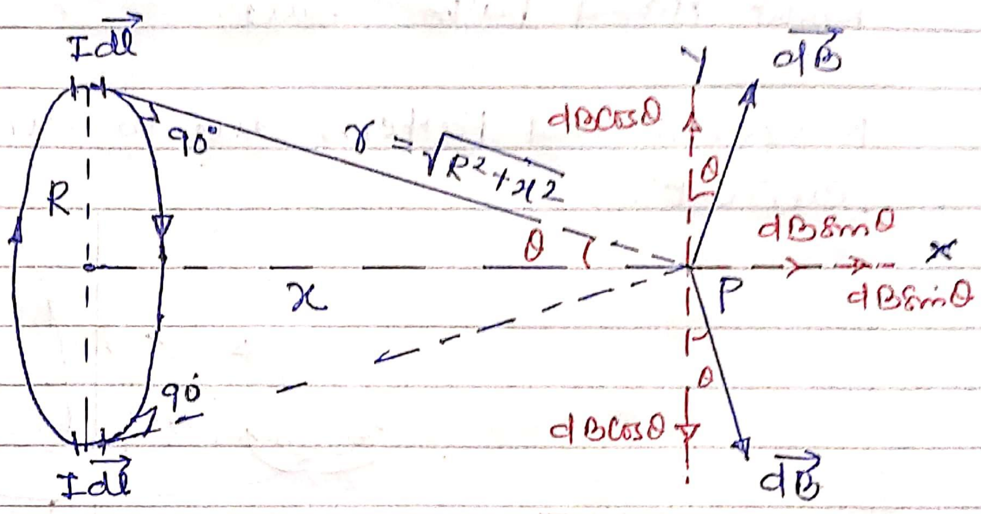
(Dot field at Centre)



(Cross field at Centre)

(C) Magnetic field due to current carrying circular coil at any point on its axis :-

Considering two diametrically opposite current elements.



The magnetic field due to these elements at P -

$$dB = \frac{\mu_0 I}{4\pi}\frac{dl \sin 90°}{r^2} = \frac{\mu_0 I}{4\pi}\frac{dl}{(R^2+x^2)}$$

Resolving these vectors along X and Y axis -
Being equal and opposite all the components along Y-axis will cancel each other.

$\therefore \int dB\cos\theta = 0$ Hence Net Magnetic field at P.

$$B = \int dB\sin\theta = \int \frac{\mu_0 I}{4\pi}\frac{dl}{(R^2+x^2)} \times \frac{R}{\sqrt{(R^2+x^2)}}$$

$$\therefore B = \frac{\mu_0 I}{4\pi}\frac{R}{(R^2+x^2)^{3/2}} \times \int_0^{2\pi R} dl$$

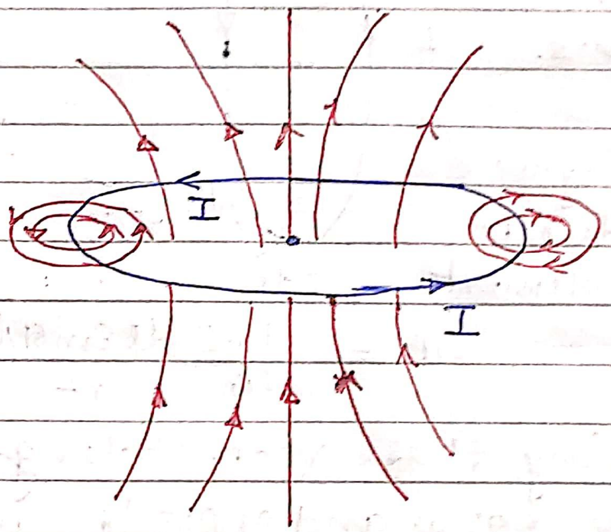
$$\therefore B = \frac{\mu_0 I}{4\pi}\frac{2\pi R^2}{(R^2+x^2)^{3/2}}$$, If there are N turns in coil, then

$$= \frac{\mu_0}{2\pi}\frac{NI(\pi R^2)}{(R^2+x^2)^{3/2}}$$ $A = \pi R^2 =$ Area of loop

$$\therefore \quad \boxed{\vec{B} = \frac{\mu_0 \vec{M}}{2\pi (R^2+x^2)^{3/2}}}$$ where $\vec{M}$ = Dipole Moment $= NI\vec{A}$

Direction of Magnetic field is given by Right Hand Palm Rule.

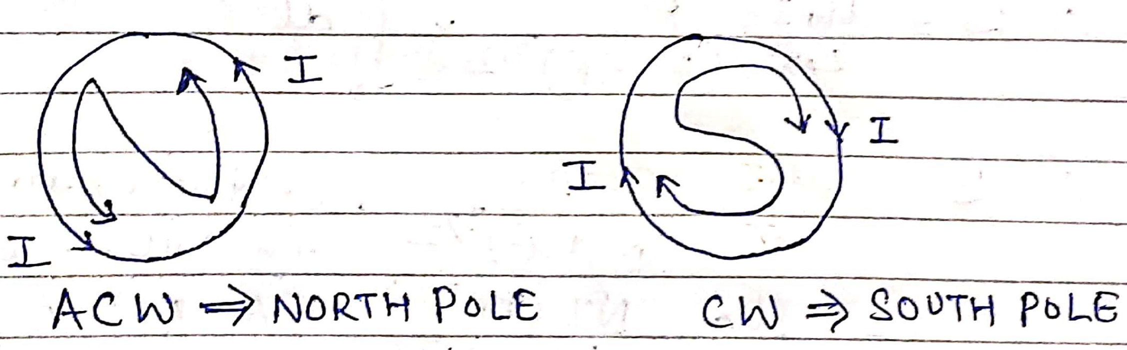
Magnetic field Pattern due to Circular Coil Carrying current.



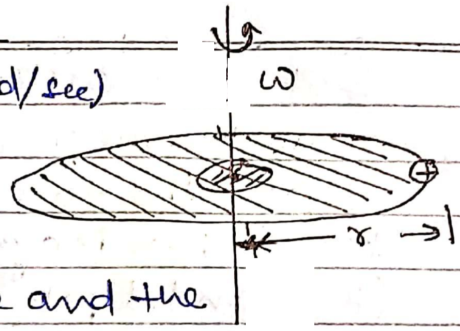
MAGNETIC DIPOLE MOMENT DUE TO CURRENT CARRYING LOOP –

A current carrying loop acts as Magnetic dipole. The dipole moment of loop of area A and Number of turns N carrying current I is given as.

$$\vec{M} = NI\vec{A} \quad \text{Ampere turn} \cdot m^2$$



ATOM AS A MAGNETIC DIPOLE



Let ω = angular velocity of e^- (rad/sec)

r = Radius of orbit.

I = Current = $\frac{\text{Charge}}{\text{Time}} = \frac{e\omega}{2\pi}$

This circular orbit behaves as dipole and the Magnetic dipole moment of the orbit $M = IA = \frac{e\omega}{2\pi} \times \pi r^2$

$\therefore \quad M = \frac{1}{2} e\omega r^2$ ---- (i)

According to Bohr's Postulate for a stable orbit, angular momentum of electron must be integral multiple of $h/2\pi$

$\therefore \quad mvr = \frac{nh}{2\pi} \Rightarrow mr^2\omega = \frac{nh}{2\pi}$ ($\because v = r\omega$) -- (ii)

From equation (i) and (ii) $M = \frac{neh}{4\pi m}$

For, $n = 1$, $h = 6.6 \times 10^{-34}$ J-s $m = 9.1 \times 10^{-31}$ kg

$\boxed{\mu_B = 9.27 \times 10^{-24} \text{ Amp}\cdot m^2}$ ***(Bohr's Magneton)***

Bohr's Magneton is defined as Magnetic moment of electron in its 1^{st} stable orbit.

Gyromagnetic Ratio

It is the ratio of Magnetic Moment of revolving electron to its angular momentum.

$$G.M.R. = M/L = \frac{1}{2} e\omega r^2 / m\omega r^2 = \frac{e}{2m}$$

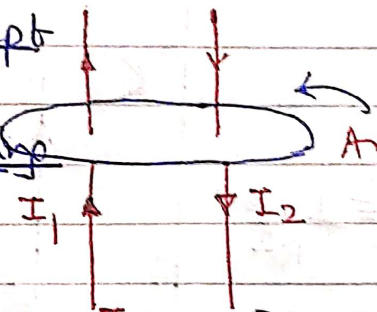
$$= \frac{1.6 \times 10^{-19} C}{2 \times 9.1 \times 10^{-31}} = 8.8 \times 10^{10} \text{ C/kg}$$

AMPERE CIRCUITAL LAW :-

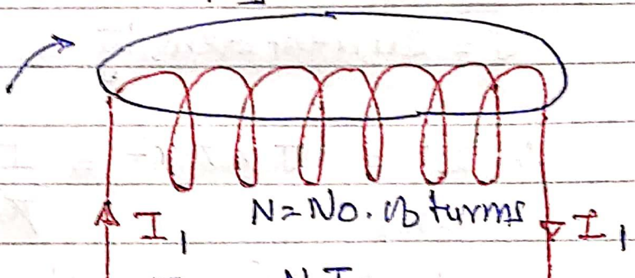
It states that the line integral of Magnetic field over a closed loop is μ_0 times the total current threading the loop.

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I \quad [I = \text{Total current Threading the loop}]$$

Concept of Threading



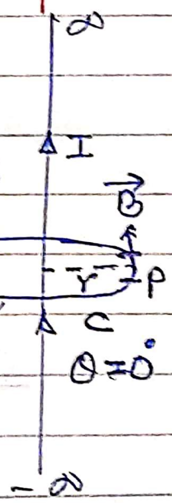
$I = I_1 - I_2$ Fig.1



$I = N I_1$

Fig.2

PROOF OF ACL using B-S-law :-



Considering a current carrying wire of infinite Length.

According B-S law Magnetic field at r distance from wire $= B = \dfrac{\mu_0 I}{2\pi r}$

Now using this value to prove ACL.

$$\oint_C \vec{B}\cdot\vec{dl} = \int_0^{2\pi r} B\,dl\cos 0^\circ$$

$$\Rightarrow \int_0^{2\pi r} B\,dl = B \times 2\pi r = \frac{\mu_0 I}{\cancel{2\pi r}} \times \cancel{2\pi r} = \mu_0 I$$

$$\therefore \oint \vec{B}\cdot\vec{dl} = \mu_0 I$$

which proves Ampere Circuital law by B-S. law.