VECTORS

VECTORS KA TEER BNAYEGA VEER!!

Prepared by Neha Agrawal Mathematically Inclined!

I) **VECTOR:** A quantity which has both magnitude & direction.

$\overrightarrow{AB}$ — Initial point, Terminal point. Directed line segment.

MAGNITUDE OF A VECTOR: $|\overrightarrow{AB}|$

II) **SCALAR:** A quantity which has ONLY magnitude but no Direction.

(IDHAR CHALA MAIN UDHAR CHALA ♫)

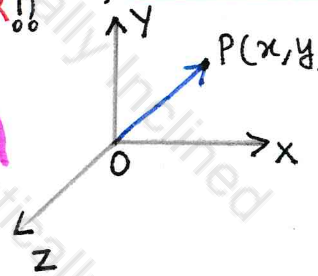
III) **POSITION VECTOR OF A POINT:** (p.v.)

let O: fixed point

p.v. of a Point P is $\overrightarrow{OP}$

(Position of pt. P w.r.t. a reference pt. i.e. Origin)

IV) **COMPONENTS OF A VECTOR:**



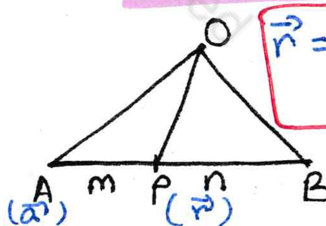
$\overrightarrow{OP} = x\hat{i} + y\hat{j} + z\hat{k}$

SCALAR COMPONENTS: x, y, z

VECTOR COMPONENTS: $x\hat{i}, y\hat{j}, z\hat{k}$

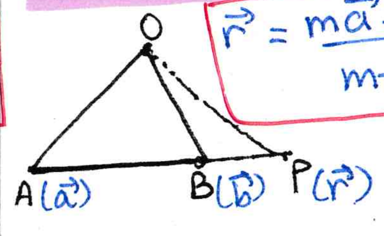
V) **SECTION FORMULA:**

INTERNAL



$$\boxed{\vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n}}$$

EXTERNAL



$$\boxed{\vec{r} = \frac{m\vec{a} - n\vec{b}}{m-n}}$$

VI) **MID-POINT FORMULA:**

$$\vec{r} = \frac{\vec{a} + \vec{b}}{2}$$

VII) **TYPES OF VECTORS:**

1) Zero: A vector whose magnitude is 0 & no specific direction. $\overrightarrow{AA}, \overrightarrow{BB}$

2. Unit: $\boxed{\hat{a} = \frac{\vec{a}}{|\vec{a}|}}$ A vector whose magnitude is UNITY.

VERY IMPORTANT to GIVE THE DIRECTION

3. Co-Initial Vectors: Vectors having same initial point.
   Co-terminal: Same terminal point
4. Collinear/Parallel: vectors having the same/parallel line of support. $\vec{a} \parallel \vec{b} \Rightarrow \boxed{\vec{b} = \lambda \vec{a}}$
5. Co-planar: 3 or more vectors are said to be co-planar (lying in the same plane.
6. Negative of a vector: A vector having the same magnitude, but opposite in direction.
7. Equal vectors: ↳ Equal magnitudes ↳ Same direction

Neha Agrawal Mathematically Inclined! KDS ho gya! ♡♡

③ PRODUCT OF 2 VECTORS

DOT (SCALAR)

- $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

  $0 \le \theta \le \pi$

  θ : acute $\Rightarrow \vec{a}\cdot\vec{b} > 0$

  θ : obtuse $\Rightarrow \vec{a}\cdot\vec{b} < 0$
- $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$
- $(m\vec{a})\cdot\vec{b} = m(\vec{a}\cdot\vec{b})$
- $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$
- $\boxed{\vec{a}\cdot\vec{b} = 0 \Leftrightarrow \vec{a}\perp\vec{b} \quad (\vec{a}\neq 0, \vec{b}\neq 0)}$
- $\vec{a}\cdot\vec{a} = |\vec{a}|^2$
- $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

  $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$
- Projection of $\vec{a}$ on $\vec{b}$

  $\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$ GEOMETRICAL INTERPRETATION
- $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

  Gives ∠s b/w 2 vectors/Lines.

CROSS (VECTOR)

- $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\sin\theta\cdot\hat{n}$

  $\hat{n}$: Unit vector ⊥ to both $\vec{a}$ & $\vec{b}$

  SUPER STAR!!
- $\vec{a}\times\vec{b} \neq \vec{b}\times\vec{a}$
- $(m\vec{a})\times\vec{b} = \vec{a}\times(m\vec{b}) = m(\vec{a}\times\vec{b})$
- $\vec{a}\times(\vec{b}+\vec{c}) = (\vec{a}\times\vec{b})+(\vec{a}\times\vec{c})$
- $\boxed{\vec{a}\times\vec{b} = 0 \Leftrightarrow \vec{a}\ \&\ \vec{b} \text{ are parallel}}$

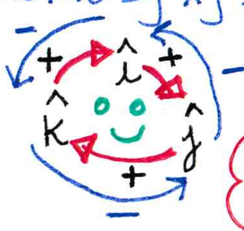
  $(\vec{a}\neq 0, \vec{b}\neq 0)$

  $\vec{a} = \lambda\vec{b}$, λ: scalar
- $\boxed{|\vec{a}\times\vec{b}|}$ = AREA OF PARALLELOGRAM → $\vec{a}$ & $\vec{b}$ are adjacent sides of //gm

  Also, $\boxed{\frac{1}{2}|\vec{d_1}\times\vec{d_2}|}$ Diagonals of //gm

  $\frac{1}{2}|\vec{a}\times\vec{b}|$ = Area of △

  GEOMETRICAL INTERPRETATION
- $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$



Clockwise +ve

Anti-clockwise −ve

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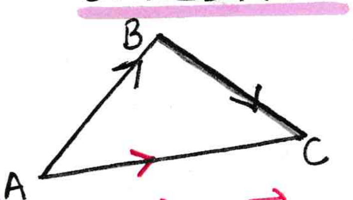
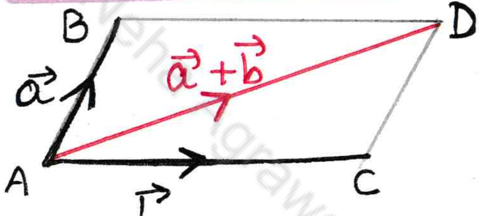
VIII) ALGEBRA OF 2 VECTORS :-

① TRIANGLE LAW OF ADDITION



$\vec{AB}+\vec{BC} = \vec{AC}$

PARALLELOGRAM LAW OF ADDITION



$\vec{AB}+\vec{AC} = \vec{AD}$

PROPERTIES :-

1) $\vec{a}+\vec{b} = \vec{b}+\vec{a}$ — Commutative
2) $(\vec{a}+\vec{b})+\vec{c} = \vec{a}+(\vec{b}+\vec{c})$ — Associative
3) $\vec{a}+\vec{0} = \vec{0}+\vec{a} = \vec{a}$
4) $\vec{a}+(-\vec{a}) = \vec{0} = (-\vec{a})+\vec{a}$

COMPONENT FORM :-

$\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

$+\ \vec{b} = b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

$\vec{a}\pm\vec{b} = (a_1\pm b_1)\hat{i}+(a_2\pm b_2)\hat{j}+(a_3\pm b_3)\hat{k}$

② MULTIPLICATION of a VECTOR by a SCALAR

If $\vec{a}$: vector, m : scalar

$m\vec{a}$: vector // $\vec{a}$ & magnitude |m| times $\vec{a}$.

1) $m(\vec{a}) = (\vec{a})m = m\vec{a}$
2) $m(n\vec{a}) = n(m\vec{a}) = (mn)\vec{a}$
3) $(m+n)\vec{a} = m\vec{a}+n\vec{a}$
4) $m(\vec{a}+\vec{b}) = m\vec{a}+m\vec{b}$

STP (contd.)

③ GEOMETRICAL INTERPRETATION

Volume of Parallelopiped = $\boxed{V = [\vec{a}\ \vec{b}\ \vec{c}]}$

$\vec{a}, \vec{b}, \vec{c}$: coterminous

$\boxed{\frac{1}{6}[\vec{a}\ \vec{b}\ \vec{c}] = \text{Vol. of Tetrahedron}}$

④ DOT & CROSS can be interchanged

$\vec{a}\cdot(\vec{b}\times\vec{c}) = (\vec{a}\times\vec{b})\cdot\vec{c}$

$[\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = [\vec{c}\ \vec{a}\ \vec{b}]$

Moving in CYCLIC ORDER is the SAME!

$[\vec{a}\ \vec{b}\ \vec{c}] = -[\vec{a}\ \vec{c}\ \vec{b}]$ NO CYCLIC ORDER ⇒ ⊖

⑤ $\vec{a}, \vec{b}, \vec{c}$ are COPLANAR

$\Leftrightarrow [\vec{a}\ \vec{b}\ \vec{c}] = 0$

⑥ $[\vec{a}\ \vec{b}\ \vec{c}] = 0$, if any two vectors are equal

⑦ $[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{c}-\vec{a}] = 0$

$[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}] = 2[\vec{a}\ \vec{b}\ \vec{c}]$

IX) Fundamental Theorem

let $\vec{a}, \vec{b}, \vec{c}$ be non-zero, non-collinear vectors. → UNIQUE LINEAR COMBINATION

$\boxed{\vec{r} = x\vec{a} + y\vec{b} + z\vec{c}}$; $x, y, z \in R$

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DOT (SCALAR)

COMPONENT FORM:

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

CROSS (VECTOR)

COMPONENT FORM:

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

④ PRODUCT OF 3 VECTORS

SCALAR TRIPLE

✫ DABBA PRODUCT (STP)

① • STP of $\vec{a}, \vec{b}, \vec{c}$ is

$(\vec{a}\times\vec{b})\cdot\vec{c} = |\vec{a}||\vec{b}||\vec{c}|\sin\theta\cdot\cos\phi$

θ: ∠ b/w $\vec{a}$ & $\vec{b}$

ϕ: ∠ b/w $(\vec{a}\times\vec{b})$ & $\vec{c}$

Also called ✫BOX PRODUCT $[\vec{a}\ \vec{b}\ \vec{c}]$

② • COMPONENT FORM

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

VECTOR TRIPLE (VTP)

• $\vec{a}\times(\vec{b}\times\vec{c})$

$=$

$(\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$

DUUR SE PAAS!!

• For $(\vec{a}\times\vec{b})\times\vec{c}$

$= -\vec{c}\times(\vec{a}\times\vec{b})$

$= -((\vec{c}\cdot\vec{b})\vec{a} - (\vec{c}\cdot\vec{a})\vec{b})$

$= (\vec{c}\cdot\vec{a})\vec{b} - (\vec{c}\cdot\vec{b})\vec{a}$

• $(\vec{a}\times\vec{b})\times\vec{c} \neq \vec{a}\times(\vec{b}\times\vec{c})$