SEQUENCES and SERIES

Prepared by Neha Agrawal Mathematically Inclined

(I) 1. SEQUENCE:

A function whose domain is set of all Natural numbers. $a_1, a_2, \cdots, a_n$

2. SERIES:

$a_1 + a_2 + \cdots + a_n$ (Replace , with +)

A sequence or series can be FINITE or INFINITE $(a_1, a_2, \cdots)$

3. PROGRESSION:

Sequences whose terms follow a pattern.

- ARITHMETIC PROGRESSION
- GEOMETRIC PROGRESSION
- HARMONIC PROGRESSION

(II) ARITHMETIC PROGRESSION (A.P.)

1. $a_1, a_2, \cdots, a_n$ such that

$a_2 - a_1 = a_3 - a_2 = \cdots = a_n - a_{n-1} = d$

d: common difference

2. GENERAL TERM:

$$a_n = a + (n-1)d$$

3. Sum to n TERMS:

$$S_n = \frac{n}{2}(2a + (n-1)d) \text{ or } \frac{n}{2}(a_1 + a_n)$$

4. SELECTION of TERMS:

No.	TERMS	d
3	$a-d,\ a,\ a+d$	d
4	$a-3d,\ a-d,\ a+d,\ a+3d$	$2d$
5	$a-2d,\ a-d,\ a,\ a+d,\ a+2d$	d

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(III) GEOMETRIC PROGRESSION (G.P.)

1. $a_1, a_2, \cdots, a_n$ such that

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \cdots = \frac{a_n}{a_{n-1}} = r$$

r: Common ratio

2. GENERAL TERM:

$$a_n = a r^{n-1}$$

3. SUM OF TERMS:

① n terms

$$S_n = \frac{a(r^n - 1)}{r - 1}, \quad r > 1$$

$$S_n = \frac{a(1 - r^n)}{1 - r}, \quad r < 1$$

$$S_n = na \qquad r = 1$$

② INFINITE Terms

$$S_\infty = \frac{a}{1 - r}$$

$|r| < 1$ (or) $-1 < r < 1$

4. SELECTION OF TERMS:

No.	TERMS	r : COMMON RATIO
3	$\frac{a}{r}, a, ar$	r
4	$\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$	r^2
5	$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$	r

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(VII) HARMONIC MEAN (H.M.)

If a, b, c are in HP, b is the HM between a & c. $\boxed{b = \frac{2ac}{a+c}}$

RELATION BETWEEN AM, GM, HM

* $AM > GM > HM$
* $AM \cdot HM = GM^2$

(VIII) SUM to n terms : SPECIAL SERIES

1. $S_n = 1 + 2 + 3 + \cdots + n = \sum_{K=1}^{n} K$

$$= \frac{n(n+1)}{2}$$

2. $S_n = 1^2 + 2^2 + \cdots + n^2 = \sum_{K=1}^{n} K^2$

$$= \frac{n(n+1)(2n+1)}{6}$$

3. $S_n = 1^3 + 2^3 + \cdots + n^3 = \sum_{K=1}^{n} K^3 = \left(\frac{n(n+1)}{2}\right)^2$

Just SQUARE IT!

(IX) ARITHMETICO-GEOMETRIC SERIES (A.G.P.)

$1 + 3x + 5x^2 + 7x^3 + \cdots$ AP = 1, 3, 5, ... GP = 1, x, x^2, ...

- METHOD OF DIFFERENCE: Multiply with common ratio & PUSH THE SERIES : DHAKKA MAAR METHOD!!
- V_n METHOD: Find general term T_n. Use $T_1 + T_2 + \cdots + T_n$

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(IV) HARMONIC PROGRESSION (H.P.)

1. $a_1, a_2, \cdots, a_n$ is in H.P. if and only if $\frac{1}{a_1}, \frac{1}{a_2}, \cdots, \frac{1}{a_n}$ is in A.P.

2. GENERAL TERM

$$\boxed{T_n = \frac{ab}{b + (n-1)(a-b)}}$$

a: 1st term
b: 2nd term

(V) ARITHMETIC MEAN (A.M.)

1.

3 TERMS	n-TERMS
a, b, c : AP	$a_1, a_2, \cdots, a_n$: n +ve nos. in AP
AM	$AM = \frac{a_1 + a_2 + \cdots + a_n}{n}$
$2b = a + c$	

2. INSERTION of n AMs between a & b

$a, A_1, A_2, \cdots, A_n, b$ $\quad \boxed{d = \frac{b-a}{n+1}}$

(VI) GEOMETRIC MEAN (G.M.)

1.

3 TERMS	n-TERMS
a, b, c : GP	$a_1, a_2, \cdots, a_n$: n +ve nos. in GP
GM : $b = \sqrt{ac}$	$GM = (a_1 a_2 \cdots a_n)^{1/n}$
a, c > 0	

2. INSERTION of n GMs between a & b

$\boxed{r = (b/a)^{\frac{1}{n+1}}}$