SPECTRUM
CAREER INSTITUTE
JEE/NEET EXPERT

PHYSICS

BY

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MAGNETIC EFFECTS OF CURRENT
(SP-13)

Student's Name: ______________________

Batch: ______________________

Magnetic Effects of Current

3
CHAPTER

JEE/NEET Syllabus

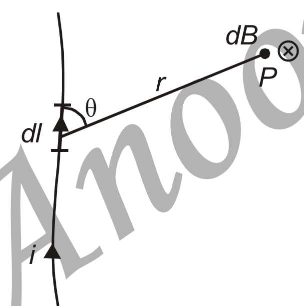
Biot - Savart law and its application to current carrying circular loop. Ampere's law and its applications to infinitely long current carrying straight wire and solenoid. Force on a moving charge in uniform magnetic and electric fields. Cyclotron. Force on a current-carrying conductor in a uniform magnetic field. Force between two parallel current-carrying conductors-definition of ampere. Torque experienced by a current loop in uniform magnetic field; Moving coil galvanometer, its current sensitivity and conversion to ammeter and voltmeter

THIS CHAPTER COVERS :

- Biot-Savart Law
- Straight Current Carrying Conductor
- Ampere Circuital Law
- Circular Loop
- Solenoid
- Motion of Charged Particle in Magnetic Field
- Force and Torque on Current Carrying Loop
- Moving Coil Galvanometer
- Conversion to Ammeter and Voltmeter

BIOT-SAVART LAW

Magnetic field due to current carrying wire is given by

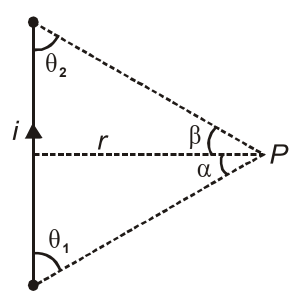


$$dB = \frac{\mu_0}{4\pi}\frac{idl\sin\theta}{r^2}$$

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{i(\overrightarrow{dl}\times\vec{r})}{r^3}$$

Magnetic Field Due to Straight Current Carrying Wire

Magnetic field at P



$$B = \frac{\mu_0 i}{4\pi r}(\cos\theta_1 + \cos\theta_2)$$

or $$B = \frac{\mu_0 i}{4\pi r}(\sin\alpha + \sin\beta)$$

1. For an infinite long wire

   $\theta_1 = \theta_2 = 0$ or $\alpha = \beta = \frac{\pi}{2}$, $B = \frac{\mu_0 i}{2\pi r}$ ‘r’

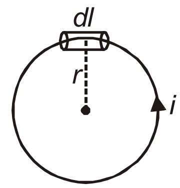
2. For semi infinite long wire

   $\theta_1 = 0°$, $\theta_2 = 90°$, $B = \frac{\mu_0 i}{4\pi r}$

CIRCULAR LOOP

1. At the centre of a current loop

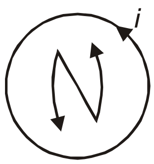
$$dB = \frac{\mu_0 i dl \sin 90^\circ}{4\pi r^2}$$



$$B = \frac{\mu_0 i}{4\pi r^2} \int dl$$

$$= \frac{\mu_0 i}{4\pi r^2} \times 2\pi r$$

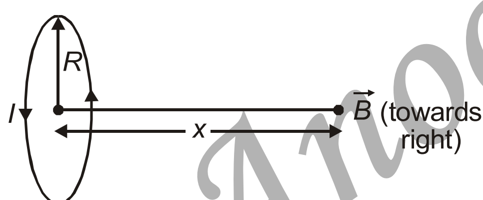
$$B = \frac{\mu_0 i}{2r} \text{ (Outward)}$$



(Outward field)

(Inward field)

2. On the axis of a loop



$$B = \frac{\mu_0}{4\pi} \frac{2 \times I \times \pi R^2}{(R^2 + x^2)^{3/2}} = \frac{\mu_0}{4\pi} \frac{2M}{(R^2 + x^2)^{3/2}}$$

For $x >> R$, $B = \frac{\mu_0}{4\pi} \frac{2M}{x^3}$ [Current carrying loop acts as an magnetic dipole]

where $M = I \times \pi R^2$ is called magnetic moment

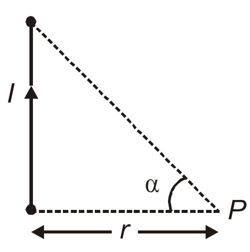
$\vec{M} = I\vec{A}$ units (A-m^2)

└→ outwards for anticlockwise current

└→ inwards for clockwise current

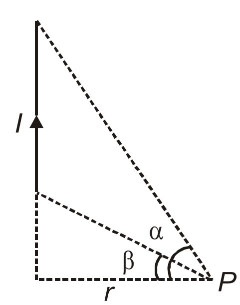
Applications

1.



$$B_0 = \frac{\mu_0 I}{4\pi r}[\sin \alpha] \otimes$$

2.



$$B_P = \frac{\mu_0 I}{4\pi r}[\sin \alpha - \sin \beta] \otimes$$

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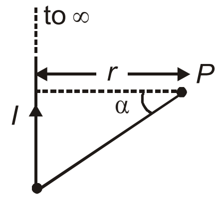
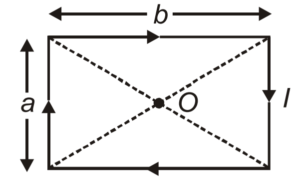
3.



$$B_P = \frac{\mu_0 I}{4\pi r}[\sin\alpha + 1] \otimes$$

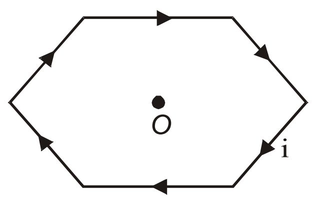
4.



$$B_0 = \frac{8\mu_0 I}{4\pi r}\frac{\sqrt{a^2+b^2}}{ab} \otimes$$

when $a = b$ $B_0 = \frac{8\sqrt{2}\,\mu_0 I}{4\pi a} \otimes$

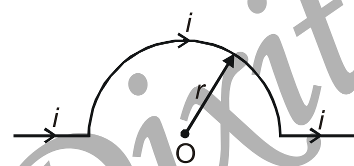
5.



Magnetic field at O

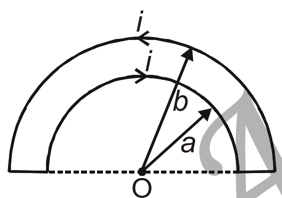
$$B_0 = \frac{\sqrt{3}\,\mu_0 i}{\pi a} \otimes$$

6.



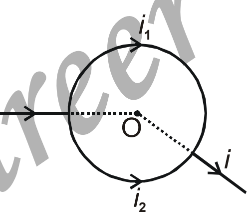
$$B_0 = \frac{\mu_0 i}{4r} \otimes$$

7.



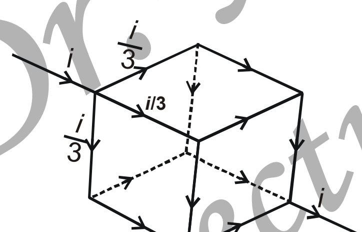
$$B_0 = \frac{\mu_0 i}{4}\left(\frac{1}{a} - \frac{1}{b}\right)$$

8.



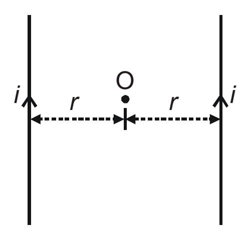
$$B_0 = 0$$

9.



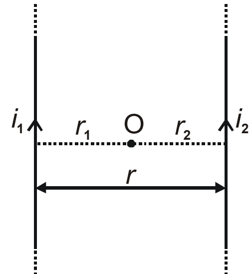
At the centre of cube, $B = 0$

10.



At O, $B = 0$

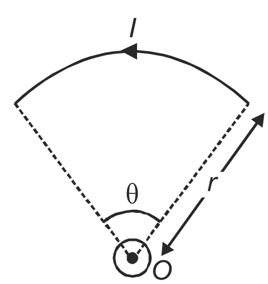
11.



At O, $B = 0$, such that

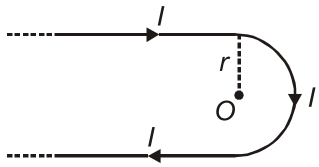
$$r_1 = \frac{i_1}{i_1 + i_2}r; r_2 = \frac{i_2 r}{i_1 + i_2}$$

12.



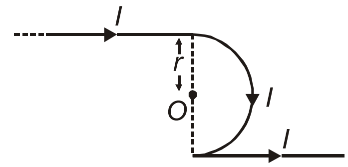
$$B_0 = \frac{\mu_0 I}{2r} \times \frac{\theta}{2\pi}$$

13.



$B_0 = \frac{\mu_0 I}{2\pi r} + \frac{\mu_0 I}{4r}$ ⊗

14.



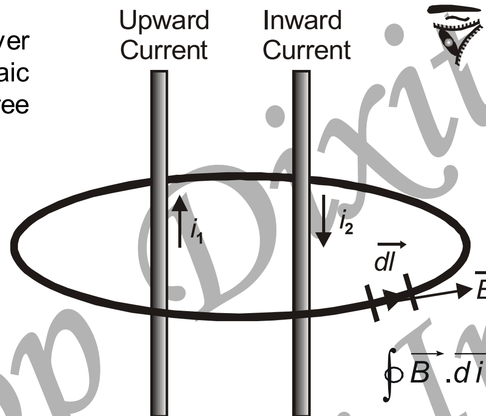
$B_0 = \frac{\mu_0 I}{4r}$ ⊗

AMPERE CIRCUITAL LAW

It states that the line integral of magnetic field over a closed path is equal μ_0 the times the algebraic sum of the current threading the closed path in free space. Ampere's circuital law has the form

$$\oint \vec{B}.\vec{dl} = \mu_0\, i_{enc}$$

Here $\oint \vec{B}.\vec{dl}$ implies the integration of scalar product $\vec{B}.\vec{dl}$ around a closed loop called an **Amperian loop**. The current i_{enc} is the net current encircled by the loop.

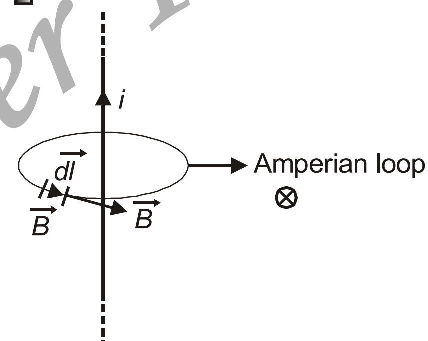


$\oint \vec{B}.\vec{dl} = \mu_0 (i_1 - i_2)$

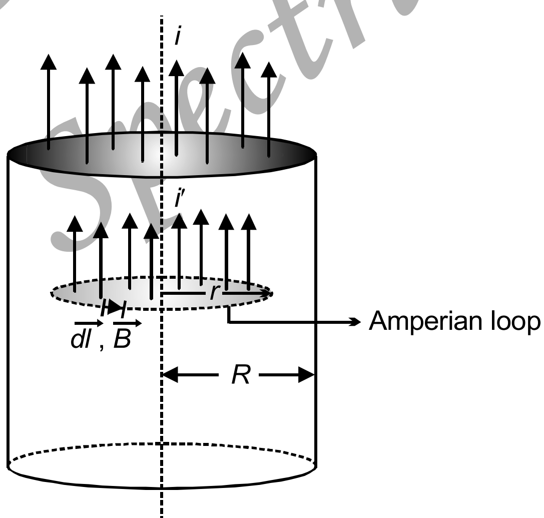
Applications of Ampere's Circuital Law

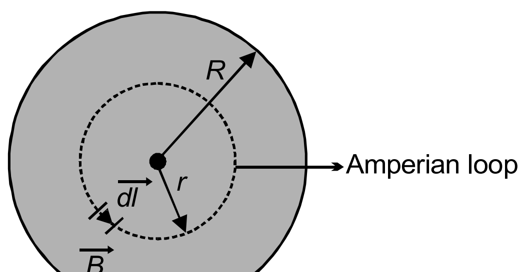
(1) Magnetic field due to a long thin current carrying wire

or $B = \frac{\mu_0 i}{2\pi r}$



(2) Magnetic field inside a long straight current carrying conductor



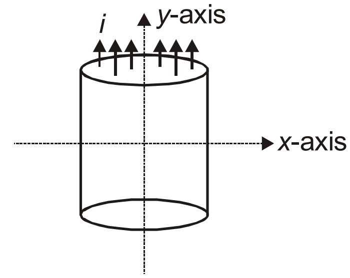


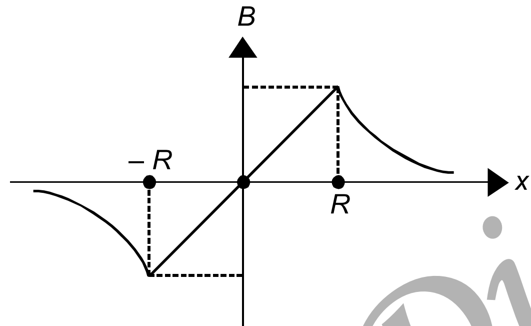
$\therefore \quad B = \frac{\mu_0 i r}{2\pi R^2} \quad$ *i.e.* $B_{in} \propto r$

Graphical variation of magnetic field

At a point in x-y plane, $\vec{B} = \frac{-\mu_0 i}{2\pi x}\hat{K}$ for $|x| > R$

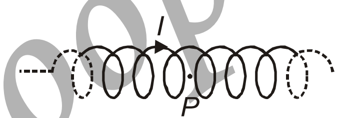
$$\vec{B} = \frac{-\mu_0 i x}{2\pi R^2}\hat{K} \text{ for } |x| < R$$





SOLENOID

A long solenoid having number of turns/length 'n' carries a current I.

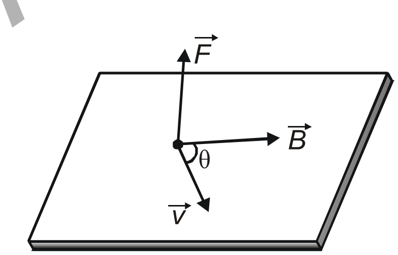


The magnetic field B is given by,

$B_P = \mu_0 nI$ (in between)

$B_{end} = \frac{\mu_0 nI}{2}$ (near one end)

Force on a Moving Charge in Magnetic Field



$\vec{F} = q(\vec{v} \times \vec{B})$

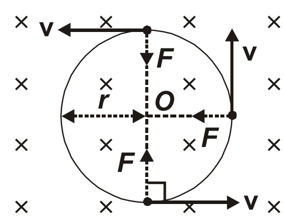
$F = qvB \sin\theta$

$\vec{F}$ is perpendicular to both $\vec{v}$ & $\vec{B}$.

***Note :** If there are electric field $\vec{E}$ and magnetic field $\vec{B}$ both present in a region, then Lorentz force is given by $\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]$. When $\vec{E} = -(\vec{v} \times \vec{B})$, net force because zero. In this situation, the particle can pass undeviated through the region.*

MOTION OF CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD $\vec{B}$.

Case 1 : $\vec{v} \perp \vec{B}$



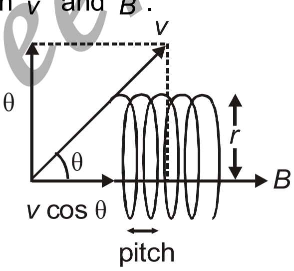
$$F = qvB = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{qB} = \frac{\sqrt{2mE}}{qB} \text{ (} E \text{ is kinetic energy)}$$

Results

1. Speed is constant
2. Kinetic energy is constant
3. Work done by the magnetic force is zero
4. Velocity and momentum change continuously in direction, not in magnitude.
5. $T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$ (Independent of speed and radius)
6. Frequency $f = \frac{qB}{2\pi m}$ (Cyclotron frequency)

Case 2 : $\vec{v}$ is not perpendicular to $\vec{B}$. Let be the 'θ' is angle between $\vec{v}$ and $\vec{B}$.

The particle moves in a helical path such that



$r = \frac{mv\sin\theta}{qB}, T = \frac{2\pi m}{qB}$, Pitch = $v \cos\theta \times T$

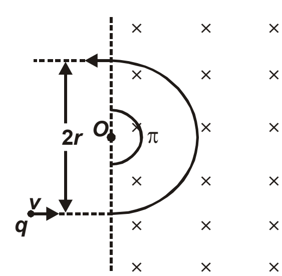
Case 3 : If charge particle is moving parallel or antiparallel to field $\vec{B}$ then force is zero and it moves in a straight line

Applications

1. $r = \frac{mv}{qB}$

   Time spent in magnetic field

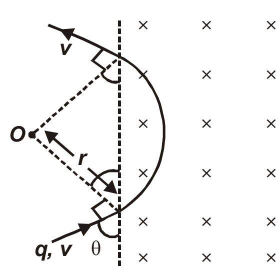
   $t = \frac{\pi m}{qB}$



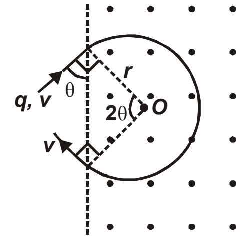
2. $r = \frac{mv}{qB}$

   Time spent in magnetic field

   $t = \frac{2\theta m}{qB}$



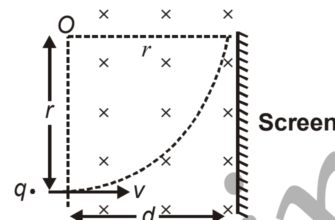
3. $r = \dfrac{mv}{qB}, t = \dfrac{(2\pi - 2\theta)\,m}{qB}$



4. To prevent the charge from hitting the screen,

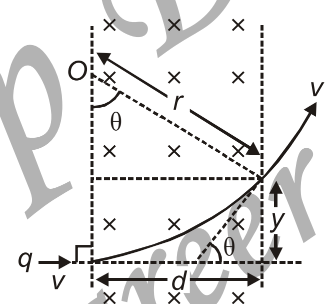
$d > r$ where 'r' is radius of circular path

i.e., $d > \dfrac{mv}{qB}$ or $B > \dfrac{mv}{qd}$



5. $\sin\theta = \dfrac{d}{r} = \dfrac{dqB}{mv}$

Deflection $y = r - r\cos\theta = r(1 - \cos\theta)$



Cyclotron

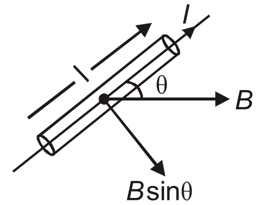
The cyclotron is a machine used to accelerate charge particle (like proton, deutron, α-particle). It uses both electric & magnetic field. Speed of particle is increased by electric field.

Cyclotron frequency $= \dfrac{qB}{2\pi m}$

$$K_{\max} = \frac{q^2B^2R_{\max}^2}{2m}$$

To accelerate electron Betatron and Synchrom are used.

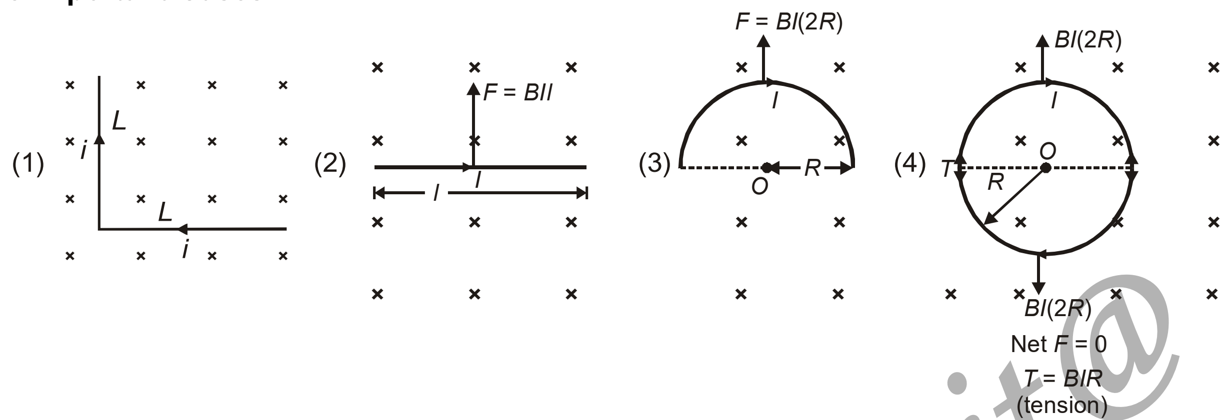
Force on a current carrying conductor in uniform field



$$F = IBl\sin\theta$$

$$\vec{F} = I(\vec{l} \times \vec{B})$$

Some Important Cases



$(F = \sqrt{2}iBL)$

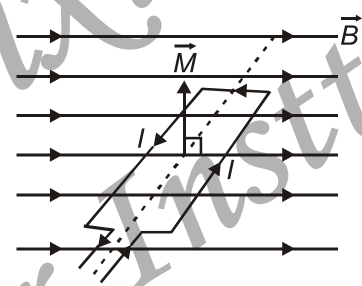
Torque on a Current Carrying Loop

***Case* 1 :** $\vec{M} = I\vec{A}$

Here $\vec{M} \perp \vec{B}$

$\vec{\tau} = \vec{M} \times \vec{B} = MB$ (maximum)

Potential energy $U = -\vec{M} \cdot \vec{B} = 0$



(a)

***Case* 2 :** Here $\vec{M} \parallel \vec{B}$

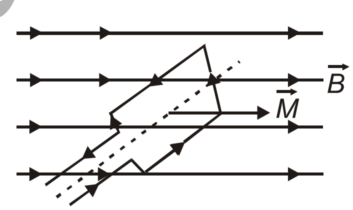
$\vec{\tau} = \vec{M} \times \vec{B} = 0$ (minimum)

Potential energy = $-MB$ (minimum)

(a) If $\theta = 0°$ (Stable equilibrium)

(b) If $\theta = 180°$ Potential energy = MB (maximum)

$\Rightarrow$ Unstable equilibrium



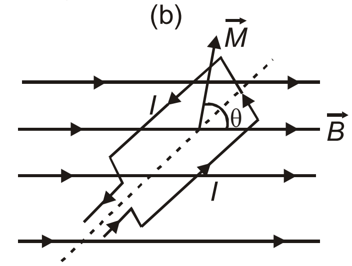
(b)

(c) Here angle between $\vec{M}$ and $\vec{B}$ is θ

$\tau = MB\sin\theta$, $U = -MB\cos\theta$

Work done in rotating from θ_1 to θ_2

is $W = MB(\cos\theta_1 - \cos\theta_2)$



(c)

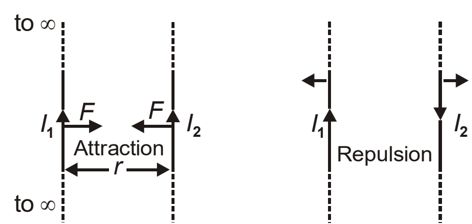
FORCE BETWEEN TWO CURRENT CARRYING WIRES

F (force/unit length) is given by

$$F = \frac{\mu_0 I_1 I_2}{2\pi r}.$$

The force on a segment of length 'l' is

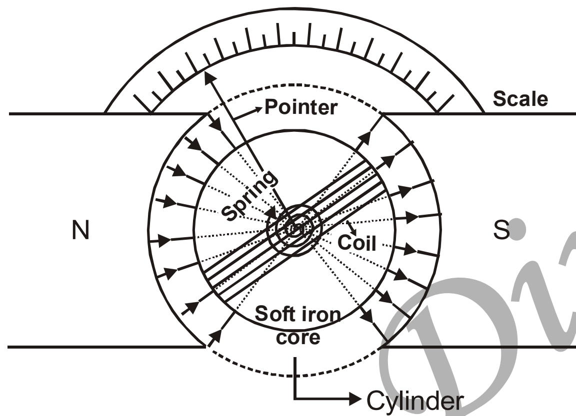
$$= \frac{\mu_0 I_1 I_2}{2\pi r} \times l$$



MOVING COIL GALVANOMETER

Principle

When a current carrying coil is placed in a magnetic field, torque acts on it. In moving coil galvanometer radial field is used which is obtained from magnet having concave shape poles. In this type of field plane of the coil is always parallel to the magnetic field so maximum torque acts on it.



$\tau = NIAB$ [$\theta = 90°$ due to radial field]

$= C\theta$

where θ is angle formed by the pointer and C is restoring

torque/twist in the suspension wire.

$I = \frac{C\theta}{NBA}, \frac{\theta}{I} = \frac{NBA}{C}$ current sensitivity

current sensitivity $\frac{\theta}{I} = \frac{NBA}{C}$

voltage sensitivity $\frac{\theta}{V} = \frac{\theta}{IR} = \frac{NBA}{CR}$

Conversion to Ammeter

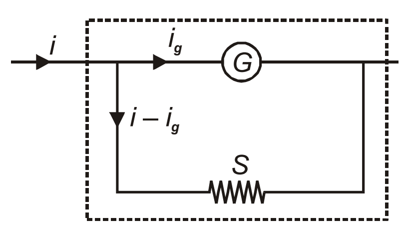
A small resistance called shunt S is connected in parallel with the galvanometer.

$i_g =$ Full scale deflection current

$i_g \times G = (i - i_g)\, S$

$i_g = \left(\frac{S}{S+G}\right) i$

$S = \frac{G}{\left(\frac{i}{i_g} - 1\right)}$



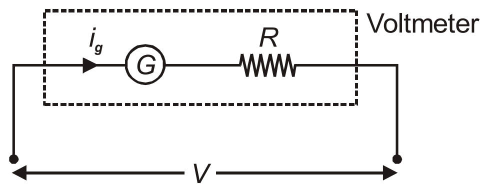
Ammeter

Conversion to Voltmeter

A high resistance *R* is connected in series with the galvanometer.

$V = i_g (G + R)$

$$R = \frac{V}{i_g} - G$$



Some Important Points :

1. An ideal ammeter has zero resistance.
2. An ideal voltmeter has infinite resistance.
3. An ammeter is connected in series in an electric circuit
4. A voltmeter is connected in parallel across the device whose potential difference is to be measured.

❑ ❑ ❑