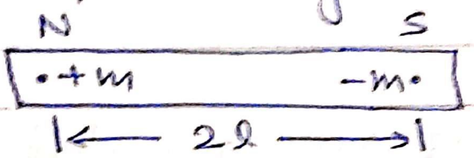
MAGNETISM & MATTER LECTURE-24

MAGNETISM AND MATTER
(Classical Magnetism)

BAR MAGNET

Dipole Moment of Bar Magnet : ($\vec{M}$)

It is defined as the pole strength and magnetic length of a bar Magnet.



$\pm m$ = Pole strength (Ampere-Metre)

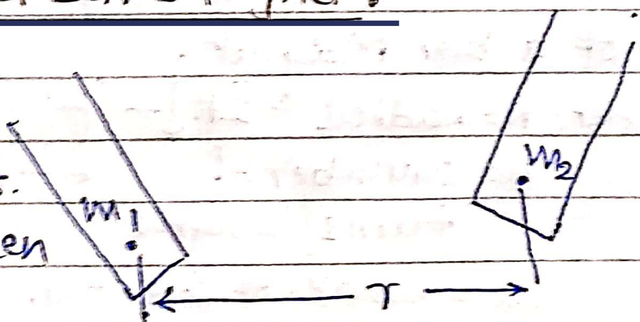
$\therefore \quad \vec{M} = m(\vec{2l})$

Dipole moment is a vector quantity its direction is From South to North pole of Bar Magnet.

SI unit : Ampere-~~turn~~-m2 (Ampere-m2)

Force Between Two Poles of a Bar Magnet :

Let m_1 and m_2 are pole strengths of two poles of different bar Magnets. The Magnetic force between them



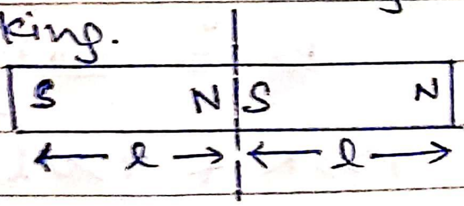
$$\boxed{F_m = \frac{\mu_0}{4\pi} \frac{m_1 m_2}{r^2}}$$ where $\frac{\mu_0}{4\pi} = 10^{-7} \frac{T\text{-}m}{\text{Ampere}}$

Breaking of Magnets :-

When a Bar Magnet is cut into two pieces, there can be following cases -

$\vec{M}$ = Dipole Moment before breaking.

$= m(\vec{2l})$

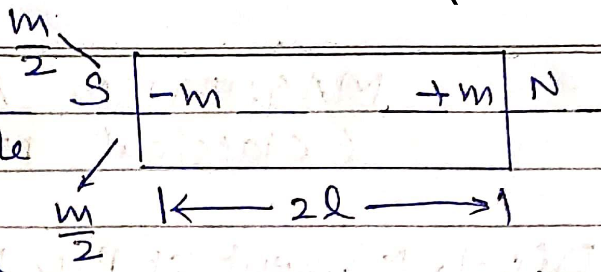


Case-1

$\vec{M'}$ = Dipole Moment of each piece $= m(\vec{l})$

In this case pole strength remains same but Magnetic Length becomes half.

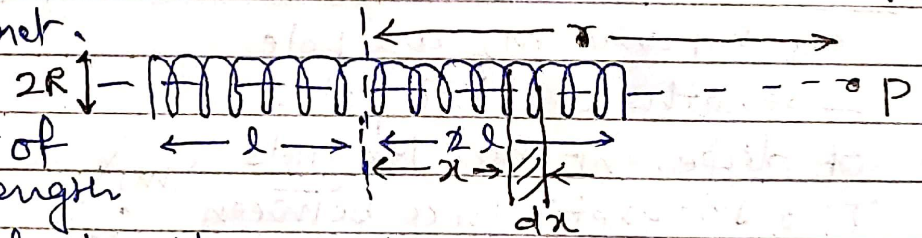
Case-II: In this case pole strength of each pole becomes half but the Magnetic length remains same.



$\vec{M} = m(\vec{2l})$ but $\vec{M'} = (2m)(\vec{l}/2) = m\vec{l}$

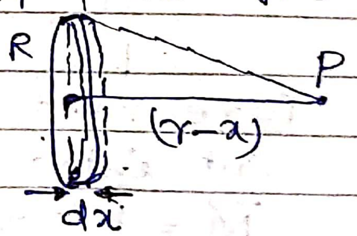
Bar Magnet as an Equivalent Solenoid:

- The resemblance of magnetic field lines for Bar Magnet and solenoid suggest that a Bar Magnet may be considered as equivalent to a long solenoid.
- To make this analogy more firm we can calculate the axial field of a finite solenoid. It can be then shown that at large distance this axial field resembles that of a bar Magnet.
- Let. R = Radius
  n = Number of turns/length
  $2l$ = Length of solenoid.



Magnetic field due to an elemental solenoid of dx length at P.

$$dB = \frac{\mu_0 (n\,dx) I R^2}{2\left[(r-x)^2 + R^2\right]^{3/2}}$$



$\therefore$ Total Magnetic field at P

$$B = \frac{\mu_0 n I R^2}{2} \int_{-l}^{l} \frac{dx}{\left[(r-x)^2 + R^2\right]^{3/2}}$$

If $r \gg R$ & $r \gg l$ then $\left[(r-x)^2 + R^2\right]^{3/2} \simeq r^3$.

$$\therefore B = \frac{\mu_0 n I R^2}{2r^3} \int_{-l}^{l} dx = \frac{\mu_0 n I}{2} \times \frac{2 l R^2}{r^3}$$

But $M = n(2l) I \pi R^2$ $\therefore B = \frac{\mu_0}{4\pi} \frac{2M}{r^3}$

This Magnetic field is same as that due to a Bar Magnet at far distance from it. Hence Bar Magnet and solenoid produce same Magnetic field and dipole moment, Hence both are equivalent.

Electrostatic Analogy for a Dipole

Properties	Electrostatics	Magnetism
1. Medium Constant	$1/\epsilon_0$	μ_0
2. Force	$\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$	$\frac{\mu_0}{4\pi} \frac{m_1 m_2}{r^2}$
3. Charge / Pole strength	Charge (q)	Pole strength (m)
4. Dipole moment	$\vec{p} = q(\vec{2a})$	$\vec{M} = m(\vec{2l})$
5. Equatorial field (Short dipole)	$E_{eq} = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$	$B_{eq} = \frac{\mu_0}{4\pi} \frac{M}{r^3}$
6. Axial Field (Short Dipole)	$E_{ax} = \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3}$	$B_{ax} = \frac{\mu_0}{4\pi} \frac{2M}{r^3}$
7. Torque on Dipole	$\vec{\tau} = \vec{p} \times \vec{E}$	$\vec{\tau} = \vec{M} \times \vec{B}$
8. Energy	$U_E = -\vec{p} \cdot \vec{E}$	$U_B = -\vec{M} \cdot \vec{B}$
9. Workdone in turning Dipole	$W = pE(\cos\theta_1 - \cos\theta_2)$	$W = MB(\cos\theta_1 - \cos\theta_2)$
10. Time Period of oscillations	$T = 2\pi\sqrt{\frac{I}{pE}}$	$T = 2\pi\sqrt{\frac{I}{MB}}$

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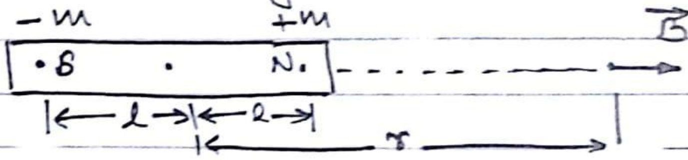
Magnetic Field due to Bar Magnet: considering a bar magnet $(\pm m, 2l)$.

a) **At any point on Axial Line** (Tangent A-Position)



Magnetic field due to North pole at P –

$$\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{m}{(r-l)^2}\hat{i}$$

Magnetic field due to South Pole at P $\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{m}{(r+l)^2}(-\hat{i})$

$\therefore$ Net Magnetic field due to Bar Magnet –

$$B_{ax} = B_1 - B_2 = \frac{\mu_0 m}{4\pi}\left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right]$$

$$B_{ax} = \frac{\mu_0 m}{4\pi}\left[\frac{\cancel{r^2}+\cancel{l^2}+2rl-\cancel{r^2}-\cancel{l^2}+2rl}{(r^2-l^2)^2}\right] = \frac{\mu_0}{4\pi}\frac{2[m(2l)]r}{(r^2-l^2)^2}$$

$$\boxed{B_{ax} = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2-l^2)^2}}$$ where $M = m(2l)$

If $r \gg l$ $\Rightarrow$ For short dipole, $\boxed{B_{ax} = \frac{\mu_0}{4\pi}\frac{2M}{r^3}}$

b) **At any Point on Equatorial Lines**:-

Magnetic Field due to North pole

$$B_1 = \frac{\mu_0}{4\pi}\frac{m}{(r^2+l^2)}$$

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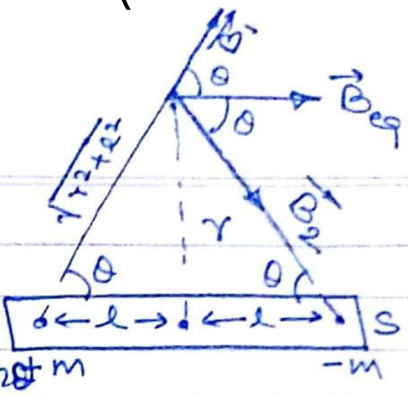


Magnetic field due to south Pole -

$$B_2 = \frac{\mu_0}{4\pi}\frac{m}{(r^2+l^2)} = B_1$$

$\therefore$ Net Magnetic field due to Bar Magnet - $B_{eq} = \sqrt{B_1^2+B_2^2+2B_1B_2\cos 2\theta}$

$$B_{eq} = \sqrt{2B_1^2+2B_1^2\cos 2\theta} = \sqrt{2B_1^2(1+\cos 2\theta)}$$
$$= \sqrt{4B_1^2\cos^2\theta} = 2B_1\cos\theta$$
$$= 2\times\frac{\mu_0}{4\pi}\frac{m}{(r^2+l^2)}\times\frac{l}{\sqrt{r^2+l^2}} = \frac{\mu_0}{4\pi}\frac{m(2l)}{(r^2+l^2)^{3/2}}$$

$$\boxed{B_{eq} = \frac{\mu_0}{4\pi}\frac{M}{(r^2+l^2)^{3/2}}}$$

For short Bar Magnet. $(r \gg l)$, $\boxed{B_{eq} = \frac{\mu_0}{4\pi}\frac{M}{r^3}}$

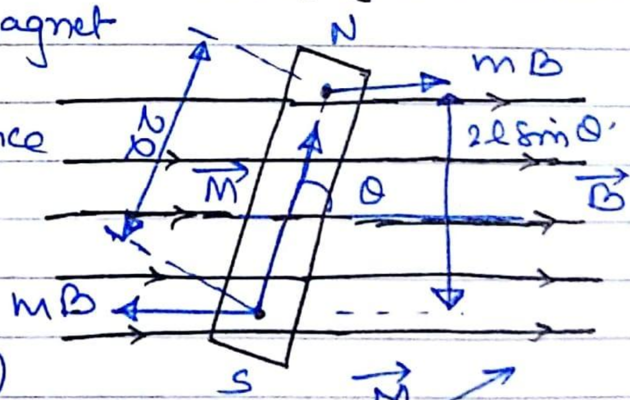
$\therefore$ For short Magnet, $\boxed{B_{ax} = 2B_{eq}}$

DIRECTION OF MAGNETIC FIELD —

* In axial Line ⇒ South → North pole (S → N) along axis
* On Equatorial Line ⇒ Antiparallel to $\vec{M}$.

Torque ON Bar Magnet Placed in Magnetic field :-

Considering a bar magnet $(\pm m, 2l)$

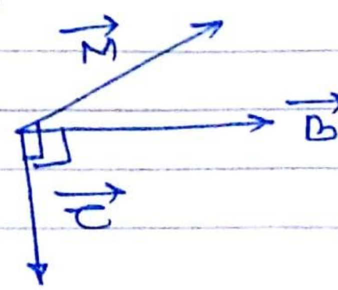


τ = Force × ⊥ distance
$= mB \times 2l\sin\theta$
$= (m2l)B\sin\theta$

$$\boxed{\tau = MB\sin\theta}$$

As $M = m(2l)$

$$\therefore \boxed{\vec{\tau} = \vec{M}\times\vec{B}}$$



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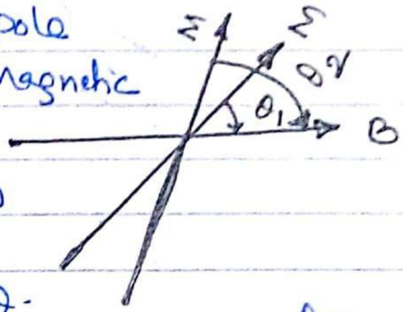
Work Done In turning the Bar Magnet in a uniform Magnetic field : considering a dipole (Bar Magnet) in a uniform Magnetic field $\vec{B}$. Turning the dipole by a $d\theta$ angle from a position of θ angle with respect to $\vec{B}$.



$$dW = \tau d\theta = MB\sin\theta\, d\theta$$

$$W = \int_{\theta_1}^{\theta_2} MB\sin\theta\, d\theta = MB[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$\boxed{W = MB[\cos\theta_1 - \cos\theta_2]}$$

This the work done in turning the dipole from θ_1 to θ_2 position.

Potential Energy of Bar Magnet Placed in Uniform Field

Work done in turning bar magnet in uniform Magnetic field is stored in the form of P.E.

$\therefore \Delta U_B = -MB[\cos\theta_2 - \cos\theta_1]$

Let $\theta_1 = 90^\circ$ (orientation for Zero P.E.)

$\theta_2 = \theta$

$U_B = -MB[\cos\theta - \cos 90^\circ]$

$$\boxed{U_B = -MB\cos\theta = -\vec{M}\cdot\vec{B}}$$

EQUILIBRIUM OF BARMAGNET IN MAGNETIC FIELD

For Eqbm. $\tau = 0$ & $F_{Net} = 0 \Rightarrow MB - MB = 0$

$\therefore MB\sin\theta = 0 \Rightarrow \sin\theta = 0 \Rightarrow \theta = 0^\circ$ or 180°

For Stable Eqbm $\Rightarrow U_B \rightarrow$ Minimum $\Rightarrow U_B = -MB$

$\therefore \theta = 0^\circ$

For Unstable Eqbm $\Rightarrow U_B \rightarrow$ Max^m $\Rightarrow U_B = +MB$

$\therefore \theta = 180^\circ$