# SPECTRUM
CAREER INSTITUTE

JEE/NEET EXPERT

# PHYSICS
## BY
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## ELECTRO MAGNETIC INDUCTION
## (SP-15)

Student's Name: ______________________

Batch: ______________________

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CHAPTER

# Electromagnetic Induction

**JEE/NEET Syllabus**

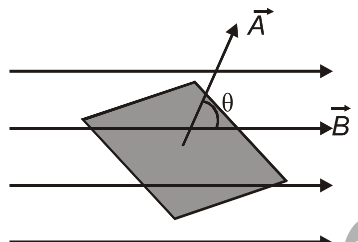
Electromagnetic induction; Faraday's law, induced emf and current; Lenz's Law, Eddy currents. Self and mutual inductance, A.C. Generator and Tranformer.

**THIS CHAPTER COVERS :**

- Magnetic flux & Faraday's Law
- Lenz's Law
- Methods of inducing emf & applications
- Mutual and self inductance
- Inductor
- LR circuit with dc source
- Eddy current
- Induced electric field
- Transformer
- LC oscillations

## MAGNETIC FLUX & FARADAY'S LAW

$\phi = \vec{B}.\vec{A} = BA\cos\theta$




**Units** : weber (SI)

Maxwell (CGS)

1 maxwell = $10^{-8}$ Wb

### Faraday's Laws of Electromagnetic Induction

1. An emf is induced in a loop when the number of magnetic field lines *i.e.*, magnetic flux passing through the loop is changing.
2. Magnitude of the emf '*e*' induced in a conducting loop is equal to the rate at which the magnetic flux $\phi_B$ through that loop changes with time.

$\Rightarrow \; e = -\dfrac{d\phi_B}{dt}$ (for a loop), for a plane coil having *N* turns

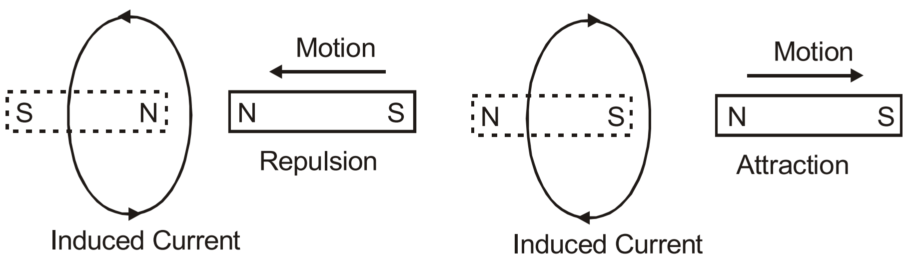
$$e = -\frac{Nd\phi_B}{dt} = -\frac{d(N\phi_B)}{dt}$$

**Note :** *Negative sign indicates opposition (explained by Lenz's law).*

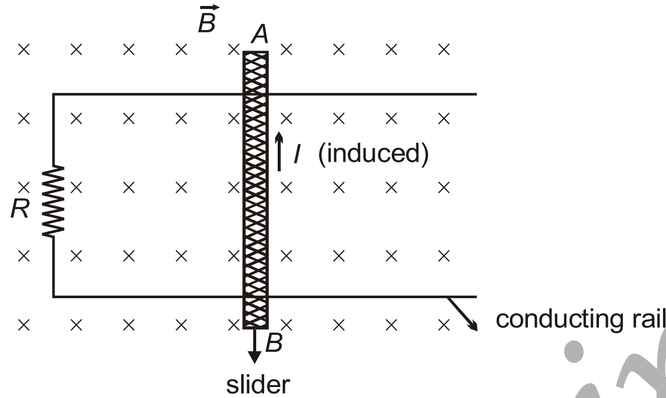
## LENZ'S LAW

The flux of the magnetic field due to the induced current opposes the change in flux that causes the induced current.

### Application of Lenz's Law



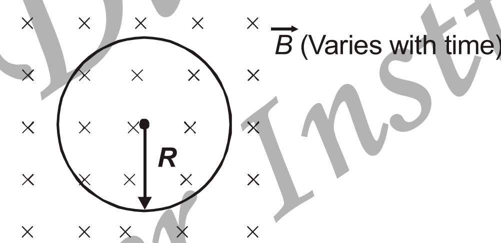
1. If loop *ABCD* is brought closed, anticlockwise current is induced. (Fig. 1)
2. If *ABCD* is moved away, clockwise current is induced. (Fig. 2)
3. As shown, if magnetic field starts increasing, an anticlockwise current starts flowing. Due to this, slider *AB* moves leftward.




## METHODS OF INDUCING EMF AND APPLICATIONS

### Methods of Inducing EMF

**(1) By Changing $\vec{B}$**




$\phi = BA = B \times \pi R^2$

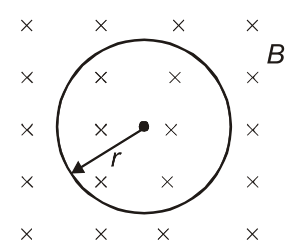
$$\frac{d\phi}{dt} = \pi R^2 \frac{dB}{dt} \quad \Rightarrow \quad e = -\pi R^2 \frac{dB}{dt}$$

(a) If $\vec{B}$ increases, current is in anticlockwise direction producing outward magnetic field.

(b) If $\vec{B}$ decreases, current is in clockwise direction.

**(2) By Changing Area**

(a) Let area changes from $A_1$ to $A_2$ in time $t$




$\phi_1 = BA_1$

$\phi_2 = BA_2$

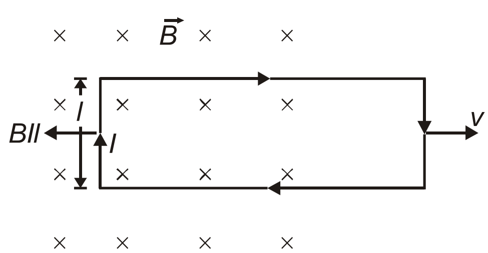
$$e = -\frac{(\phi_2 - \phi_1)}{t}$$

$$e = \frac{B(A_1 - A_2)}{t}$$

(b) Radius of the loop starts increasing at rate $\frac{dr}{dt}$

$$e = -\frac{d}{dt}(B \times \pi r^2) = -B \times 2\pi r \frac{dr}{dt}$$

(c) Moving a loop in/out of a uniform field.




$e = Bvl$, $R$ = resistance of loop, $I = \frac{e}{R}$ (clockwise)

**Some Important Points :**

(a) Force required to pull the loop with constant velocity

$$F = BIl = \frac{Bl \times e}{R} = \frac{B^2l^2v}{R}$$

(b) Power of external force = $Fv = \dfrac{B^2l^2v^2}{R}$

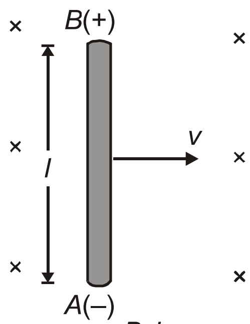
(c) Rate of heat loss = $I^2R = \dfrac{B^2l^2v^2}{R}$

$\therefore$ External power = thermal power dissipated

If the loop is pushed inside, current will be anticlockwise.

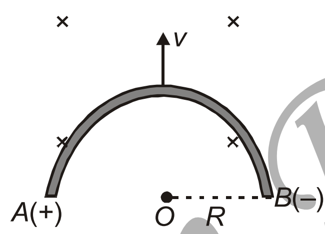
## Applications

1.



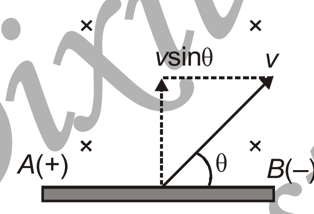

$e = Bvl$

2.



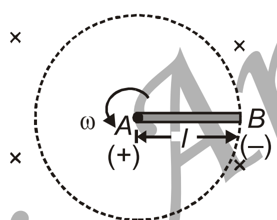

$e = Bv(2R)$

3.



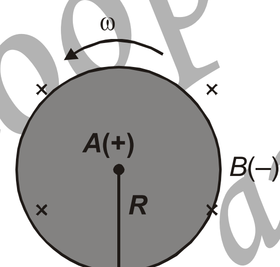

$e = Bvl\sin\theta$

4.




$e = \frac{1}{2}Bl^2\omega$

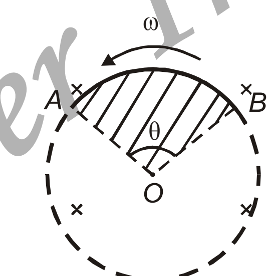
5.




$e = \frac{1}{2}Br^2\omega$

Disc

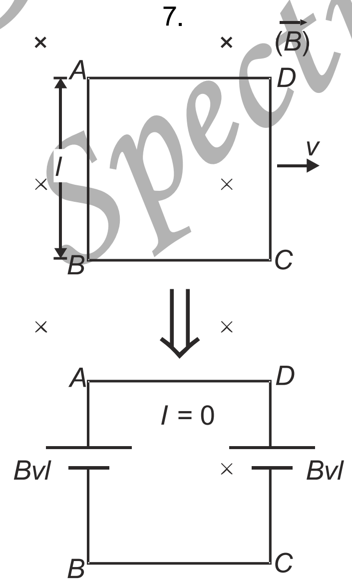
6.




$\varepsilon_{AB} = 0 \quad e_{OA} = \frac{1}{2}Br^2\omega$

Part of Disc

7.

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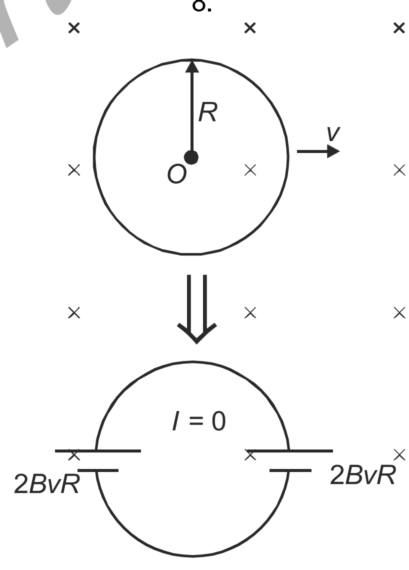


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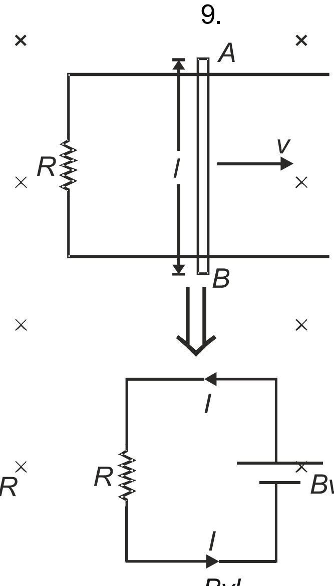
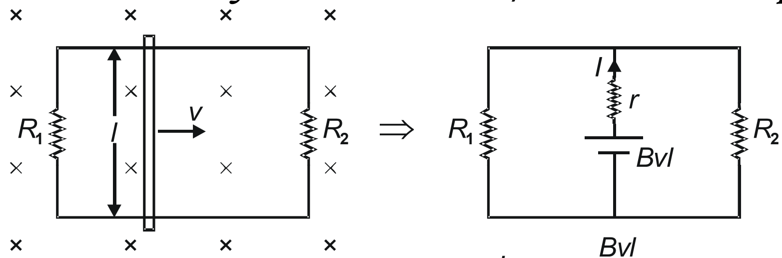



$I = \dfrac{Bvl}{R}$

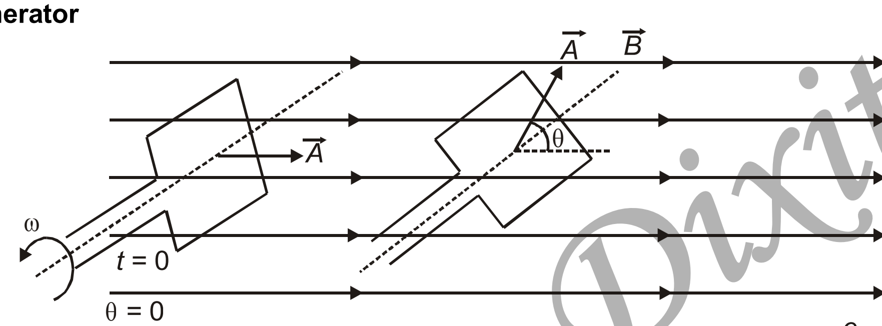
10.

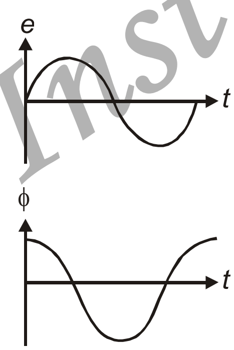



Sliding rod also has a resistance '$r$'

$$I = \frac{Bvl}{r + \dfrac{R_1R_2}{R_1 + R_2}}$$

**(3) A.C. Generator**




$e = e_0 \sin\omega t$

Alternating emf is produced in the coil, given by

$e = NBA\omega.\sin\omega t$

[N = No. of turns, B = magnetic field,

A = Area of loop, $\omega$ = angular speed of rotation]

## MUTUAL AND SELF INDUCTANCE

### Mutual Induction

Property of two coils by virtue of which each opposes any change in the magnitude of current flowing through the other by inducing an emf in itself provided magnetic flux of one coil is linked with other.

Let $I_1$ is current through one coil, $\phi_2$ is flux linked with other coil, then

$\phi_2 \propto I_1$

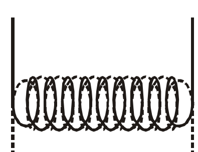
$\Rightarrow$ $\phi_2 = MI_1$, where $M$ is mutual induction

$\Rightarrow$ $e = \dfrac{-d\phi_2}{dt} = -M\dfrac{dI_1}{dt}$

**Important cases :**

1. Mutual inductance of two solenoids :

$M = \dfrac{\mu_0 N_1 N_2 A}{l}$



$N_1$ = Number of turns in one solenoid

$N_2$ = Number of turns in other solenoid

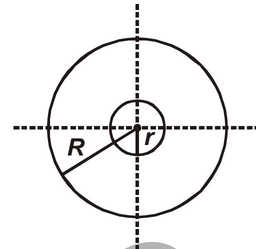
$A$ = Area of cross-section of narrower solenoid

$l$ = Length of solenoid

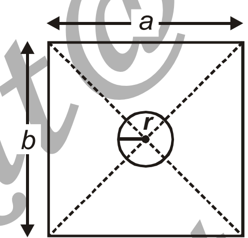
2. Two loops :

   $R >> r$

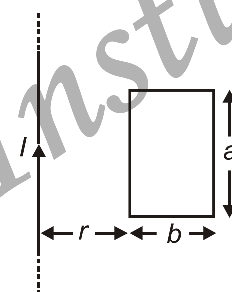
   (a) $M = \dfrac{\mu_0 \pi r^2}{2R}$




   (b) $M = \dfrac{\mu}{4\pi}\dfrac{8\sqrt{a^2+b^2}}{ab}\pi r^2$




   (c) $M = \dfrac{\mu_0}{2\pi} a \log\left(\dfrac{r+b}{r}\right)$




## Self Induction

Property of a coil by which it opposes any change in the magnitude of current flowing through it by inducing an emf in itself.

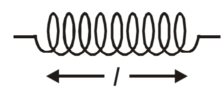
Here $\phi = LI$, $e = \dfrac{-LdI}{dt}$, $L$ = Coefficient of self-induction.

Both $M$ and $L$ have unit Henry. 1 Henry = $\dfrac{\text{1 Volt - sec}}{\text{Ampere}}$ = 1 Joule/(Amp)$^2$

## INDUCTOR

**Ideal Inductor :** A part of long solenoid having zero resistance, inductance or coefficient of self induction

$L = \mu_0 n^2 Al = \dfrac{\mu_0 N^2 A}{l}$

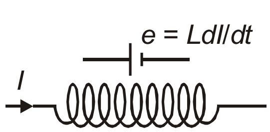



$l$ = length, $A$ = area of cross-section

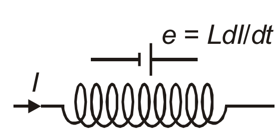
$n$ = number of turns/length, $N$ = total number of turns

## Direction of Induced emf

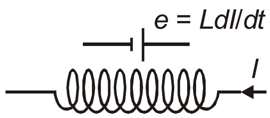
(a) $I$ is increasing



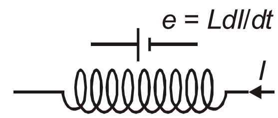

(b) $I$ is decreasing



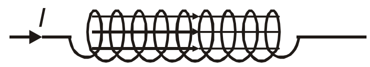
(c) $I$ is increasing




(d) $I$ is decreasing



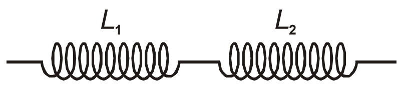

**Energy in Inductor**




Energy $U_B = \frac{1}{2}LI^2$, Energy Density = $\frac{1}{2}\frac{B^2}{\mu_0}$

**Combination of Inductors**

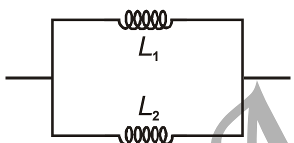
**1. Inductor in series**




(a) $L = L_1 + L_2$

(b) $L = L_1 + L_2 \pm 2M$ (If mutual inductance is also considered)

**2. Inductor in parallel**




$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$ (Neglecting mutual induction)

**Relation between Self Inductance and Mutual Inductance**

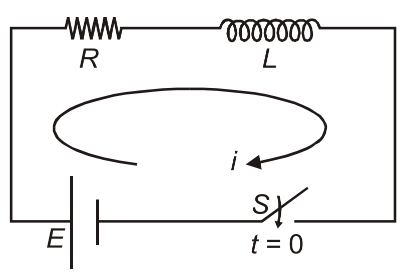
$M = K\sqrt{L_1 L_2}$

$M$ = Mutual inductance of two inductors $L_1$ and $L_2$, $K$ = Coefficient of coupling.

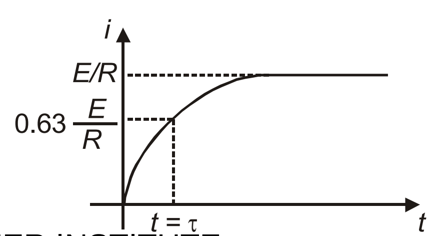
For a tight (perfect) coupling $K = 1$, otherwise $K < 1$.

**L-R- CIRCUIT WITH D.C. SOURCE**

**Growth of Current**




At $t = 0$ the switch $S$ is closed



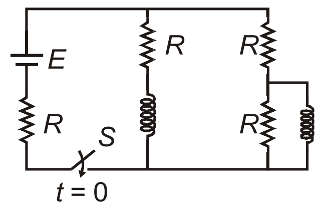

$$i = \frac{E}{R}\left(i - e^{-t/\tau}\right).$$

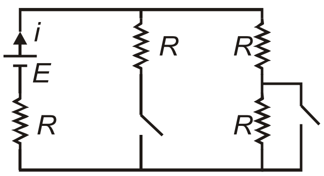
Graph for variation of current ($i$) with time ($t$) is shown here.

(1) Inductor behaves like open circuit. No current through it, as $t = 0$, $i = 0$.

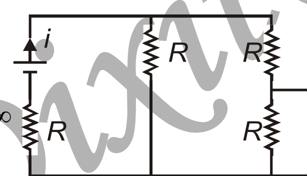
(2) After long time, as $t \to \infty$, $i_0 = \frac{E}{R}$. (Inductor behaves as closed switch)

**Illustration :**



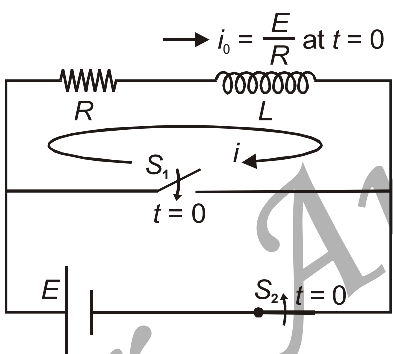


at $t = 0$ $\quad i = \frac{E}{3R}$

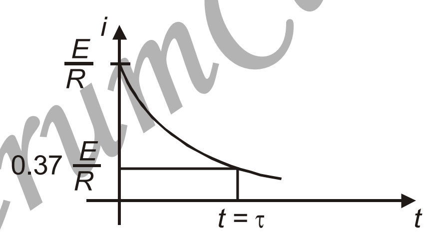


at $t = \infty$ $\quad i = \frac{2E}{3R}$

### Decay of Current



At $t = 0$, $i = \frac{E}{R}$ is flowing through the inductor. Now $S_1$ is closed and $S_2$ is opened.

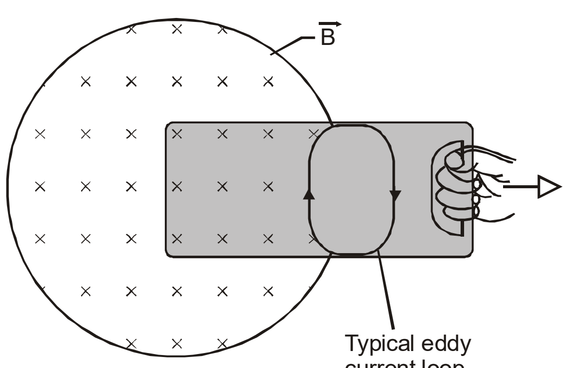


$$i = \frac{E}{R}\left(e^{-t/\tau}\right)$$

Graph for variation of current ($i$) with time ($t$) is shown here.

### EDDY CURRENTS

When the magnetic flux through a large piece of conducting material changes, induced currents appear in the material. These current are called eddy currents.



The magnitude of eddy currents is given by $i = \frac{-\varepsilon}{R} = \frac{d\phi/dt}{R}$ where $R$ is resistance of the conductor. The direction of eddy currents is given by Lenz's law or Flemming's right hand rule.

Some of the important applications of eddy currents are: Induction motor and in diathermy *i.e.*, deep heat treatment of parts of human body.

Some of the undesirable effects of eddy currents are that they oppose the relative motion, involve loss of energy in the form of heat and reduce the life of electrical devices. To minimise eddy currents, we use laminated cores.

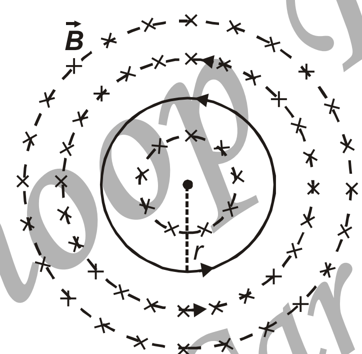
### Uses of Eddy Currents

Induction furnace, magnetic brakes, speedometers, damping in galvanometers, dead beat galvanometer.

## INDUCED ELECTRIC FIELD

A changing magnetic field produces an electric field. This field is non-conservative and always forms closed lop. Consider a region of magnetic field. The magnetic field strength is increasing at a rate $\frac{d\vec{B}}{dt}$. This creates anticlockwise electric field lines. Electric field strength at distance $r$ from point $O$ is given by

$$E = \frac{dB}{dt} \times \frac{r}{2}.$$




## TRANSFORMER

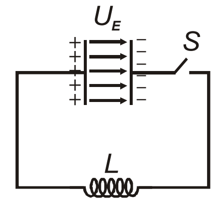
1. A transformer is a device for converting high voltage into low voltage and vice versa, without change in power.
2. There are two types of transformers.
   (a) Step up Transformer : It converts low voltage into high voltage.
   (b) Step down transformer : It converts high voltage into low voltage.
3. The principle of a transformer is based on mutual induction and a transformer always works on AC. The input is applied across primary terminals and output is obtained across secondary terminals.
4. The ratio of number of turns in secondary and primary is called the turn ratio *i.e.*, $\frac{n_s}{n_p}$ **=** turn ratio $n$.
5. If $E_p$ and $E_s$ are alternating voltage, $I_p$ and $I_s$ the alternating currents across primary and secondary terminals respectively, then for ideal transformer

$$\frac{E_s}{E_p} = \frac{I_p}{I_s} = \frac{n_s}{n_p} = n.$$

6. Efficiency of practical transformer

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{P_{out}}{P_{in}} = \frac{E_s I_s}{E_p I_p} < 1$$

**LC OSCILLATIONS**



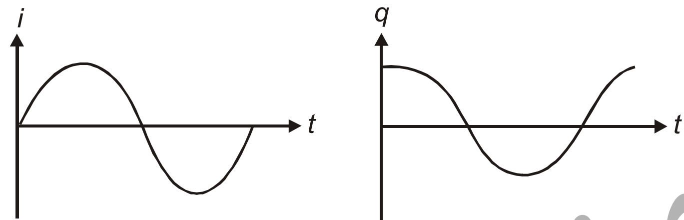

A charged capacitor is connected to an inductor and switch is closed at $t = 0$.

The charge and current vary sinusoidally as,

$q = q_0 \cos \omega t$ [$\because$ at $t = 0$, $q = q_0$]

$i = i_0 \sin \omega t$ [$\because$ at $t = 0$, $i = 0$]

Graphical representation of this variation is as shown.




$i_0 = \frac{q_0}{\sqrt{LC}}, \omega = \frac{1}{\sqrt{LC}}, \nu = \frac{1}{2\pi\sqrt{LC}}$ is frequency of $LC$ oscillations

**Some Important Points :**

1. If there is some resistance, there is a continuous loss of energy. $\therefore$ Amplitude of charge or current decays with time.
2. During oscillations, voltage across capacitor at any instant = emf induced in the inductor.
3. Energy stored in capacitor or inductor oscillates with frequency $2\nu$.

❑ ❑ ❑