SPECTRUM
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JEE/NEET EXPERT

MATHEMATICS
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THREE DIMENSIONAL GEOMETRY
(MSP-18)

Student's Name: ____________________
Batch: ____________________

Three - Dimensional Geometry

14 CHAPTER

JEE Syllabus

Coordinates of a point in space, distance between two points, Section formula, direction ratios and direction cosines, angle between two intersecting lines. Skew lines, the shortest distance between them and its equation. Equations of a line and a plane in different forms; intersection of a line and a plane, coplanar lines.

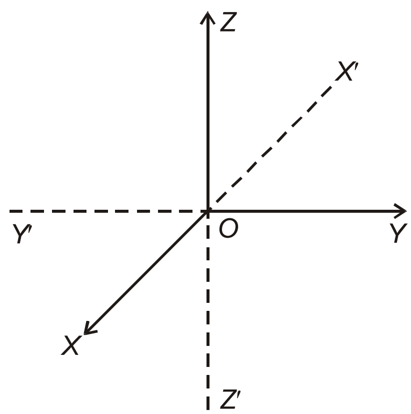
THIS CHAPTER INCLUDES :

- Co-ordinate axes and origin
- Co-ordinate planes
- Coordinates and their sign.
- Distance formula
- Section formula
- Centroid of triangle & tetrahedron
- Direction cosines of a line and direction ratio
- Equation of straight line
- Angle between two lines
- Skew lines and shortest distance between them
- Plane and its equation
- Angle between planes
- Angle between a line and a plane
- Perpendicular distance of plane from a point
- Plane through the intersection of planes
- Solved examples

In the practical life different objects do not lie in same plane but in space. If we are to locate any object in universe three co-ordinates are required. Hence, three co-ordinate axes namely X, Y & Z intersecting at a point, called origin are chosen in mutually perpendicular directions. In terms of 3 co-ordinates i.e. (X, Y, Z) any point in universe can be specified exactly.

CO-ORDINATE AXES AND ORIGIN

Let XOX', YOY' and ZOZ' are three mutually perpendicular lines intersecting at O. 'O' is called origin of co-ordinate system.



$OX \rightarrow$ Positive direction of x-axis

$OX' \rightarrow$ Negative direction of x-axis

$OY \rightarrow$ Positive direction of y-axis

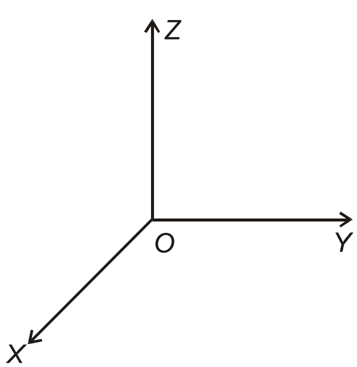
$OY' \rightarrow$ Negative direction of y-axis

$OZ \rightarrow$ Positive direction of z-axis

$OZ' \rightarrow$ Negative direction of z-axis

COORDINATE PLANES

XOY is called xy-plane which is perpendicular to Z-axis. YOZ is called yz-plane which is perpendicular to X-axis. ZOX is called zx-plane which is perpendicular to Y-axis. These three planes divide the space into 8 octants, namely $XOYZ$, $X'OYZ$, $XOYZ'$, $XOY'Z$, $X'OYZ'$, $XOY'Z'$, $X'OY'Z'$ & $X'OY'Z$.



CO-ORDINATES OF A POINT AND THEIR SIGNS

A point P (x, y, z) is such that

x = Perpendicular distance of point P from yz plane, with proper signs

y = Perpendicular distance of point P from zx plane, with proper signs

z = Perpendicular distance of point P from xy plane, with proper signs

The signs of coordinates would be taken as given below :

Octant → Co-ordinate ↓	**OXYZ**	**OX′YZ**	**OXY′Z**	**OXYZ′**	**OX′Y′Z**	**OX′YZ′**	**OXY′Z′**	**OX′Y′Z′**
x	+	–	+	+	–	–	+	–
y	+	+	–	+	–	+	–	–
z	+	+	+	–	+	–	–	–

When a point lies on	**Co-ordinates**
(i) x – axis	$(\alpha, 0, 0)$
(ii) y – axis	$(0, \beta, 0)$
(iii) z – axis	$(0, 0, \gamma)$
(iv) XY – plane	$(\alpha, \beta, 0)$
(v) XZ – plane	$(\alpha, 0, \gamma)$
(vi) YZ – plane	$(0, \beta, \gamma)$

Illustration 1 :

Give the co-ordinates of a point P such that P is at a distance 3 units from all the three planes (XY, YZ & ZX) and P lies in $OX'Y'Z$ octant.

Solution :

Let P is (x, y, z)

P is equidistant from three planes, hence its x, y, z all three would be equal in magnitude *i.e.*, 3. In quadrant $OX'Y'Z$, x & y co-ordinate would be negative and z co-ordinate is positive.

So P = (–3, –3, 3)

DISTANCE FORMULA

Distance PQ in cartesian co-ordinates where P is (x_1, y_1, z_1) and Q is (x_2, y_2, z_2)

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \qquad \text{... (i)}$$

In vector form

$$\overrightarrow{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$\therefore |\overrightarrow{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \qquad \text{same as ...(i)}$$

Note : Distance of P (x_1, y_1, z_1) from origin O (0, 0, 0) = $\sqrt{x_1^2 + y_1^2 + z_1^2}$

$$\overrightarrow{OP} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$|\overrightarrow{OP}| = \sqrt{x_1^2 + y_1^2 + z_1^2}$$

Illustration 2 :

The points $A(0, 7, 10)$, $B(-1, 6, 6)$ and $C(-4, 9, 6)$ form

(1) A straight line (2) An equilateral triangle

(3) An isosceles right angled triangle (4) Are non-coplanar

Solution :

Three points in space are always coplanar, hence (4) is wrong

Now $AB = \sqrt{1+1+16} = 3\sqrt{2}$

$AC = \sqrt{16+4+16} = 6$

$BC = \sqrt{9+9+0} = 3\sqrt{2}$

$AB = BC \neq AC$ option (2) is wrong

$AB + BC \neq AC$ option (1) is wrong

$AB = BC$ & $AB^2 + BC^2 = AC^2$ hence ABC is an isosceles right angled triangle

option (3) is correct

SECTION FORMULA

I. Internal division

If point $R(x, y, z)$ divides the line joining $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ internally in the ratio $m : n$ then

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$y = \frac{my_2 + ny_1}{m+n}$$

$$z = \frac{mz_2 + nz_1}{m+n}$$

$$\therefore R = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

Vectorially ; $\overrightarrow{OR} = \dfrac{m\overrightarrow{OQ} + n\overrightarrow{OP}}{m+n}$

II. External division

If $R(x, y, z)$ divides PQ externally in $m : n$ then

$$R = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$$

Vectorially ; $\overrightarrow{OR} = \dfrac{m\overrightarrow{OQ} - n\overrightarrow{OP}}{m-n}$

Mid point is given by $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}, \dfrac{z_1 + z_2}{2}\right)$

Centroid of the Triangle

If (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) are the vertices of a triangle, then centroid is given by

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

Centroid of a Tetrahedron

If (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) and (x_4, y_4, z_4) are the vertices of a tetrahedron, then the centroid G of tetrahedron is given by $\left(\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}, \frac{z_1+z_2+z_3+z_4}{4}\right)$.

Direction-Cosines of a Line

If α, β, γ are the angles that a given line makes with the positive directions of the co-ordinate axes, then $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are called the Direction-cosines (or d.c.'s) of the given line.

Usually the direction cosines of a line are denoted by l, m, n. Thus $[l, m, n]$ denote d.c.'s of a line.

Cor. Direction-cosines of Co-ordinate Axes : The x-axis makes angles of 0°, 90° and 90° with axes of X, Y and Z respectively. Hence direction-cosines of X-axis are [cos 0°, cos 90°, cos 90°], i.e., [1, 0, 0].

Similarly d.c.'s of Y-axis are [0, 1, 0] and those of Z-axis are [0, 0, 1].

Position of a point by Radius Vector and Direction-Cosines

Let $P(x, y, z)$ be a point in the space and O the origin. Then length $OP = \vec{r}$ is the radius vector of the point P.

Also let $(\cos\alpha, \cos\beta, \cos\gamma)$ be the d.c.'s of OP. Drop PA perpendicular to OX. Then

$x = OA = OP\cos\alpha = r\cos\alpha$,

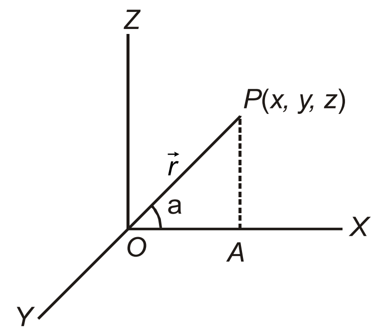
i.e., $x = lr$ if $\cos\alpha = l$.

Similarly $y = mr$, $z = nr$.

Hence co-ordinates of P are (lr, mr, nr).

Note : *In vector notations* $\overrightarrow{OP} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Since $\cos\alpha = \frac{x}{r}, \cos\beta = \frac{y}{r}$ and $\cos\gamma = \frac{z}{r}$.



Relation between Direction-Cosines

If l, m, n are the direction-cosines of a line then $l^2 + m^2 + n^2 = 1$.

Direction-Ratios : Quantities proportional to direction-cosines of a line are called direction-ratios of that line. Thus if l, m, n are direction-cosines and a, b, c direction-ratios of the line, then

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \frac{\sqrt{l^2+m^2+n^2}}{\sqrt{a^2+b^2+c^2}} = \pm\frac{1}{\sqrt{a^2+b^2+c^2}}$$

as a, b, c are proportional to l, m, n respectively.

Thus $l = \frac{\pm a}{\sqrt{a^2+b^2+c^2}}$ etc.

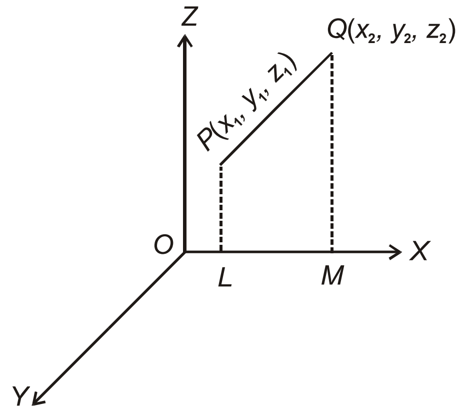
Hence if a, b, c are direction-ratios, then actual direction-cosines are

$$\left(\frac{\pm a}{\sqrt{a^2+b^2+c^2}}, \frac{\pm b}{\sqrt{a^2+b^2+c^2}}, \frac{\pm c}{\sqrt{a^2+b^2+c^2}}\right)$$

Direction-cosines of line joining $P\ (x_1, y_1, z_1)$ and $Q\ (x_2, y_2, z_2)$

Let l, m, n be the direction-cosines of PQ; then projection of PQ on X-axis = $PQ . l$

$PQ . l = x_2 - x_1$



$$\therefore \quad l = \frac{x_2 - x_1}{PQ}$$

Similarly $m = \dfrac{y_2 - y_1}{PQ}$

and $n = \dfrac{z_2 - z_1}{PQ}$

$$\therefore \quad \frac{x_2 - x_1}{l} = \frac{y_2 - y_1}{m} = \frac{z_2 - z_1}{n} = PQ$$

Therefore direction-cosines of line joining (x_1, y_1, z_1) and (x_2, y_2, z_2) are proportional to $x_2 - x_1, y_2 - y_1, z_2 - z_1$.

Illustration 3 :

The projections of a line on the axes are 3, 4, 12. Find the length and the direction-cosines of the line.

Solution :

Let PQ be the length of the line and (l, m, n) be its direction-cosines.

Then projection on X-axis, $PQ \cdot l = 3$

projection on Y-axis, $PQ \cdot m = 4$

projection on Z-axis, $PQ \cdot n = 12$.

Squaring and adding, $PQ^2 = 3^2 + 4^2 + 12^2 = 169$

$\Rightarrow \quad PQ = 13$

$$\therefore \quad l = \frac{3}{13}, m = \frac{4}{13}, n = \frac{12}{13}$$

Hence direction cosines are $\left(\dfrac{3}{13}, \dfrac{4}{13}, \dfrac{12}{13}\right)$.

EQUATION OF A STRAIGHT LINE

1. Vector equation of a line passing through a point with position vector $\vec{a}$ and parallel to the vector $\vec{b}$ is

   $\vec{r} = \vec{a} + t\vec{b}$ (**One Point Form**)

2. **Cartesian Form :**

   Let $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$, $\vec{b} = l\,\hat{i} + m\,\hat{j} + n\,\hat{k}$ and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

   This gives $\dfrac{x - x_1}{l} = \dfrac{y - y_1}{m} = \dfrac{z - z_1}{n} (= t)$.

3. Vector equation of a line through the point $A(\vec{a})$ and $B(\vec{b})$ is

$$\vec{r} = \vec{a} + t(\vec{b} - \vec{a}) \text{ (Two Point Form)}$$

4. If $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and $\vec{b} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$, then parametric form are

$x = x_1 + t(x_2 - x_1)$, $y = y_1 + t(y_2 - y_1)$, $z = z_1 + t(z_2 - z_1)$

Cartesian form $\dfrac{x - x_1}{x_2 - x_1} = \dfrac{y - y_1}{y_2 - y_1} = \dfrac{z - z_1}{z_2 - z_1}$.

ANGLE BETWEEN TWO LINES

$\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$

where θ is the angle between the lines whose direction-cosines are $[l_1, m_1, n_1]$ and $[l_2, m_2, n_2]$.

Perpendicular and Parallel Lines

Two lines whose direction-cosines are $[l_1, m_1, n_1]$ and $[l_2, m_2, n_2]$ are perpendicular if

$l_1l_2 + m_1m_2 + n_1n_2 = 0$ and parallel if $\dfrac{l_1}{l_2} = \dfrac{m_1}{m_2} = \dfrac{n_1}{n_2}$

Two lines $\dfrac{x - x_1}{a_1} = \dfrac{y - y_1}{b_1} = \dfrac{z - z_1}{c_1}$ and $\dfrac{x - x_2}{a_2} = \dfrac{y - y_2}{b_2} = \dfrac{z - z_2}{c_2}$ are **coplanar** iff

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Shortest Distance between Two Skew Lines

1. The shortest distance between two non-parallel lines $\vec{r} = \vec{a_1} + t\vec{b_1}$ and $\vec{r} = \vec{a_2} + s\vec{b_2}$ is $\left|\dfrac{(\vec{b_1} \times \vec{b_2})\cdot(\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|}\right|$

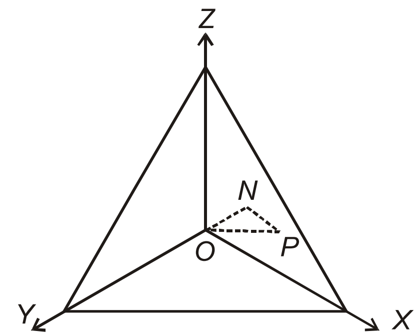
Cor. These lines are coplanar iff $[\vec{a}_2 - \vec{a}_1 \;\; \vec{b}_1 - \vec{b}_2] = 0$

2. The distance (i.e., the shortest distance) between two parallel lines $\vec{r} = \vec{a_1} + t\vec{b}$ and $\vec{r} = \vec{a_2} + s\vec{b}$ is $\dfrac{|\vec{b} \times (\vec{a_2} - \vec{a_1})|}{|\vec{b}|}$

THE PLANE

1. Every equation of the first degree in x, y, z represents a plane i.e., $ax + by + cz + d = 0$
2. $\vec{r} \cdot \hat{n} = P$ is the vector equation of a plane.
3. **Intercept Form :** $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$ represents a plane and that its intercepts on the three axes of co-ordinates are a, b, c respectively.

4. **Normal Form :** Let $ON = p$ be the perpendicular from O on the plane ABC. Also let $[l, m, n]$ be the d.c.'s of normal ON.
Take a point $P(x, y, z)$ on the plane.
Clearly angle ONP is a right angle because ON is perpendicular to the plane and PN lies in the plane.
Also ON = Projection of OP on a line ON
$= lx + my + nz$
But $ON = p$, Perpendicular distance.
Hence $lx + my + nz = p$ is the required equation of the plane.



5. **One–Point Form :** Vector equation of the plane through the point $A(\vec{a})$ and perpendicular to the direction $\vec{n}$ is
$(\vec{r} - \vec{a}).\vec{n} = 0$

6. Equation of the plane passing through three non-collinear points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0 \text{ or } \begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0 \quad (\textbf{Three Point form})$$

Angle between Two Planes

1. If θ is the angle between the planes $\vec{r}.\vec{n}_1 = d_1$ and $\vec{r}.\vec{n}_2 = d_2$, then
$$\cos\theta = \frac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$$
where $\vec{n}_1$ and $\vec{n}_2$ are vectors along normals to the two planes.

2. If $\vec{n}_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$
and $\vec{n}_2 = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$, then
$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

3. Two planes are perpendicular iff
$a_1a_2 + b_1b_2 + c_1c_2 = 0$ & Parallel if $\vec{n}_1 \times \vec{n}_2 = 0$

Angle between a Line and a Plane

$$\sin\theta = \frac{al + bm + cn}{\sqrt{a^2 + b^2 + c^2}\sqrt{l^2 + m^2 + n^2}},$$

where (a, b, c) are the direction rations of a line perpendicular to the plane and (l, m, n) are the direction ratios of the line.

This gives the angle between the line and the plane.

1. If line is parallel to the plane, $\theta = 0$ (or the line is perpendicular to the normal AN),
$\therefore \quad al + bm + cn = 0$

2. The line will be perpendicular to the plane, if it is parallel to the normal to the plane, *i.e,* $\frac{a}{l} = \frac{b}{m} = \frac{c}{n}$.

Perpendicular Distance of a Point from a Plane

1. **Plane in the normal form :**

Let the equation of the plane be

$$x \cos \alpha + y \cos \beta + z \cos \gamma = P \quad ...(1)$$

P being the perpendicular distance of the plane from origin

Any plane parallel to (1) is

$$x \cos \alpha + y \cos \beta + z \cos \gamma = P' \quad ...(2)$$

It passes through (x_1, y_1, z_1)

$$\therefore \quad x_1 \cos \alpha + y_1 \cos \beta + z_1 \cos \gamma = P' \quad ...(3)$$

Now perpendicular distance of the point (x_1, y_1, z_1) from (1) = $P' - P$

$$= x_1 \cos \alpha + y_1 \cos \beta + z_1 \cos \gamma - P$$

2. **Plane given by general equation.** Let the equation of the plane be

$$ax + by + cz + d = 0$$

Changing it into normal form the equation becomes $\dfrac{ax+by+cz+d}{\pm\sqrt{(a^2+b^2+c^2)}} = 0$ ...(1)

Perpendicular distance of the (x_1, y_1, z_1) from (1)

$$= \left|\frac{ax_1+by_1+cz_1+d}{\sqrt{(a^2+b^2+c^2)}}\right|$$

PLANE THROUGH THE INTERSECTION OF TWO PLANES

1. Equation of any plane passing through the intersection of the planes $\vec{r}.\vec{n}_1 = d_1$ and $\vec{r}.\vec{n}_2 = d_2$ is $\vec{r}.(\lambda\vec{n}_1 + \mu\vec{n}_2) = \lambda d_1 + \mu d_2, \lambda^2 + \mu^2 \neq 0$
2. Equation of any plane through the intersection of the planes

$$P \equiv a_1x + b_1y + c_1z + d_1 = 0$$

and $\quad Q \equiv a_2x + b_2y + c_2z + d_2 = 0$

Then the equation $P + \lambda Q = 0$

i.e., $(a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0$

SOLVED EXAMPLES

Example 1 :

A prototype model of a molecule in a lab, consists of 4 equal masses kept at (0, 0, 5), (3, 0, 0), $\left(\frac{-3}{2}, \frac{\sqrt{3}}{2}, 0\right)$ *and* $\left(\frac{-3}{2}, \frac{-\sqrt{3}}{2}, 0\right)$*. Find the coordinates of centroid of this system.*

Solution :

Centroid is $\left(\dfrac{x_1+x_2+x_3+x_4}{4}, \dfrac{y_1+y_2+y_3+y_4}{4}, \dfrac{z_1+z_2+z_3+z_4}{4}\right)$

$$= \left(\frac{0+3-\frac{3}{2}-\frac{3}{2}}{4}, \frac{0+0+\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}}{4}, \frac{5+0+0+0}{4} \right)$$

$= \left(0, 0, \frac{5}{4}\right)$ *i.e.*, on *z*-axis

Example 2 :

Point P(3, 4, 5) is at a maximum distance from

(1) *x-axis* (2) *y-axis*

(3) *z-axis* (4) *xz-plane*

Solution :

Distance of P from x-axis = $\sqrt{4^2+5^2} = \sqrt{41}$

Distance of P from y-axis = $\sqrt{3^2+5^2} = \sqrt{34}$

Distance of P from z-axis = $\sqrt{3^2+4^2} = \sqrt{25}$

Distance of P from xz-plane = y coordinate = 4

Hence maximum distance is from x-axis ($\sqrt{41}$)

Example 3 :

Find the locus of a variable point such that sum of its distance from two point (5, 0, 0) and (0, 0, 5) remains 10 units.

Solution :

Let P (x, y, z) be the variable point.

Then, $\sqrt{(x-5)^2+y^2+z^2} + \sqrt{x^2+y^2+(z-5)^2} = 10$

$\Rightarrow (x-5)^2+y^2+z^2 = 100+x^2+y^2+(z-5)^2 - 20\sqrt{x^2+y^2+(z-5)^2}$

$\Rightarrow x^2-10x+25+z^2 = 100+x^2+z^2+25-10z-20\sqrt{x^2+y^2+(z-5)^2}$

$\Rightarrow 10z-10x-100 = -20\sqrt{x^2+y^2+(z-5)^2}$

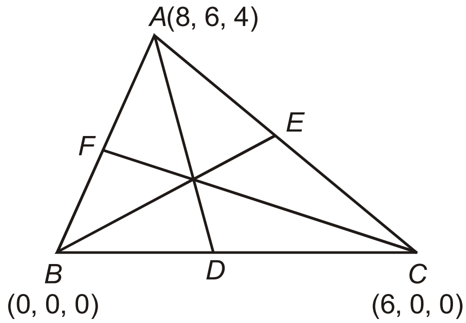
$\Rightarrow z-x-10 = -2\sqrt{x^2+y^2+(z-5)^2}$

$\Rightarrow z^2+x^2-2zx+100-20z+20x = 4x^2+4y^2+4z^2-40z+100$

$\Rightarrow 3x^2+3z^2+4y^2-20x-20z+2zx = 0$ is the required locus.

Example 4 :

Find the lengths of three medians of triangle ABC shown below



Solution :

Point D = (3, 0, 0)

E = (7, 3, 2)

F = (4, 3, 2)

$$\therefore AD = \sqrt{(8-3)^2 + (6-0)^2 + (4-0)^2}$$

$$= \sqrt{25+36+16}$$

$$= \sqrt{77}$$

$$BE = \sqrt{49+9+4} = \sqrt{62}$$

$$CF = \sqrt{4+9+4} = \sqrt{17}$$

Example 5 :

If the axes are rectangular, find the distance from the point (3, 4, 5) to the point where the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ *meets the plane x + y + z = 17.*

Solution :

Any point on the line is (r + 3, 2r + 4, 2r + 5). It lies on the plane $x + y + z = 17$

$\therefore$ (r + 3) + (2r + 4) + (2r + 5) = 17, i.e., r = 1.

Thus the point of intersection of the plane and the line is (4, 6, 7).

Required distance = distance between (3, 4, 5) and (4, 6, 7)

$$= \sqrt{\{(4-3)^2 + (6-4)^2 + (7-5)^2\}}$$

$$= \sqrt{1+4+4}$$

= 3.

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