Spectrum Gateway to IITs

MATHEMATICS (CBSE-XI) GEOMETRIC PROGRESSIONS-I DPP-11

1. Find the 9th term and the general terms of the progression $\frac{1}{2}, -\frac{1}{2}, 1, -2 \ldots\ldots\ldots\ldots\ldots$ [64, $(-1)^{n-1}2^{n-3}$]
2. Which terms of the G.P 2, 1, $\frac{1}{2}, \frac{1}{4}$, ………… is $\frac{1}{128}$? [9]
3. The fourth, seventh, and the last term of a G.P are 10, 80 and 2560 respectively. Find the first terms and the number of terms in the G.P. [10/8, 12]
4. The first term of G.P is 1 the sum of the third and fifth terms is 90. Find the common ratio of the G.P. [∓ 3]
5. If the 4th and 9th terms of the G.P be 54 and 13122 respectively. Find the G.P. [2,6,18,54…]
6. If the first and nth term of a G.P are a and b respectively and if P is the product of the first n terms, prove that $P^2 = (ab)^n$
7. If the 3rd term of a G.P is 4. Find the product of its first five terms. [4^5]
8. The (m+n)th and (m−n)th terms of a G.P are p and q respectively show that the mth and nth are $\sqrt{pq}$ and $p\left(\frac{q}{p}\right)^{m/2n}$ respectively.
9. If a, b and c are respectively the pth qth and rth terms of a G.P., show that $(q-r)\log a + (r-p)\log b + (p-q)\log c = 0$.
10. In a finite G.P. the product of the terms equidistance from the beginning and the end is always same and equal to the product of first and last term.
11. If a, b, c, d and p are different real number such that: $(a^2+b^2+c^2) - 2(ab+bc+cd) + (b^2+c^2+d^2) = 0$, then show that a, b, c, d are in G.P..
12. If $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$ $(x \neq 0)$, then show that a, b, c and d are in G.P.
13. If the continued product of three numbers in G.P is 216 and the sum of their product in pairs is 156, find the numbers. [18,6,2 or 2,6,18]
14. Find the three numbers in G.P whose sum is 13 and the sum of whose square is 91. [1,3,9 or 9,3,1]
15. Three numbers are in G.P. whose sum is 70. If the extremes be each multiplied by 4 and the means by 5, they will be in A.P. find the numbers. [10,20,40 or 40,20,10]
16. Find three numbers in G.P. whose sum is 52 and the sum of whose products in pairs is 624. [4,12,36 or 36,12,4]
17. The product of the first three terms of a G.P is 1000. If 6 is added to its second term and 7 added to its third term, then terms becomes in A.P. find the G.P. [5,10,20……]
18. Find the number of terms in G.P. in which the third term is greater than the first by 9 and the second terms is greater than the fourth by 18. [$3, -6, 12, -24$]
19. The sum of three numbers in G.P. is 14. If the first two terms are each increased by 1 and the third term decreased by 1, the resulting number are in A.P. find the number. [10, 5, 5/2]
20. The product of three numbers in G.P. is 216. If 2,8,6 be added to them, The results are in A.P. find the numbers.
21. Find the sum of 10 terms of the G.P. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\ldots\ldots\ldots\ldots$ [1023/512] Ans: Q20 [18,6,2]
22. Find the sum to 7 terms of the sequence $\left(\frac{1}{5}+\frac{2}{5^2}+\frac{3}{5^3}\right), \left(\frac{1}{5^4}+\frac{2}{5^5}+\frac{3}{5^6}\right), \left(\frac{1}{5^7}+\frac{2}{5^8}+\frac{3}{5^9}\right), \ldots\ldots$ $\left[\frac{19}{62}\left(1-\frac{1}{5^{21}}\right)\right]$
23. Find the sum of n terms of the sequence $\left(x+\frac{1}{x}\right)^2, \left(x^2+\frac{1}{x^2}\right)^2, \left(x^3+\frac{1}{x^3}\right)^2 \ldots\ldots$ $\left[\left(\frac{x^{2n}-1}{x^2-1}\right)\left(x^2+\frac{1}{x^{2n}}\right)+2n\right]$
24. Sum of the series: $x(x+y) + x^2(x^2+y^2) + x^3(x^3+y^3) + \cdots$ ....to n terms. $\left[x^2\left(\frac{x^{2n}-1}{x^2-1}\right) + xy\left(\frac{(xy)^n-1}{xy-1}\right)\right]$

25. Find the sum of the following series: (i) $5+55+555+\cdots$ .... to n terms (ii) $0.7+0.77+0.777+\cdots$ ...to n terms
` $\left[\frac{5}{18}(10^{n+1}-10-9n), \frac{7}{81}\left(9n-1+\frac{1}{10^n}\right)\right]$
26. Prove that the sum to terms $11+103+1005+\cdots$ ... .is $\frac{10}{9}(10^n-1)+n^2$.
27. The sum of first three of a G.P. is 16 and the sum of the next three terms is 128.find the sum of n terms of the G.P. $\left[\frac{16}{7}(2^n-1)\right]$
28. Determine the number of terms in G.P.$< a_n >$, if $a_1=3, a_n=96$ and $S_n=189$ $n=6$
29. Find the least value of n for which the sum $1+3+3^2+\cdots$ ... . to n terms is greater than 7000. $n>8.69$
30. How many terms of the geometrical series $1+4+16+64+\cdots$ ... . will make the sum 5461? [7]
31. In an increasing G.P., the sum of the first and last term is 66, the product of the second and the last but one is 128 and the sum of the terms is 126. How many terms are there in the progression? [6]
32. If $S_1, S_2, and\ S_3$ be respective the sum of $n, 2n$ and $3n$ terms of the G.P., prove that $S_1(S_3-S_2)=(S_2-S_1)^2$
33. If S be the sum, P the product and R the sum of the reciprocal of n terms of a G.P., prove that $\left(\frac{S}{R}\right)^n=P^2$.
34. If f is a function satisfying $f(x+y)=f(x)+f(y)$ for all $x,y\in N$ $f(1)=3$ and $\sum_{x=1}^{n} f(x)=120$,find the value of n. $n=4$
35. If S_1, S_2, S_3 be respectively the sums of $n, 2n, 3n$ terms of a G.P., then prove that $S_1^2+S_2^2=S_1(S_2+S_2)$.
36. If $S_1, S_2, S_3 \dots\dots\dots\dots\dots S_n$are the sum of n terms of n G.P.'s whose first term is 1 in each and common ratio are 1,2,3…………n respectively, then prove that $S_1+S_2+2S_3+3S_4+\cdots\dots\dots(n-1)S_n=1^n+2^n+3^n+\cdots..+n^n$.
37. Let $a_n$be the nth terms of the G.P. of positive number. Let $\sum_{n=1}^{100} a_{2n}=\alpha$ and $\sum_{n=1}^{100} a_{2n-1}=\beta$, such that $\alpha\neq\beta$. prove that the common ratio of the G.P. is α/β .
38. If a and b are roots of $x^2-3x+p=0$ and c,d are the roots $x^2-12x+q=0$ where a,b,c,d form a G.P. prove that $(q+p):(q-p)=17:15$.
39. Sum of the geometrical series to infinity: $\frac{1}{2}+\frac{1}{3^2}+\frac{1}{2^3}+\frac{1}{3^4}\dots\dots\dots..\infty$. [19/24]
40. Prove that $6^{1/2}.6^{1/4}.6^{1/8}\dots\dots\dots\dots.\infty=6$.
41. Find the sum of an infinity decreasing G.P. whose first term is equal to $b+2$ and the common ratio 10 $\frac{2}{c}$, where b is the least value of the product of the roots of the equation $(m^2+1)x^2-3x+(m^2+1)^2=0$, and c is the greatest value of the sum of its roots. [9]
42. If $x=1+a+a^2\dots\dots\dots\dots..\infty$, where $|a|<1$ and $y=1+b+b^2\dots\dots\dots..\infty$ where $|b|<1$. Prove that: $1+ab+a^2b^2+\cdots\dots\dots\dots\dots\dots\infty=\frac{xy}{x+y-1}$
43. If $A=1+r^a+r^{2a}+\cdots\dots\dots.to\ \infty$ and $B=1+r^b+r^{2b}+\cdots\dots..\infty$, prove that $r=\left(\frac{A-1}{A}\right)^{1/a}=\left(\frac{B-1}{B}\right)^{1/b}$
44. If $x=\sum_{n=0}^{\infty} cos^{2n}A, y=\sum_{n=0}^{\infty} sin^{2n}B, z=\sum_{n=0}^{\infty} cos^{2n}Asin^{2n}B$, where $0<A,B<\pi/2$ then prove that $xz+yz-z=xy$.
45. If $|x|<1$ and $|y|<1$, find the sum to infinity of the series $(x+y)+(x^2+xy+y^2)+(x^3+x^2y+xy^2+y^3)+\cdots$ ... ... $\left[\frac{x+y-xy}{(1-x)(1-y)}\right]$
46. The sum of an infinite G.P. is 57 and the sum of their cubes is 9747, find the G.P. $\left[19, \frac{38}{3}, \frac{76}{9}\dots..\right]$
47. Which is the rational number having the decimal expansion $0.\overline{359}$? [353/990]

48. Insert 5 geometric means between 576 and 9. [288, 144, 72, 36, 18]
49. Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be geometric mean between a and b.
50. If A.M and G.M between two numbers are in the ratio $m:n$, then prove that the number are in the ratio $m+\sqrt{m^2-n^2}:m-\sqrt{m^2-n^2}$.
51. Let x be the arithmetic mean and y, z be two geometric means between any two positive. Then prove that $\frac{y^3+z^3}{xyz}=2$.
52. If a is the A.M of b and c and the two geometric mean are G_1 and G_2, then prove that $G_1^3+G_2^3=2abc$.
53. Prove that $2^{\frac{1}{4}}.4^{\frac{1}{8}}.8^{\frac{1}{16}}.16^{\frac{1}{32}}\ldots\ldots\ldots.\infty=2$.