Alternating Currents

6

CHAPTER

JEE/NEET Syllabus

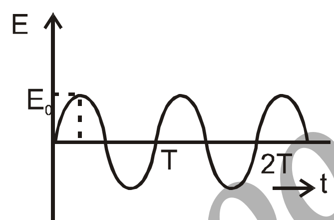
Alternating currents, peak and rms value of alternating current/ voltage; reactance and impedance; LCR series circuit, resonance; Quality factor, power in AC circuits, wattless current

THIS CHAPTER COVERS :

- Alternating current
- Average and rms value of AC
- Phasors and component of AC circuit
- Series LCR circuit
- Power consumed in AC circuit

ALTERNATING CURRENT

A time varying, periodic current is called an ac when its amplitude is constant and alternate half cycles are positive and negative.



The alternating emf E at any instant may be expressed as $E = E_0 \sin \omega t$ where ω is angular frequency of alternating emf and E_0 is the peak value or amplitude of alternating emf.

The frequency of alternating emf, $f = \omega/2\pi$ and time period of alternating emf., $T = 1/f = 2\pi/\omega$.

The alternating current in a circuit, fed by an alternating source of emf may be controlled by inductance L, resistance R and capacitance C. Due to presence of elements L and C, the current is not necessarily in phase with the applied emf. Therefore alternating current is, in general expressed as $I = I_0 \sin(\omega t + \phi)$ where ϕ is the phase which may be positive, zero or negative depending on the value of reactive components L and C.

AVERAGE AND RMS VALUE OF AC

1. Mean or Average value for time 't'

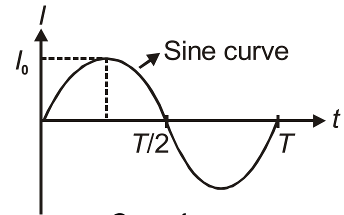
$$E_{\text{mean}} = \frac{1}{t}\int_0^t E\,dt,\quad I_{\text{mean}} = \frac{1}{t}\int_0^t I\,dt$$

2. Root Mean Square (RMS) Value

RMS value of ac is equal to that value of dc, which when passed through a resistance for a given time will produce the same amount of heat as produced by the ac when passed through the same resistance for same time. RMS values are also known as **virtual or effective value**.

Cases :

$$E_{rms}^2 = \frac{1}{t}\int_0^t E^2 dt,\ I_{rms}^2 = \frac{1}{t}\int_0^t I^2 dt$$



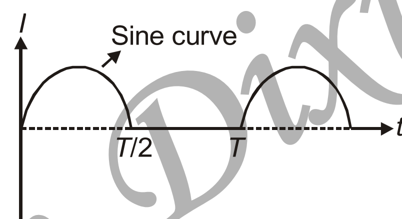
Case-1

1. $I_{mean} = 0$ for $t = T$

   $I_{mean} = \frac{2I_0}{\pi}$ for $t = T/2$

   $I_{rms} = \frac{I_0}{\sqrt{2}}$ for $t = T$

   $I_{rms} = \frac{I_0}{\sqrt{2}}$ for $t = T/2$



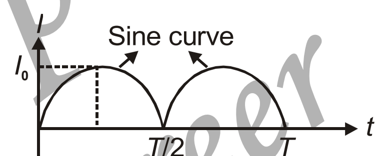
Case-2

2. $I_{mean} = \frac{I_0}{\pi}$ $t = T$

   $I_{mean} = \frac{2I_0}{\pi}$ $t = T/2$

   $I_{rms} = \frac{I_0}{2}$ for $t = T$

   $I_{rms} = \frac{I_0}{\sqrt{2}}$ for $t = T/2$



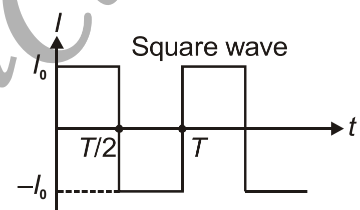
Case-3

3. $I_{mean} = \frac{2I_0}{\pi}$ for $t = T$

   $I_{mean} = \frac{2I_0}{\pi}$ for $t = T/2$

   $I_{rms} = \frac{I_0}{\sqrt{2}}$ for $t = T$

   $I_{rms} = \frac{I_0}{\sqrt{2}}$ for $t = T/2$



Case-4

4. $I_{mean} = 0$ for $t = T$

   $I_{mean} = I_0$ for $t = T/2$

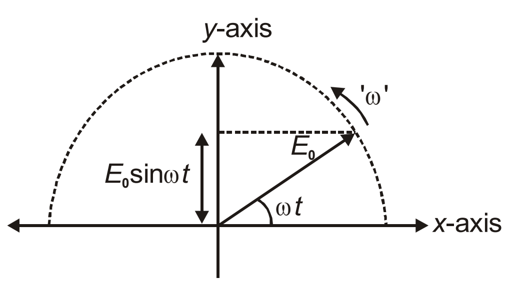
   $I_{rms} = I_0$ for $t = T$

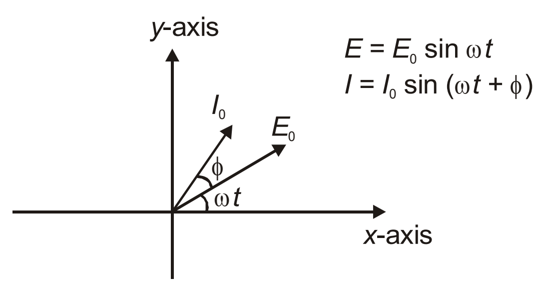
   $I_{rms} = I_0$ for $t = T$

PHASOR AND AC CIRCUITS

Phasor

1. A vector rotating in anticlockwise direction with angular velocity 'ω'.





2. Its length is equal to amplitude of alternating quantity.

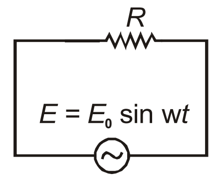
3. Projection of vector on y-axis gives the instantaneous value of alternating quantity.

Different ac Circuits

1. Resistive Circuit

$I = I_0 \sin \omega t$

$$I_0 = \frac{E_0}{R}$$

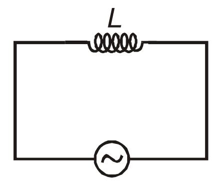


Generator

2. Inductive Circuit

$I = I_0 \sin (\omega t - \pi/2)$

$$I_0 = \frac{E_0}{X_L}, \text{ where } X_L = \omega L = 2\pi f L$$

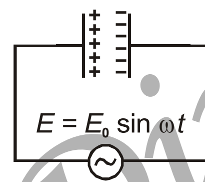


$E = E_0 \sin \omega t$

Capacitive Circuit

1. $I = I_0 \sin (\omega t + \pi/2)$

2. $I_0 = \dfrac{E_0}{X_C}$, where $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi\nu C}$

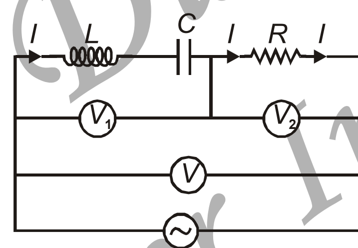


SERIES LCR CIRCUIT

$V = \dfrac{E_0}{\sqrt{2}}$ = rms value of applied voltage

V_1 = rms voltage across L-C = $V_L - V_C$

V_2 = rms voltage across R = V_R



$E = E_0 \sin \omega t$

Phase Relationship

I and V_R are in same phase.

V_L leads I by 90°.

V_C lags behind I by 90°.

Case 1 :

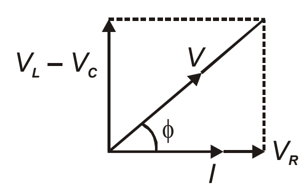
$V_L > V_C$

$\Rightarrow$ V leads I by ϕ

where $\tan\phi = \dfrac{V_L - V_C}{V_R}$

$$\tan\phi = \frac{X_L - X_C}{R}$$

Here $X_L > X_C$ *i.e.,* $\omega > \dfrac{1}{\sqrt{LC}}$



Phasor diagram

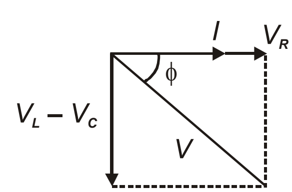
Case 2 :

$V_L < V_C$

i.e., V lags behind I by ϕ

$$\tan\phi = \frac{V_C - V_L}{V_R} = \frac{X_C - X_L}{R}$$

Here $X_C > X_L$ *i.e.,* $\omega < \dfrac{1}{\sqrt{LC}}$



Phasor diagram

In both cases $V = \sqrt{V_R^2 + (V_L - V_C)^2} = I\sqrt{R^2 + (X_L - X_C)^2}$

(a) Impedance = $Z = \sqrt{R^2 + (X_L - X_C)^2}$

(b) Power factor = $\cos\phi = \dfrac{R}{Z} = \dfrac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$

Case 3 :

$V_L = V_C$ *i.e.,* $X_L = X_C$ *i.e.,* $\omega = \dfrac{1}{\sqrt{LC}}$ [Resonance]

In this case

(a) $V_2 = V = \dfrac{E_0}{\sqrt{2}}$

(b) $V_1 = V_L - V_C = 0$

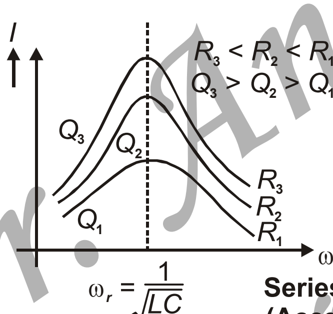
(c) $\tan\phi = 0$, or $\phi = 0$

(d) $\cos\phi = 1$

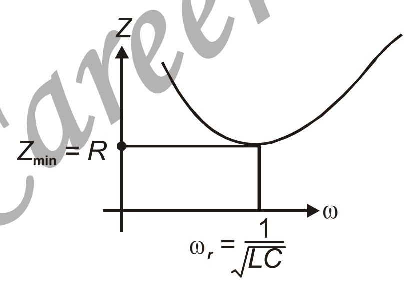
(e) $Z = R$ (minimum)

(f) Power consumed is maximum

(g) Graphs :



Series *LCR* circuit
(Acceptor Circuit)



Series *LCR* circuit

(h) In a series *LCR* circuit,

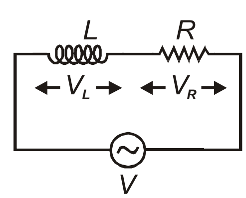
(i) When voltage leads current, then to bring resonance state, either *L* or *C* should be decreased.

(ii) If voltage lags behind current, then to bring resonance state either *L* or *C* should be increased.

(iii) Quality factor *Q* represents the sharpness of tuning at resonance

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad i.e., \qquad Q \propto \frac{1}{R}$$

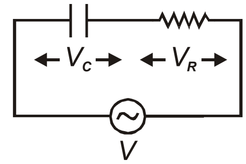
Case 4 : Series *LR* Circuit



(a) $Z = \sqrt{R^2 + X_L^2}$, $V = \sqrt{V_R^2 + V_L^2}$, $I = \dfrac{V}{Z}$

(b) $\cos\phi = \dfrac{R}{\sqrt{R^2 + X_L^2}}$, $\tan\phi = \dfrac{X_L}{R}$. Voltage leads current

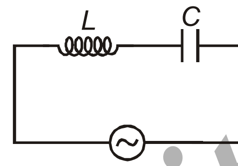
Case 5 : Series *CR* Circuit



(a) $V = \sqrt{V_R^2 + V_C^2}\ ,\ Z = \sqrt{R^2 + X_C^2}\ ,\ I = \dfrac{V}{Z}$

(b) $\cos\phi = \dfrac{R}{Z} = \dfrac{R}{\sqrt{R^2 + X_C^2}}, \tan\phi = \dfrac{X_C}{R}$. Voltage lags behind current

Case 6 : Series *LC* Circuit

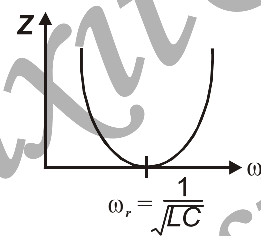


(a) $V = V_L - V_C$, $Z = X_L - X_C$, $I = \dfrac{V}{Z}$

(b) $\phi = \dfrac{\pi}{2}$ $(X_L > X_C)$, $\phi = \dfrac{-\pi}{2}$ $(X_L < X_C)$

(c) When $X_L = X_C$, $Z = 0$

i.e., $\omega = \dfrac{1}{\sqrt{LC}}$



POWER CONSUMED IN AN A.C. CIRCUIT

$$P_{av} = \frac{1}{T}\int_0^T EIdt$$

If $E = E_0 \sin \omega t$ and $I = I_0 \sin(\omega t + \phi)$

$$P_{av} = \frac{E_0 I_0}{2}\cos\phi = \frac{E_0}{\sqrt{2}}.\frac{I_0}{\sqrt{2}}\cos\phi = E_v I_v \cos\phi$$

[E_v = Virtual or rms voltage, I_v = Virtual or rms current]

Some Important Points :

1. For pure resistor $\phi = 0$, $P_{av} = \dfrac{E_0 I_0}{2} = E_{rms}\ I_{rms}$
2. For pure inductor or capacitor, $P_{av} = 0$ [Non **resistive** circuit]
3. Power consumed is independent of $I_v \sin\phi$. This is called wattless component.
4. $\cos\phi$ = Power factor $= \dfrac{R}{Z}$
5. In a series *LCR* circuit

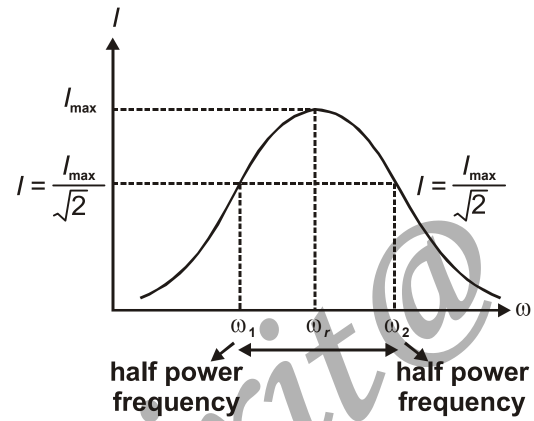
   $P_{av} = E_v I_v \cos\ \phi = \dfrac{E_v^2}{Z}\cos\phi = I_v^2 R$

   Wattless current $I_v \sin\phi$
6. At resonance *i.e.*, at $\omega_r = \dfrac{1}{\sqrt{LC}}$, $Z = R$ power is maximum
7. At frequencies other than $\omega_r = \dfrac{1}{\sqrt{LC}}$, power consumed is less.

8. At $\omega = \omega_1$ or ω_2, power = half the maximum power then $\omega_r = \sqrt{\omega_1\omega_2}$

$P_{\max} = I_{\max}^2 R$

$$P_{1/2} = \frac{I_{\max}^2 R}{2} = \left(\frac{I_{\max}}{\sqrt{2}}\right)^2 R$$



i.e., when $I = \frac{I_{\max}}{\sqrt{2}}$, power is half

$$I_{\max} = \frac{E_v}{R}, I_v = \frac{E_v}{Z}$$

$$I = \frac{I_{\max}}{\sqrt{2}} \Rightarrow \boxed{Z = R\sqrt{2}}$$

or, $\sqrt{(X_L - X_C)^2 + R^2} = R\sqrt{2} \Rightarrow \boxed{X_L - X_C = R}$

$$\cos\phi = \frac{R}{Z} = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad \phi = 45^\circ$$

9. Quality factor $Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{1}{R}\sqrt{\frac{L}{C}}$

and $\Delta\omega = \omega_2 - \omega_1 = \frac{R}{L}$

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