♡ SIN and COS ki kahaani, Neha m'am ki Zubaani... ♫♫

SIN — FRIENDLY, LOYAL, ACCOMODATING

COS — STUBBORN, ANGRY, NON-Accomodating

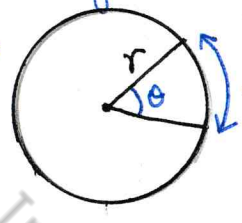
Prepared by Neha Agrawal Mathematically Inclined

TRIGONOMETRIC FUNCTIONS

(I) RECALL (BACHPAN WALI FEELING)

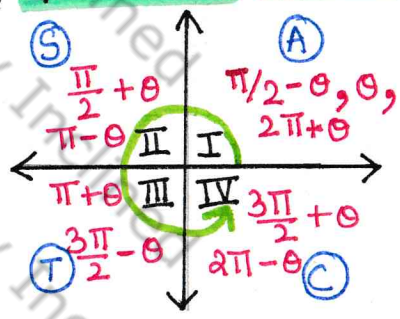
- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

(II) DEGREE and RADIAN

- $\pi^c = 180°$
- Radian measure $= \frac{\pi}{180} \times$ Deg. measure
- Degree measure $= \frac{180}{\pi} \times$ Radian meas.
- 

$l = r\theta$ NOTE: θ is in RADIANS

(III) QUADRANT SYSTEM



Ⓐ: All Trigonometric Functions

Ⓢ: Sin & Cosec

Ⓣ: Tan & Cot

Ⓒ: Cos & Sec

(IV) VALUE of ANY ANGLE (NEHA MA'AM STYLE)

① $\frac{\pi}{2} \times$ ODD multiple ⇒ sin ↔ cos, tan ↔ cot, sec ↔ cosec

$\frac{\pi}{2} \times$ EVEN multiple ⇒ T-function : Same

② Check the Quadrant for the SIGN as per the ORIGINAL FUNCTION.

(V)

- $\sin^2A - \sin^2B = \cos^2B - \cos^2A = \sin(A+B)\sin(A-B)$
- $\cos^2A - \sin^2B = \cos^2B - \sin^2A = \cos(A+B)\cos(A-B)$

(VI) DOUBLE ANGLE FORMULAE

① $\sin 2A = 2\sin A\cos A = \frac{2\tan A}{1+\tan^2 A}$

② $\cos 2A = \cos^2A - \sin^2A = 2\cos^2A - 1 = 1 - 2\sin^2A = \frac{1-\tan^2A}{1+\tan^2A}$

③ $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

(VII) TRIPLE ANGLE FORMULAE

① $\sin 3A = 3\sin A - 4\sin^3 A$

② $\cos 3A = 4\cos^3 A - 3\cos A$

③ $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

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KDS ho gya!! ♡♡

TRIGONOMETRIC FUNCTIONS

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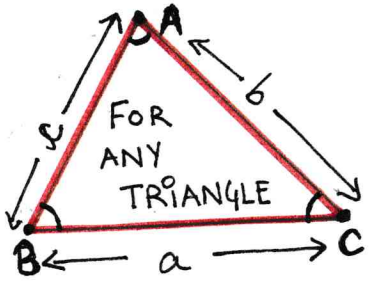
(XI) SINE and COSINE RULE



SINE RULE:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

COSINE RULE:

$$\cos A = \frac{b^2+c^2-a^2}{2bc} \qquad \cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

$$\frac{\text{ADOSI}^2 + \text{PADOSI}^2 - \text{DUSHMAN}^2}{2\,\text{ADOSI}\cdot\text{PADOSI}}$$

Neha ma'am SPECIAL

(XII)

$*\sin 18^\circ = \frac{\sqrt{5}-1}{4}$ $*\cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}$

*REST can be DERIVED EASILY

(XIII) IMPORTANT - Dom, Range, Periodicity

T-Func	DOMAIN	RANGE	PERIODICITY
$\sin x$	R	$[-1, 1]$	2π
$\cos x$	R	$[-1, 1]$	2π
$\tan x$	$\{R-(2n+1)\frac{\pi}{2}, n\in Z\}$	R	π
$\cot x$	$\{R-n\pi, n\in Z\}$	R	π
$\sec x$	$R-\{(2n+1)\pi/2\}$, $n\in Z$	$(-\infty,-1]\cup[1,\infty)$ or $R-(-1,1)$	2π
$\operatorname{cosec} x$	$\{R-n\pi, n\in Z\}$		2π

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(VIII) C and D FORMULAE

① $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

② $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

③ $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

④ $\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

(IX) NAAM KYA DU?

① $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

② $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

③ $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

④ $-2\sin A\sin B = \cos(A+B) - \cos(A-B)$

(X) TRIGONOMETRIC EQUATIONS (T-Eqns)

- **PRINCIPAL SOLUTION** $0 \le x < 2\pi$ → Every T-Eqn gives 2 solutions
- **GENERAL SOLUTION**

① $\sin x = \sin y \Rightarrow x = n\pi + (-1)^n y$

② $\cos x = \cos y \Rightarrow x = 2n\pi \pm y$

③ $\tan x = \tan y \Rightarrow x = n\pi + y$

④ $\sin^2 x = \sin^2 y$

⑤ $\cos^2 x = \cos^2 y$

⑥ $\tan^2 x = \tan^2 y$

$\Rightarrow x = n\pi \pm y$ (ALL THE SAME)

SIN $n\pi$: ISKA π USKA π = 0

COS $n\pi$: EVEN +1, ODD -1

$n \in Z$ → set of Integers