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JEE/NEET EXPERT

MATHEMATICS
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DIFFERENTIATION (ASSIGNMENT-II)

Student's Name: ____________________

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EXERCISE – I SINGLE CORRECT (OBJECTIVE QUESTIONS)

1. If y = f(x) is an odd differentiable function defined on $(-\infty, \infty)$ such that $f'(3) = -2$, then $f'(-3)$ equals

(A) 4 (B) 2 (C) – 2 (D) 0

Sol.

2. If $f(x) = \log_x (\ln x)$ then $f'(x)$ at x = e is

(A) 1/e (B) e (C) 1 (D) zero

Sol.

3 . If $y = \cos^{-1}(\cos x)$ then $\frac{dy}{dt}$ at $x = \frac{5\pi}{4}$ is equal to

(A) 1 (B) – 1 (C) $\frac{1}{\sqrt{2}}$ (D) $-\frac{1}{\sqrt{2}}$

Sol.

4. If $x = \frac{1+t}{t^3}$, $y = \frac{3}{2t^2} + \frac{2}{t}$ then, $x\left(\frac{dy}{dx}\right)^3 - \frac{dy}{dx} =$

(A) 0 (B) – 1 (C) 1 (D) 2

Sol.

5. If sin (xy) + cos (xy) = 0 then $\frac{dy}{dx}$ =

(A) $\frac{y}{x}$ (B) $-\frac{y}{x}$ (C) $-\frac{x}{y}$ (D) $\frac{x}{y}$

Sol.

6. If $y = x^{x^2}$ then $\frac{dy}{dx}$ =

(A) $2\ln x \cdot x^{x^2}$ (B) $(2\ln x + 1) \cdot x^{x^2}$

(C) $(2\ln x + 1) \cdot x^{x^2+1}$ (D) none of these

Sol.

7. If $f(x) = |x|^{|\sin x|}$ then $f'(\pi/4)$ equals

(A) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\ln\frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}\right)$

(B) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\ln\frac{4}{\pi} + \frac{2\sqrt{2}}{\pi}\right)$

(C) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\ln\frac{\pi}{4} - \frac{2\sqrt{2}}{\pi}\right)$

(D) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\ln\frac{\pi}{4} + \frac{2\sqrt{2}}{\pi}\right)$

Sol.

METHOD OF DIFFERENTIATION

8. If $y = \sin^{-1}\dfrac{x^2-1}{x^2+1} + \sec^{-1}\dfrac{x^2+1}{x^2-1}$, $|x| > 1$ then $\dfrac{dy}{dx}$ is equal to

(A) $\dfrac{x}{x^4-1}$ (B) $\dfrac{x^2}{x^4-1}$ (C) 0 (D) 1

Sol.

9. If $y = x - x^2$, then the derivative of y^2 w.r.t. x^2 is
(A) $2x^2 + 3x - 1$ (B) $2x^2 - 3x + 1$
(C) $2x^2 + 3x + 1$ (D) none of these
Sol.

10. Let f(x) be a polynomial in x. Then the second derivative of $f(e^x)$, is
(A) $f''(e^x) \cdot e^x + f'(e^x)$ (B) $f''(e^x) \cdot e^{2x} + f'(e^x) \cdot e^{2x}$
(C) $f''(e^x)\, e^{2x}$ (D) $f''(e^x) \cdot e^{2x} + f'(e^x) \cdot e^x$
Sol.

11. If $x = at^2$, $y = 2at$, then $\dfrac{d^2y}{dx^2}$ is

(A) $-\dfrac{1}{t^2}$ (B) $\dfrac{1}{2at^2}$ (C) $-\dfrac{1}{t^3}$ (D) $-\dfrac{1}{2at^3}$

Sol.

12. If f(x), g(x), h(x) are polynomials in x of degree 2 and $F(x) = \begin{vmatrix} f & g & h \\ f' & g' & h' \\ f'' & g'' & h'' \end{vmatrix}$, then $F'(x)$ is equal to

(A) 1 (B) 0
(C) −1 (D) f(x) . g(x) . h(x)
Sol.

13. If $y = \sin^{-1}\left(x\sqrt{1-x} + \sqrt{x}\sqrt{1-x^2}\right)$ and $\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x(1-x)}} + p$, then p =

(A) 0 (B) $\dfrac{1}{\sqrt{1-x}}$ (C) $\sin^{-1}\sqrt{x}$ (D) $\dfrac{1}{\sqrt{1-x^2}}$

Sol.

14. If $\dfrac{d}{dx}\left(\dfrac{1+x^2+x^4}{1+x+x^2}\right) = ax + b$ then the value of a and b are respectively
(A) 2 and 1 (B) −2 and 1
(C) 2 and −1 (D) none of these
Sol.

15. Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$. Then $\displaystyle\lim_{x\to 0}\frac{f'(x)}{x} =$

(A) 2 (B) −2 (C) −1 (D) 0

Sol.

16. If u = ax + b then $\frac{d^n}{dx^n}$ (f(ax + b)) is equal to

(A) $\frac{d^n}{du^n}$ (f(u)) (B) a$\frac{d^n}{du^n}$(f(u))

(C) $a^n \frac{d^n}{du^n}$ (f(u)) (D) $a^{-n} \frac{d^n}{du^n}$ (f(u))

Sol.

17. If y = x + e^x then $\frac{d^2x}{dy^2}$ is

(A) e^x (B) $\frac{-e^x}{(1+e^x)^3}$ (C) $-\frac{e^x}{(1+e^x)^2}$ (D) $\frac{e^x}{(1+e^x)^2}$

Sol.

18. If y = f $\left(\frac{2x-1}{x^2+1}\right)$ and f′(x) = sin x then $\frac{dy}{dx}$ =

(A) $\frac{1+x-x^2}{(1+x^2)^2}\sin\left(\frac{2x-1}{x^2+1}\right)$ (B) $\frac{2(1+x-x^2)}{(1+x^2)^2}\sin\left(\frac{2x-1}{x^2+1}\right)$

(C) $\frac{1-x+x^2}{(1+x^2)^2}\sin\left(\frac{2x-1}{x^2+1}\right)$ (D) none of these

Sol.

19. If 8 f(x) + 6 f $\left(\frac{1}{x}\right)$ = x +5 and y = x^2 f(x), then $\frac{dy}{dx}$ at x = −1 is equal to

(A) 0 (B) $\frac{1}{14}$ (C) – $\frac{1}{14}$ (D) none of these

Sol.

20. If x = $e^{y+e^{y+....to\,\infty}}$, x > 0, then $\frac{dy}{dx}$

(A) $\frac{x}{1+x}$ (B) $\frac{1}{x}$ (C) $\frac{1-x}{x}$ (D) $\frac{1+x}{x}$

Sol.

21. If f(x) = x^n, then the value of

$f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} + ... + \frac{(-1)^n f^n(1)}{n!}$ is

(A) 2^n (B) 2^{n-1} (C) 0 (D) 1

Sol.

22. If $y = \frac{a + bx^{3/2}}{x^{5/4}}$ & $\frac{dy}{dx}$ vanishes when x = 5 then $\frac{a}{b} =$

(A) $\sqrt{3}$ (B) 2 (C) $\sqrt{5}$ (D) None of these

Sol.

23. If f(x) = f′(x) + f″(x) + f‴(x) + f″″(x) ....∞ also f(0) = 1 and f(x) is a differentiable function indefinitely then f(x) has the value

(A) e^x (B) $e^{x/2}$ (C) e^{2x} (D) e^{4x}

Sol.

24. If $y = \sin^{-1}\frac{2x}{1+x^2}$ then $\left.\frac{dy}{dx}\right]_{x=-2}$ is

(A) $\frac{2}{5}$ (B) $\frac{2}{\sqrt{5}}$ (C) $-\frac{2}{5}$ (D) None of these

Sol.

25. If $y = \sqrt{\sin x + y}$, then $\frac{dy}{dx} =$

(A) $\frac{\sin x}{2y-1}$ (B) $\frac{\sin x}{1-2y}$ (C) $\frac{\cos x}{1-2y}$ (D) $\frac{\cos x}{2y-1}$

Sol.

26. If $y = e^{-x}\cos x$ and $y_4 + ky = 0$, where $y_4 = \frac{d^4y}{dx^4}$, then k =

(A) 4 (B) −4 (C) 2 (D) −2

Sol.

27. If y = a cos (*l*n x) + b sin (*l*n x), then $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx}$

(A) 0 (B) y (C) −y (D) None of these

Sol.

28. If y = sin mx then the value of $\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$ (where subscripts of y shows the order of derivative) is

(A) independent of x but dependent on m

(B) dependent of x but independent of m

(C) dependent on both m & x

(D) independent of m & x

Sol.

29. If f is differentiable in (0, 6) & $f'(4) = 5$ then $\lim\limits_{x\to 2}\dfrac{f(4)-f(x^2)}{2-x}$ =

(A) 5 (B) 5/4 (C) 10 (D) 20

Sol.

30. Let $y = e^{2x}$. Then $\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{d^2x}{dy^2}\right)$ is

(A) 1 (B) e^{-2x} (C) $2e^{-2x}$ (D) $-2e^{-2x}$

Sol.

31. If g is the inverse function of f an $f'(x) = \dfrac{x^5}{1+x^4}$.

If $g(2) = a$, then $f'(2)$ is equal to

(A) $\dfrac{a^5}{1+a^4}$ (B) $\dfrac{1+a^4}{a^5}$ (C) $\dfrac{1+a^5}{a^4}$ (D) $\dfrac{a^4}{1+a^5}$

Sol.

32. If $y = (1 + x)(1 + x^2)(1 + x^4) \ldots\ldots (1 + x^{2n})$, then $\dfrac{dy}{dx}$ at x = 0 is

(A) −1 (B) 1 (C) 0 (D) 2^n

Sol.

33. The derivative of $\sec^{-1}\left(\dfrac{1}{2x^2-1}\right)$ w.r.t. $\sqrt{1-x^2}$ at $x = \dfrac{1}{2}$ is

(A) 4 (B) 1/4 (C) 1 (D) None of these

Sol.

34. Let $f(x) = \begin{vmatrix} \cos x & \sin x & \cos x \\ \cos 2x & \sin 2x & 2\cos 2x \\ \cos 3x & \sin 3x & 3\cos 3x \end{vmatrix}$ then $f'\left(\dfrac{\pi}{2}\right)$ =

(A) 0 (B) 1 (C) 4 (D) None of these

Sol.

EXERCISE – II MULTIPLE CORRECT (OBJECTIVE QUESTIONS)

1. The differential coefficient of $\sin^{-1}\frac{t}{\sqrt{1+t^2}}$ w.r.t $\cos^{-1}\frac{1}{\sqrt{1+t^2}}$ is

(A) $1 \ \forall \ t > 0$ (B) $-1 \ \forall \ t < 0$
(C) $1 \ \forall \ t \in R$ (D) none of these

Sol.

2. If $f(x) = |(x - 4)(x - 5)|$, then $f'(x)$ is
(A) $-2x + 9$, for all $x \in R$ (B) $2x - 9$ if $x > 5$
(C) $-2x + 9$ if $4 < x < 5$ (D) not defined for $x = 4, 5$

Sol.

3. If $x^p \cdot y^q = (x + y)^{p+q}$ then $\frac{dy}{dx}$ is

(A) independent of p (B) independent of q

(C) dependent on both p and q (D) $\frac{y}{x}$

Sol.

4. The functions $u = e^x \sin x$; $v = e^x \cos x$ satisfy the equation

(A) $v\frac{du}{dx} - u\frac{dv}{dx} = u^2 + v^2$ (B) $\frac{d^2u}{dx^2} = 2v$

(C) $\frac{d^2v}{dx^2} = -2u$ (D) $\frac{du}{dx} + \frac{dv}{dx} = 2v$

Sol.

5. If $\sqrt{x^2 + y^2} = e^t$ where $t = \sin^{-1}\left(\frac{y}{\sqrt{x^2+y^2}}\right)$ then $\frac{dy}{dx} =$

(A) $\frac{x-y}{x+y}$ (B) $\frac{x+y}{x-y}$ (C) $\frac{y-x}{y+x}$ (D) $\frac{x-y}{2x+y}$

Sol.

6. If $f_n(x) = e^{f_{n-1}(x)}$ for all $n \in N$ and $f_0(x) = x$, then $\frac{d}{dx}\{f_n(x)\}$ is equal to

(A) $f_n(x) \cdot \frac{d}{dx}\{f_{n-1}(x)\}$ (B) $f_n(x) \cdot f_{n-1}(x)$
(C) $f_n(x) \cdot f_{n-1}(x) \ldots\ldots f_2(x) \cdot f_1(x)$
(D) none of these

Sol.

7. If f is twice differentiable such that $f''(x) = -f(x)$ and $f'(x) = g(x)$. If h(x) is twice differentiable function such that $h'(x) = [f(x)]^2 + [g(x)]^2$. If $h(0) = 2$, $h(1) = 4$, then the equation $y = h(x)$ represents
(A) a curve of degree 2
(B) a curve passing through the origin
(C) a straight line with slope 2
(D) a straight line with y intercept equal to 2.
Sol.

8. If $f(x) = \begin{vmatrix} \cos(x+x^2) & \sin(x+x^2) & -\cos(x+x^2) \\ \sin(x-x^2) & \cos(x-x^2) & \sin(x-x^2) \\ \sin 2x & 0 & \sin 2x^2 \end{vmatrix}$ then
(A) $f(-2) = 0$ (B) $f'(-1/2) = 0$
(C) $f'(-1) = 2$ (D) $f''(0) = 4$
Sol.

9. If $f(x) = (ax + b)\sin x + (cx + d)\cos x$, then the values of a, b, c and d such that $f'(x) = x \cos x$ for all x are
(A) $a = d = 1$ (B) $b = 0$ (C) $c = 0$ (D) $b = c$
Sol.

10. $y = \cos^{-1}\sqrt{\dfrac{\sqrt{1+x^2}+1}{2\sqrt{1+x^2}}}$ then $\dfrac{dy}{dx}$ is
(A) $\dfrac{1}{2(1+x^2)}$, $x \in R$ (B) $\dfrac{1}{2(1+x^2)}$, $x > 0$
(C) $\dfrac{-1}{2(1+x^2)}$, $x < 0$ (D) $\dfrac{1}{2(1+x^2)} < 0$
Sol.

11. Two functions f & g have first & second derivatives at x = 0 satisfy the relations, $f(0) = \dfrac{2}{g(0)}$, $f'(0) = 2$ $g'(0) = 4g(0)$, $g''(0) = 5 f''(0) = g(0) = 3$ then
(A) if $h(x) = \dfrac{f(x)}{g(x)}$ then $h'(0) = \dfrac{15}{4}$
(B) if $k(x) = f(x) . g(x) \sin x$ then $k'(0) = 2$
(C) $\lim\limits_{x\to 0}\dfrac{g'(x)}{f'(x)} = \dfrac{1}{2}$ (D) None of these
Sol.

12. If $y = \tan^{-1}\left(\dfrac{\ln\dfrac{e}{x^2}}{\ln ex^2}\right) + \tan^{-1}\dfrac{3+2\ln x}{1-6\ln x}$ then
(A) $\dfrac{dy}{dx} = 0$ (B) $\dfrac{d^2y}{dx^2} = 0$
(C) $\dfrac{dy}{dx} = \dfrac{2}{x(1+\ln^2 x)}$ (D) $\dfrac{dy}{dx} = 1$
Sol.

METHOD OF DIFFERENTIATION

EXERCISE – III | SUBJECTIVE QUESTIONS

1. Find the derivative of following functions with respect to x from the first principle (ab – initio method).

(i) $f(x) = \sin x^2$

Sol.

(ii) $f(x) = e^{2x+3}$

Sol.

2. Differentiate the following functions with respect to x.

(i) $x^{2/3} + 7e - \frac{5}{x} + 7 \tan x$

Sol.

(ii) $x^2 . \ln x. e^x$

Sol.

(iii) $\ln \tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$

Sol.

(iv) $\frac{\sin x - x\cos x}{x\sin x + \cos x}$

Sol.

(v) $\tan\left(\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$

Sol.

3. If $f(x) = 2\, \ln(x - 2) - x^2 + 4x + 1$, then find the solution set of the inequality $f'(x) \geq 0$.

Sol.

4. Find $\frac{dy}{dx}$ when x and y are connected by the following relations

(i) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

Sol.

(ii) $xy + xe^{-y} + y . e^x = x^2$

Sol.

5. Differentiate the given functions w.r.t.x.

(i) $(\ln x)^{\cos x}$

Sol.

(ii) $x^x - 2^{\sin x}$

Sol.

(iii) $y = (x \ln x)^{\ln \ln x}$

Sol.

6. If P_n is the sum of GP upon n terms. Show that

$(1 - r)\dfrac{dP_n}{dr} = n \cdot P_{n-1} - (n-1)P_n$.

Sol.

7. If $x = a t^3$ and $y = b t^2$, where t is a parameter, then prove that $\dfrac{d^3y}{dx^3} = \dfrac{8b}{27a^3 . t^7}$

Sol.

8. Show that the substitution $z = \ln\left(\tan\dfrac{x}{2}\right)$ changes the equation $\dfrac{d^2y}{dx^2} + \cot x\dfrac{dy}{dx} + 4y\,\text{cosec}^2 x = 0$ to $(d^2y/dz^2) + 4y = 0$.

Sol.

9. If $f(x) = x^n$ then find the value of

$f(1) + \dfrac{f^1(1)}{1!} + \dfrac{f^2(1)}{2!} + \ldots + \dfrac{f^n(1)}{n!}$ where $f^r(x)$ denotes the r^{th} derivative of f(x) w.r.t. x

Sol.

10. If $\underset{x\to 0}{\text{Limit}}\dfrac{a\sin x - bx + cx^2 + x^3}{2x^2.\ln(1+x) - 2x^3 + x^4}$ exists and is finite, find the values of a, b, c and the limit.

Sol.

11. If $\cos\dfrac{x}{2}.\cos\dfrac{x}{2^2}.\cos\dfrac{x}{2^3}\ldots\infty = \dfrac{\sin x}{x}$ then find the value of $\dfrac{1}{2^2}\sec^2\dfrac{x}{2} + \dfrac{1}{2^4}\sec^2\dfrac{x}{2^2} + \dfrac{1}{2^6}\sec^2\dfrac{x}{2^3}\ldots\infty$.

Sol.

12. Show that the function $y = f(x)$ defined by the parametric equations $x = e^t \sin t$, $y = e^t \cos t$ satisfies the relation $y''(x + y)^2 = 2(xy' - y)$.

Sol.

13. If y = x log $\left(\frac{x}{a+bx}\right)$, then prove that $x^3 \frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$.

Sol.

14. If y = $(\cos x)^{ln\, x} + (ln\, x)^x$ find $\frac{dy}{dx}$.

Sol.

15. Suppose f(x) = tan ($\sin^{-1}$ (2x))

(a) Find the domain and range of f.

Sol.

(b) Express f(x) as an algebraic function of x.

Sol.

(c) Find f′(1/4).

Sol.

16. Let f(x) = x + $\cfrac{1}{2x + \cfrac{1}{2x + \cfrac{1}{2x + \ldots\ldots\infty}}}$. Compute the value of f(100) . f′(100).

Sol.

17. Differentiate $\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}$ w.r.t. $\sqrt{1-x^4}$.

Sol.

18. Find the derivative with respect to x of the function

$(\log_{\cos x} \sin x)\,(\log_{\sin x} \cos x)^{-1} + \arcsin\frac{2x}{1+x^2}$ at $x = \frac{\pi}{4}$

Sol.

19. If $\sqrt{1-x^6}+\sqrt{1-y^6}$ = a^3 (x^3 – y^3), prove that

$\frac{dy}{dx} = \frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$.

Sol.

20. If $y = x + \cfrac{1}{x + \cfrac{1}{x + \cfrac{1}{x + \ldots\ldots}}}$, prove that

$$\frac{dy}{dx} = \cfrac{1}{2 - \cfrac{x}{x + \cfrac{1}{x + \cfrac{1}{x + \ldots\ldots\ldots\ldots}}}}.$$

Sol.

21. If $y = \tan^{-1}\frac{u}{\sqrt{1-u^2}}$ & $x = \sec^{-1}\frac{1}{2u^2-1}$, $u \in \left(0, \frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right)$ prove that $2\frac{dy}{dx} + 1 = 0$.

Sol.

22. If $y = \cot^{-1}\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}$, find $\frac{dy}{dx}$ if $x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$.

Sol.

23. If $y = \tan^{-1}\frac{x}{1+\sqrt{1-x^2}} + \sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$, then find $\frac{dy}{dx}$ for $x \in (-1, 1)$.

Sol.

24. (a) Let $f(x) = x^2 - 4x - 3$, $x > 2$ and let g be the inverse of f. Find the value of g' where $f(x) = 2$.

Sol.

(b) Let f, g and h are differentiable functions. If $f(0) = 1$; $g(0) = 2$; $h(0) = 3$ and the derivatives of their pair wise products at $x = 0$ are $(fg)'(0) = 6$; $(gh)'(0) = 4$ and $(hf)'(0) = 5$ then compute the value of $(fgh)'(0)$.

Sol.

25. If x = 2 cost – cos2t & y = 2sint – sin2t, find the value of $\left(\frac{d^2y}{dx^2}\right)$ when t = $\left(\frac{\pi}{2}\right)$.

Sol.

26. If f : R $\to$ R is a function such that $f(x) = x^3 + x^2 f'(1) + xf''(2) + f'''(3)$ for all x $\in$ R, then prove that f(2) = f(1) – f(0).

Sol.

27. If y = x $\ln\left[(ax)^{-1} + a^{-1}\right]$, prove that

$$x(x+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = y - 1.$$

Sol.

28. Let g(x) be a polynomial, of degree one & f(x) be defined by f(x) = $\begin{cases} g(x), & x \le 0 \\ \left(\frac{1+x}{2+x}\right)^{1/x}, & x > 0 \end{cases}$.

Find the continuous function f(x) satisfying $f'(1) = f(-1)$

Sol.

29. If sin y = x sin (a + y), show that

$$\frac{dy}{dx} = \frac{\sin a}{1 - 2x\cos a + x^2}.$$

Sol.

30. If $y = \tan^{-1}\frac{1}{x^2+x+1} + \tan^{-1}\frac{1}{x^2+3x+3} + \tan^{-1}\frac{1}{x^2+5x+7} + \tan^{-1}\frac{1}{x^2+7x+13} + \ldots$ to n terms. Find dy/dx, expressing your answer in 2 terms.

Sol.

EXERCISE – IV ADVANCED SUBJECTIVE QUESTIONS

1. Prove that
if $| a_1 \sin x + a_2 \sin 2x + ... + a_n \sin nx | \le | \sin x |$ for $x \in R$, then $|a_1 + 2a_2 + 3a_3 + ... + na_n| \le 1$
Sol.

2. The function $f : R \to R$ satisfies
$f(x^2) . f''(x) = f'(x) . f'(x^2)$ for all real x. Given that $f(1) = 1$ and $f'''(1) = 8$, compute the value of $f'(1) + f''(1)$.
Sol.

3. Let $y = x \sin kx$. Find the possible vale of k for which the differential equation $\frac{d^2y}{dx^2} + y = 2k \cos kx$ holds true for all $x \in R$.
Sol.

4. Let $f(x) = \frac{\sin x}{x}$ if $x \ne 0$ and $f(0) = 1$. Define the function $f'(x)$ for all x find $f''(0)$ if it exist.
Sol.

5. Show that the substitution $z = ln\left(\tan\frac{x}{2}\right)$ changes the equation $\frac{d^2y}{dx^2} + \cot x \frac{dy}{dx} + 4 y \operatorname{cosec}^2 x = 0$ to $\frac{d^2y}{dz^2} + 4y = 0$.
Sol.

6. Prove that $\cos x + \cos 3x + \cos 5x + ... + \cos (2n-1) x$
$= \frac{\sin 2nx}{2\sin x}$, $x \ne K\pi$, $K \in I$ and deduce from this:
$\sin x + 3 \sin 3x + 5 \sin 5x + ... + (2n - 1) \sin (2n - 1) x$
$= \frac{[(2n+1)\sin(2n-1)x - (2n-1)\sin(2n+1)x]}{4\sin^2 x}$.
Sol.

7. Find a polynomial function f(x) such that $f(2x) = f'(x) f''(x)$.
Sol.

8. (i) Let $f(x) = \begin{bmatrix} xe^x & x \le 0 \\ x + x^2 - x^3 & x < 0 \end{bmatrix}$ then prove that
(a) f is continuous and differentiable for all x.
Sol.

(b) f' is continuous and differentiable for all x.
Sol.

(ii) $f : [0, 1] \rightarrow R$ is defined as

$$f(x) = \begin{bmatrix} x^3(1-x)\sin\left(\frac{1}{x^2}\right) & \text{if} \quad 0 < x \le 1 \\ 0 & \text{if} \quad x = 0 \end{bmatrix}$$, then prove that

(a) f is differentiable in [0, 1]
Sol.

(b) f is bounded in [0, 1]
Sol.

(c) f is bounded in [0, 1]
Sol.

9. Let f(x) be a derivable function at x = 0 & $f\left(\frac{x+y}{k}\right) = \frac{f(x)+f(y)}{k}$ $(k \in R, k \neq 0, 2)$. Show that f(x) is either a zero or an odd linear function.
Sol.

10. Let $\frac{f(x)-f(y)}{2} = \frac{f(y)-a}{2}$ + xy for all real x and y. If f(x) is differentiable and $f'(0)$ exists for all real permissible values of 'a' and is equal to $\sqrt{5a-1-a^2}$. Prove that f(x) is positive for all real x.
Sol.

11. If $f(x) = \begin{vmatrix} (x-a)^4 & (x-a)^3 & 1 \\ (x-b)^4 & (x-b)^3 & 1 \\ (x-c)^4 & (x-c)^3 & 1 \end{vmatrix}$ then

$f'(x) = \lambda \cdot \begin{vmatrix} (x-a)^4 & (x-a)^2 & 1 \\ (x-b)^4 & (x-b)^2 & 1 \\ (x-c)^4 & (x-c)^2 & 1 \end{vmatrix}$. Find the value of λ.
Sol.

12. Let $f(x) = \begin{vmatrix} a+x & b+x & c+x \\ \ell+x & m+x & n+x \\ p+x & q+x & r+x \end{vmatrix}$. Show that $f''(x) = 0$ and that $f(x) = f(0) + kx$ where k denotes the sum of all the co–factors of the elements in f(0).
Sol.

13. $\lim_{x \to 0}\left[\frac{1}{x\sin^{-1}x} - \frac{1-x^2}{x^2}\right]$
Sol.

14. $\lim_{x \to 0}\frac{x\cos x - \ln(1+x)}{x^2}$
Sol.

15. If $\lim_{x\to a}\dfrac{a^x - x^a}{x^x - a^a}$ find 'a'.

Sol.

16. $\lim_{x\to 0}\dfrac{1+\sin x-\cos x+\ln(1-x)}{x.\tan^2 x}$

Sol.

17. Determine the values of a, b and c so that $\lim_{x\to 0}\dfrac{(a+b\cos x)x-c\sin x}{x^5}=1$.

Sol.

18. $\lim_{x\to\frac{\pi}{2}}\dfrac{\sin x-(\sin x)^{\sin x}}{1-\sin x+\ln(\sin x)}$

Sol.

19. $\lim_{x\to 0}\dfrac{3x\ln\left(\dfrac{\sin x}{x}\right)^2+x^3}{(x-\sin x)(1-\cos x)}$

Sol.

20. Find the value of f(0) so that the function $f(x)=\dfrac{1}{x}-\dfrac{2}{e^{2x}-1}$, $x\neq 0$ is continuous at x = 0 & examine the differentiability of f(x) at x = 0.

Sol.

21. If $\lim_{x\to 0}\dfrac{a\sin x-bx+cx^2+x^3}{2x^2.\ln(1+x)-2x^3+x^4}$ exists & is finite, find the vales of a, b, c & the limit.

Sol.

22. Evaluate : $\lim_{x\to 0}\dfrac{x^{6000}-(\sin x)^{6000}}{x^2.(\sin x)^{6000}}$

Sol.

23. If $\lim_{x\to 0}\dfrac{1-\cos x.\cos 2x.\cos 3x.....\cos nx}{x^2}$ has the value equal to 253, find the value of n (where $n\in N$)

Sol.

EXERCISE – V | JEE PROBLEMS

1. If $f(x) = \frac{x^2 - x}{x^2 + 2x}$, then find the domain and the range of f. Show that f is one-one. Also find the function $\frac{df^{-1}(x)}{dx}$ and its domain. **[REE 99,6]**

Sol.

2. (a) If $x^2 + y^2 = 1$ then **[JEE 2000 (Scr.), 1]**

(A) $yy'' - 2(y')^2 + 1 = 0$ (B) $yy'' + (y')^2 + 1 = 0$

(C) $yy'' - (y')^2 - 1 = 0$ (D) $yy'' + 2(y')^2 + 1 = 0$

Sol.

(b) Suppose $p(x) = a_0 + a_1 x + a_2x^2 + \ldots\ldots + a_nx^n$. If $|p(x)| \le |e^{x-1} - 1|$ for all $x \ge 0$ prove that $|a_1 + 2a_2 + \ldots + na_n| \le 1$. **[JEE 2000 (Mains), 5]**

Sol.

3. (a) If $\ell n(x + y) = 2xy$, then $y'(0) =$ **[JEE 2004 (Scr.)]**

(A) 1 (B) –1 (C) 2 (D) 0

Sol.

(b) $f(x) = \begin{cases} b\sin^{-1}\left(\frac{x+c}{2}\right), & -\frac{1}{2} < x < 0 \\ \frac{1}{2} & \text{at } x = 0 \\ \frac{e^{ax/2} - 1}{x}, & 0 < x < \frac{1}{2} \end{cases}$ **[JEE 2004, 4]**

If f(x) is differentiable at x = 0 and $|c| < 1/2$ then find the value of 'a' and prove that $64b^2 = 4 - c^2$.

Sol.

4. (a) If y = y(x) and it follows the relation $x \cos y + y \cos x = \pi$, then $y''(0)$ **[JEE 2005 (Scr.)]**

(A) 1 (B) –1 (C) π (D) $-\pi$

Sol.

(b) If P(x) is a polynomial of degree less than or equal to 2 and S is the set of all such polynomials so that $P(1) = 1$, $P(0) = 0$ and $P'(x) > 0\ \forall\ x \in [0, 1]$, then

(A) $S = \phi$

(B) $S = \{(1 - a)x^2 + ax,\ 0 < a < 2\}$

(C) $(1 - a)x^2 + ax,\ a \in (0, \infty)$

(D) $S = \{(1 - a)x^2 + ax,\ 0 < a < 1\}$

Sol.

(c) If f(x) is a continuous and differentiable function and $f(1/n) = 0,\ \forall\ n \ge 1$ and $n \in I$, then

(A) $f(x) = 0,\ x \in (0, 1]$

(B) $f(0) = 0,\ f'(0) = 0$

(C) $f'(x) = 0 = f''(x),\ x \in (0, 1]$

(D) $f(0) = 0$ and $f'(0)$ need not to be zero

Sol.

(d) If $f(x - y) = f(x) \cdot g(y) - f(y) \cdot g(x)$ and $g(x - y) = g(x) \cdot g(y) + f(x) \cdot f(y)$ for all $x, y \in R$. If right hand derivative at x = 0 exists for f(x). Find derivative of g(x) and x = 0. **[JEE 2005 (Mains), 4]**

Sol.

5. For x > 0, $\lim_{x\to 0}\left((\sin x)^{1/x}+(1/x)^{\sin x}\right)$ is **[JEE 2006, 3]**
(A) 0 (B) –1 (C) 1 (D) 2
Sol.

6. $\frac{d^2x}{dy^2}$ equals **[JEE 2007, 3]**
(A) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (B) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$
(C) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (D) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$
Sol.

7. (a) Let g(x) = ℓn f(x) where f(x) is a twice differentiable positive function on (0, ∞) such that f(x + 1) = x f(x). Then for N = 1, 2, 3 ;
$g''\left(N+\frac{1}{2}\right) - g''\left(\frac{1}{2}\right) =$ **[JEE 2008, 3 + 3]**
(A) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+.....+\frac{1}{(2N-1)^2}\right\}$
(B) $4\left\{1+\frac{1}{9}+\frac{1}{25}+.....+\frac{1}{(2N-1)^2}\right\}$
(C) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+.....+\frac{1}{(2N+1)^2}\right\}$
(D) $4\left\{1+\frac{1}{9}+\frac{1}{25}+.....+\frac{1}{(2N+1)^2}\right\}$
Sol.

(b) Let f and g be real valued functions defined on interval (–1, 1) such that g″(x) is continuous, g(0) ≠ 0, g′(0) = 0, g″(0) ≠ 0, and f(x) = g(x) sin x.

Statement–1 : $\lim_{x\to 0}$ [g(x) cot x – g(0) cosec x] = f″(0).
and
Statement–2 : f′(0) = g(0).
(A) Statement (1) is correct and statement (2) is correct and statement (2) is correct explanation for (1)
(B) Statement (1) is correct and statement (2) is correct and statement (2) is NOT correct explanation for (1)
(C) Statement (1) is true but (2) is false
(D) Statement (1) is false but (2) is true
Sol.

8. If the function f(x) = $x^3 + e^{x/2}$ and g(x) = f^{-1} (x), then the value of g′(1) is **[JEE 2009]**
Sol.

9. Let $f(\theta)=\sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4}<\theta<\frac{\pi}{4}$.
Then the value of $\frac{d}{d(\tan\theta)}$ (f(θ)) is **[JEE 2011]**
Sol.

10. Let y'(x) + y(x)g'(x) = g(x)g'(x), y(0) = 0, x ∈ R, where f'(x) denotes $\frac{d\,f(x)}{dx}$ and g(x) is a given non-constant differentiable function on R with g(0) = g(2) = 0. Then the value of y(2) is **[JEE 2011]**
Sol.