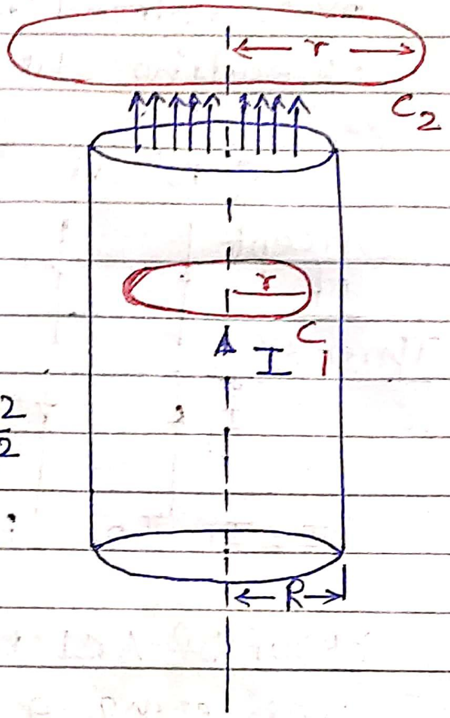
MAGNETIC FIELD- LECTURE-21

APLICATIONS OF AMPERE CIRCUITAL LAW

A. Magnetic field due to cylindrical conductor carrying uniformly distributed current.



Case-I Applying ACL over loop C_1. $(r<R)$

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I'$$

$$J = \text{Current density} = \frac{I}{\pi R^2}$$

$$\therefore I' = J \times \pi r^2 = \frac{I}{\cancel{\pi} R^2} \times \cancel{\pi} r^2 = \frac{I r^2}{R^2}$$

$$\int_0^{2\pi r} B\,dl\cos 0^\circ = \mu_0 \frac{I \times r^2}{R^2}$$

$$B \times 2\pi \cancel{r} = \frac{\mu_0 I}{R^2} \times r^{\cancel{2}}$$

$$B = \left(\frac{\mu_0 I}{2\pi R^2}\right) r$$

$\therefore$ Magnetic field at interior point

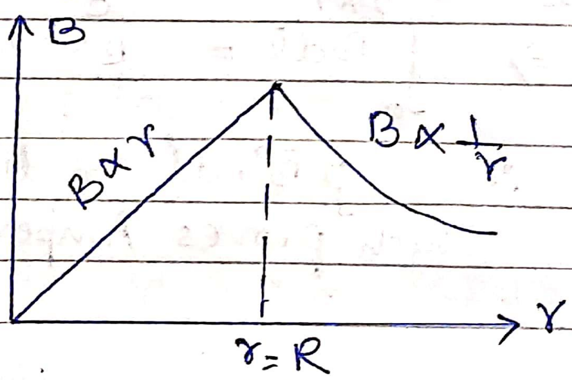
$$B \propto r$$

Case-II Applying ACL over loop C_2. $(r>R)$

$$\oint_{C_2} \vec{B}\cdot\vec{dl} = \mu_0 I \Rightarrow \int_0^{2\pi r} B\,dl\cos 0^\circ = \mu_0 I$$

$$\therefore B \times 2\pi r = \mu_0 I$$

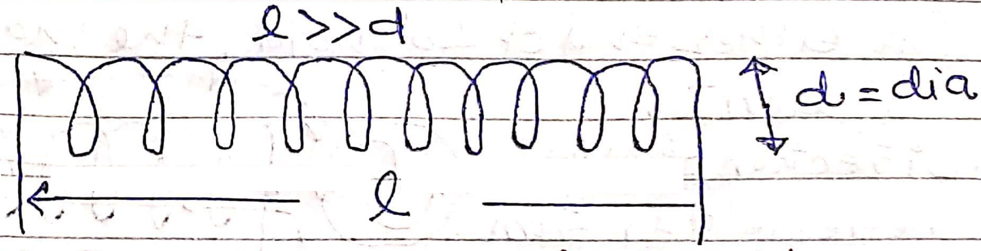
$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$



B. SOLENOID

It is a long coil made up of an insulated conducting wire wound in the form of helix.

$l >> d$

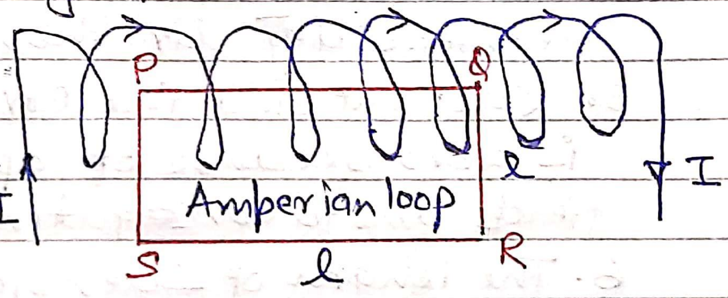


Magnetic field at the Core of solenoid –

Let us consider a solenoid of

n = number of turns / length

I = Current



Considering a square amperian loop of length l ⇒ applying ACL over this loop –

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 \times \underbrace{(n \times l) \times I}_{\text{Total current Threading loop.}}$$

$$\int_{PQ} \vec{B}\cdot\vec{dl} + \int_{QR} \vec{B}\cdot\vec{dl} + \int_{RS} \vec{B}\cdot\vec{dl} + \int_{SP} \vec{B}\cdot\vec{dl} = \mu_0 n l I$$

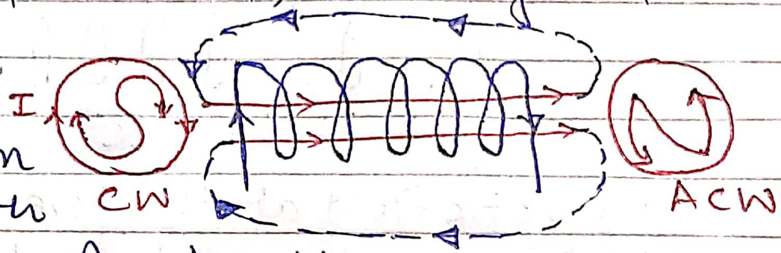
$$\int_0^l B\,dl \cos 0^\circ + \int_0^l B\,dl \cos 90^\circ + \int_0^l 0 \cdot \vec{dl} + \int_0^l B\,dl \cos 90^\circ = \mu_0 n l I$$

$$B \times l + 0 + 0 + 0 = \mu_0 n l I$$

$$\therefore \boxed{B = \mu_0 n I}$$

Important Points.

1. Inside the core of solenoid, the magnetic field is uniform.
2. On either end of solenoid, the magnetic field is $\frac{1}{2}\mu_0 n I$.
3. Direction of Magnetic field is from South pole to North pole inside the core of solenoid.



4. When core of solenoid is made of material of relative permeability μ_r then magnetic field $= \mu_0 \mu_r n I$.
5. Just outside the core of solenoid, magnetic field is zero because of opposite orientation of Magnetic fields due to consequtive loops.
6. The length of solenoid decreases, when current is passed through it. This is due to induction of North and South poles on either ends.

TOROID : An endless solenoid is called a toroid. This is formed by joining both the ends of solenoid.

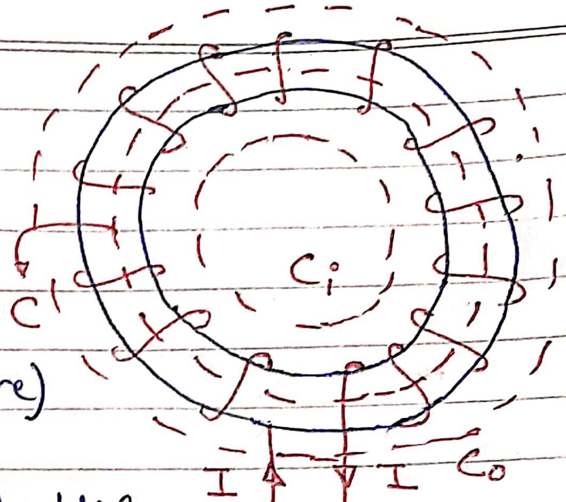
Magnetic field due to Current carrying Toroid:-

Let us consider a toroid carrying current I.

r_i = Inner Radius r_o = Outer Radius

$r_m = \frac{r_i + r_o}{2}$ (Mean Radius) n = No. of turns / length

To find out Magnetic field inside the core, Applying ACL over loop C.



$$\oint_C \vec{B}\cdot\vec{dl} = \mu_0 n \times (2\pi r_m) \times I$$

$$\int_0^{2\pi r} B\,dl\cos 0^\circ = \mu_0 n (2\pi r) I$$

$$B \times \cancel{2\pi r} = \mu_0 n I (\cancel{2\pi r})$$

$$\boxed{B_c = \mu_0 n I} \text{ (M.F. at Core)}$$

To find the magnetic field inside the Toroid applying ACL over C_i (amperian loop)

$$\oint_{C_i} \vec{B}\cdot\vec{dl} = \mu_0 \times 0 \Rightarrow \boxed{B_i = 0}$$

To find the Magnetic field outside the toroid applying ACL over C_o (amperian loop).

$$\oint_{C_o} \vec{B}\cdot\vec{dl} = \mu_0 \times 0 \Rightarrow \boxed{B_o = 0}$$

If core of Toroid is made up of material with relative permeability μ_r, the magnetic field at the core of Toroid is –

$$\boxed{B_c = \mu_0 \mu_r n I}$$