TEST YOUR SKILLS-2

Time : 1 hr. **M.M. : 40**

Section - A (1 mark each)

1. If $f = \{(5, 2), (6, 3)\}$, $g = \{(2, 5), (3, 6)\}$. Write fog.
2. Let R be the equivalence relation in the set Z of integers given by $R = \{(a, b) : 2 \text{ divides } a - b\}$. Write the equivalence class $[0]$.
3. If $A = \{a, b, c, d\}$ and $f = \{(a, b), (b, d), (c, a), (d, c)\}$, show that f is one-one from A onto A, find f^{-1}.
4. Find the number of binary operations on the set $\{a, b\}$.

Section - B (4 marks each)

5. Let $f: N \to R$ be the function defined by $f(x) = \dfrac{2x-1}{2}$ and $g : Q \to R$ be another function defined by $g(x) = x + 2$. Find $fog(x)$ and $gof(x)$ and hence find the value of $fog\left(\dfrac{3}{2}\right) + gof(-1)$.
6. Binary operation on the set R is defined as $a * b = \dfrac{a+b}{2}\ \forall\ a, b \in R$. Find whether $*$ is commutative and associative or not.
7. If $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}} + 2$ and $g(x) = \log_e\left(\dfrac{x-1}{3-x}\right)^{1/2}$, show that f and g are inverse of each other.
8. Let L be the set of all lines in XY plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1 \text{ is parallel to } L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $2x + 3y = 5$.
9. Relation R in the set R of real numbers, defined as $R = \{(a, b) : a \le b^2\}$. Find whether R is reflexive, symmetric, transitive or not.
10. Let $f: N \to N$ be defined by

$$f(n) = \begin{cases} \dfrac{n+1}{2}; & \text{if } n \text{ is odd} \\ \dfrac{n}{2}; & \text{if } n \text{ is even} \end{cases} \quad \text{for all } n \in N$$

Check the injectivity and surjectivity of f.

Section - C (6 marks each)

11. Relation S in the set $A = \{x \in Z; 0 \le x \le 12\}$ given by $S = \{(a, b) : a, b \in Z, |a - b| \text{ is divisible by } 4\}$. Show that S is an equivalence relation. Find the set of all elements related to 1.
12. Let $f: N \to R$ be function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f: N \to S$, where S is range of f is invertible. Find, also the inverse of f.