SPECTRUM
CAREER INSTITUTE
JEE/NEET EXPERT

MATHEMATICS

BY

Dr. ANOOP DIXIT

B.Tech (Mech) M.Tech (P&I)
PhD(NIT Kurukshetra)

DIFFERENTIATION (ASSIGNMENT-I)

Student's Name: _____

Batch: _____

Differentiation

Introduction

Problem: 1 If $f(2)=4,\ f'(2)=1$, then $\lim_{x\to 2}\frac{xf(2)-2f(x)}{x-2}=$ **[Rajasthan PET 1995, 2000]**

(a) 1 (b) 2 (c) 3 (d) – 2

Problem: 2 If $f(x+y)=f(x).f(y)$ for all x and y and $f(5)=2,\ f'(0)=3$, then $f'(5)$ will be

[IIT 1981; Karnataka CET 2000; UPSEAT 2002; MP PET 2002; AIEEE 2002]

(a) 2 (b) 4 (c) 6 (d) 8

Problem: 3 If $f(a)=3, f'(a)=-2, g(a)=-1, g'(a)=4$, then $\lim_{x\to a}\frac{g(x)f(a)-g(a)f(x)}{x-a}=$ **[MP PET 1997]**

(a) – 5 (b) 10 (c) – 10 (d) 5

Problem: 4 If $5f(x)+3f\left(\frac{1}{x}\right)=x+2$ and $y=xf(x)$ then $\left(\frac{dy}{dx}\right)_{x=1}$ is equal to

(a) 14 (b) $\frac{7}{8}$ (c) 1 (d) None of these

Problem: 5 The derivative of $f(x)=|x|^3$ at $x=0$ is **[Rajasthan PET 2001; Haryana CEE 2002]**

(a) 0 (b) 1 (c) –1 (d) Not defined

Problem: 6 The first derivative of the function $(\sin 2x\cos 2x\cos 3x+\log_2 2^{x+3})$ with respect to x at $x=\pi$ is

(a) 2 (b) –1 (c) $-2+2^{\pi}\log_e 2$ (d) $-2+\log_e 2$

Problem: 7 If $y=|\cos x|+|\sin x|$ then $\frac{dy}{dx}$ at $x=\frac{2\pi}{3}$ is

(a) $\frac{1-\sqrt{3}}{2}$ (b) 0 (c) $\frac{1}{2}(\sqrt{3}-1)$ (d) None of these

Problem: 8 If $f(x)=\log_x(\log x)$, then $f'(x)$ at $x=e$ is

[IIT 1985; Rajasthan PET 2000; MP PET 2000; Karnataka CET 2002]

(a) e (b) $1/e$ (c) 1 (d) None of these

Problem: 9 If $f(x)=|\log x|$, then for $x\neq 1, f'(x)$ equals

(a) $\frac{1}{x}$ (b) $\frac{1}{|x|}$ (c) $\frac{-1}{x}$ (d) None of these

Problem: 10 $\frac{d}{dx}\left[\log\left\{e^x\left(\frac{x-2}{x+2}\right)^{3/4}\right\}\right]$ equals to

(a) 1 (b) $\frac{x^2+1}{x^2-4}$ (c) $\frac{x^2-1}{x^2-4}$ (d) $e^x\frac{x^2-1}{x^2-4}$

Problem: 11 If $x=\exp\left\{\tan^{-1}\left(\frac{y-x^2}{x^2}\right)\right\}$ then $\frac{dy}{dx}$ equals **[MP PET 2002]**

(a) $2x[1+\tan(\log x)]+x\sec^2(\log x)$ (b) $x[1+\tan(\log x)]+\sec^2(\log x)$

(c) $2x[1+\tan(\log x)]+x^2\sec^2(\log x)$ (d) $2x[1+\tan(\log x)]+\sec^2(\log x)$

Problem: 12 If $y=\sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)+\sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$, then $\frac{dy}{dx}=$ **[UPSEAT 1999; AMU 2002]**

(a) 0 (b) $\frac{1}{\sqrt{x}+1}$ (c) 1 (d) None of these

Problem: 13 $\frac{d}{dx}\tan^{-1}\left[\frac{\cos x-\sin x}{\cos x+\sin x}\right]$ **[AISSE 1985, 87; DSSE 1982, 84; MNR 1985; Karnataka CET 2002; Rajasthan PET 2002, 03]**

(a) $\frac{1}{2(1+x^2)}$ (b) $\frac{1}{1+x^2}$ (c) 1 (d) – 1

Problem: 14 $\frac{d}{dx}\left[\sin^2\cot^{-1}\left\{\sqrt{\frac{1-x}{1+x}}\right\}\right]$ equal **[MP PET 2002; EAMCET 1996]**

(a) –1 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 1

Problem: 15 If $y=\cos^{-1}\left(\frac{5\cos x-12\sin x}{13}\right)$, $x\in\left(0,\frac{\pi}{2}\right)$, then $\frac{dy}{dx}$ is equal to

(a) 1 (b) – 1 (c) 0 (d) None of these

Problem: 16 $\frac{d}{dx}\cosh^{-1}(\sec x)=$ **[Rajasthan PET 1997]**

(a) $\sec x$ (b) $\sin x$ (c) $\tan x$ (d) $\operatorname{cosec} x$

Problem: 17 $\frac{d}{dx}\left[\left(\frac{\tan^2 2x-\tan^2 x}{1-\tan^2 2x\tan^2 x}\right)\cot 3x\right]$ **[AMU 2000]**

(a) $\tan 2x\tan x$ (b) $\tan 3x\tan x$ (c) $\sec^2 x$ (d) $\sec x\tan x$

Problem: 18 If $f(x)=\cot^{-1}\left(\frac{x^x-x^{-x}}{2}\right)$, then $f'(1)$ is equal to **[Rajasthan PET 2000]**

(a) – 1 (b) 1 (c) $\log 2$ (d) $-\log 2$

Problem: 19 If $y=(1+x)(1+x^2)(1+x^4).....(1+x^{2^n})$ then $\frac{dy}{dx}$ at $x=0$ is

(a) 1 (b) – 1 (c) 0 (d) None of these

Problem: 20 If $f(x)=\cos x.\cos 2x.\cos 4x.\cos 8x.\cos 16x$ then $f'\left(\frac{\pi}{4}\right)$ is

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) None of these

Problem: 21 If $xe^{xy}=y+\sin^2 x$, then at $x=0$, $\frac{dy}{dx}=$ **[IIT 1996]**

(a) – 1 (b) – 2 (c) 1 (d) 2

Problem: 22 If $\sin(x+y)=\log(x+y)$, then $\frac{dy}{dx}=$ **[Karnataka CET 1993; Rajasthan PET 1989, 1992; Roorkee 2000]**

(a) 2 (b) – 2 (c) 1 (d) – 1

Problem: 23 If $\ln(x+y)=2xy$, then $y'(0)=$ **[IIT Screening 2004]**

(a) 1 (b) – 1 (c) 2 (d) 0

Problem: 24 If $x^m y^n = 2(x+y)^{m+n}$, the value of $\frac{dy}{dx}$ is **[MP PET 2003]**

(a) $x+y$ (b) $\frac{x}{y}$ (c) $\frac{y}{x}$ (d) $x-y$

Problem: 25 If $y=(\sin x)^{\tan x}$, then $\frac{dy}{dx}$ is equal to **[IIT 1994; Rajasthan PET 1996]**

(a) $(\sin x)^{\tan x}.(1+\sec^2 x.\log\sin x)$ (b) $\tan x.(\sin x)^{\tan x-1}.\cos x$

(c) $(\sin x)^{\tan x}.,\sec^2 x\log\sin x$ (d) $\tan x.(\sin x)^{\tan x-1}$

Problem: 26 If $x=a(\cos\theta+\theta\sin\theta)$, $y=a(\sin\theta-\theta\cos\theta)$, $\frac{dy}{dx}=$ **[DCE 1999]**

(a) $\cos\theta$ (b) $\tan\theta$ (c) $\sec\theta$ (d) $\text{cosec}\,\theta$

Problem: 27 If $\cos x=\frac{1}{\sqrt{1+t^2}}$ and $\sin y=\frac{t}{\sqrt{1+t^2}}$, then $\frac{dy}{dx}=$ **[MP PET 1994]**

(a) – 1 (b) $\frac{1-t}{1+t^2}$ (c) $\frac{1}{1+t^2}$ (d) 1

Problem: 28 If $x=\frac{1-t^2}{1+t^2}$ and $y=\frac{2t}{1+t^2}$, then $\frac{dy}{dx}=$ **[Karnataka CET 2000]**

(a) $\frac{-y}{x}$ (b) $\frac{y}{x}$ (c) $\frac{-x}{y}$ (d) $\frac{x}{y}$

Problem: 29 If $y=\sqrt{x+\sqrt{x+\sqrt{x+.....\text{ to }\infty}}}$ then $\frac{dy}{dx}=$ **[Rajasthan PET 2002]**

(a) $\frac{x}{2y-1}$ (b) $\frac{2}{2y-1}$ (c) $\frac{-1}{2y-1}$ (d) $\frac{1}{2y-1}$

Problem: 30 If $y=x^{x^{x.....\infty}}$, then $x(1-y\log_e x)\frac{dy}{dx}$ is **[DCE 2000]**

(a) x^2 (b) y^2 (c) xy^2 (d) None of these

Problem: 31 If $y=x^2+\cfrac{1}{x^2+\cfrac{1}{x^2+\cfrac{1}{x^2+.....\infty}}}$, then $\frac{dy}{dx}=$

(a) $\frac{2xy}{2y-x^2}$ (b) $\frac{xy}{y+x^2}$ (c) $\frac{xy}{y-x^2}$ (d) $\frac{2x}{2+\frac{x^2}{y}}$

Problem: 32 If $x=e^{y+e^{y+.....\text{ to }\infty}}$, then $\frac{dy}{dx}$ is

(a) $\frac{1+x}{x}$ (b) $\frac{1}{x}$ (c) $\frac{1-x}{x}$ (d) $\frac{x}{1+x}$

Problem: 33 If $f(x)=|x-2|$ and $g(x)=f(f(x))$, then for $x>20$, $g'(x)$ equals

(a) –1 (b) 1 (c) 0 (d) None of these

Problem: 34 If g is inverse of f and $f'(x)=\frac{1}{1+x^n}$, then $g'(x)$ equals

(a) $1+x^n$ (b) $1+[f(x)]^n$ (c) $1+[g(x)]^n$ (d) None of these

Problem: 35 The differential coefficient of $\tan^{-1}\frac{2x}{1-x^2}$ *w.r.t.* $\sin^{-1}\frac{2x}{1+x^2}$ is

[Roorkee 1966; BIT Mesra 1996; Karnataka CET 1994; MP PET 1999; UPSEAT 1999, 2001]

(a) 1 (b) –1 (c) 0 (d) None of these

Problem: 36 The first derivative of the function $\left[\cos^{-1}\left(\sin\frac{\sqrt{1+x}}{2}\right)+x^x\right]$ with respect to x at $x=1$ is

(a) $\frac{3}{4}$ (b) 0 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

Problem: 37 If $y=\left(x+\sqrt{1+x^2}\right)^n$, then $(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}$ is **[AIEEE 2002]**

(a) n^2y (b) $-n^2y$ (c) $-y$ (d) $2x^2y$

Problem: 38 If $f(x)=x^n$, then the value of $f(1)-\frac{f'(1)}{1!}+\frac{f''(1)}{2!}-\frac{f'''(1)}{3!}+.....+\frac{(-1)^n f^n(1)}{n!}$ is **[AIEEE 2003]**

(a) 2^n (b) 2^{n-1} (c) 0 (d) 1

Problem: 39 If $f(x)=\tan^{-1}\left\{\frac{\log\left(\frac{e}{x^2}\right)}{\log(ex^2)}\right\}+\tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$, then $\frac{d^ny}{dx^n}$ is $(n\geq 1)$

(a) $\tan^{-1}\{(\log x)^n\}$ (b) 0 (c) 1/2 (d) None of these

Problem: 40 If $f(x)=(\cos x+i\sin x)(\cos 3x+i\sin 3x).....(\cos(2n-1)x+i\sin(2n-1)x)$, then $f''(x)$ is equal to

(a) $n^2f(x)$ (b) $-n^4f(x)$ (c) $-n^2f(x)$ (d) $n^4f(x)$

Problem: 41 If $y=\frac{x^4}{x^2-3x+2}$, then for $n>2$ the value of y_n is equal to

(a) $(-1)^n n![16(x-2)^{-n-1}-(x-1)^{-n-1}]$ (b) $(-1)^n n![16(x-2)^{-n-1}+(x-1)^{-n-1}]$

(c) $n![16(x-2)^{-n-1}+(x-1)^{-n-1}]$ (d) None of these

Problem: 42 If $f_r(x),g_r(x),h_r(x),r=1,2,3$ are polynomials in x such that $f_r(a)=g_r(a)=h_r(a),r=1,2,3$ and

$F(x)=\begin{vmatrix}f_1(x) & f_2(x) & f_3(x)\\ g_1(x) & g_2(x) & g_3(x)\\ h_1(x) & h_2(x) & h_3(x)\end{vmatrix}$, then find $F'(x)$ at $x=a$ **[IIT 1985]**

(a) 0 (b) $f_1(a)g_2(a)h_3(a)$ (c) 1 (d) None of these

Problem: 43 Let $f(x)=\begin{vmatrix}x^3 & \sin x & \cos x\\ 6 & -1 & 0\\ 1 & p^2 & p^3\end{vmatrix}$ where p is a constant. Then $\frac{d^3}{dx^3}[f(x)]$ at $x=0$ is **[IIT 1997]**

(a) p (b) $p+p^2$ (c) $p+p^3$ (d) Independent of p

Problem: 44 If $F(x)=\int_{x^2}^{x^3}\log t\,dt\,(x>0)$, then $F'(x)=$ **[MP PET 2001]**

(a) $(9x^2-4x)\log x$ (b) $(4x-9x^2)\log x$ (c) $(9x^2+4x)\log x$ (d) None of these

Problem: 45 If $x=\int_0^y\frac{1}{\sqrt{1+4t^2}}dt$, then $\frac{d^2y}{dx^2}$ is

(a) $2y$ (b) $4y$ (c) $8y$ (d) $6y$

Problem: 46 If $y=x^2e^x$, then value of y_n is

(a) $\{x^2-2nx+n(n-1)\}e^x$ (b) $\{x^2+2nx+n(n-1)\}e^x$ (c) $\{x^2+2nx-n(n-1)\}e^x$ (d) None of these

SPECTRUM CAREER INSTITUTE **DIFFERENTIATION-1**

Problem: 47 If $y = x^2 \log x$, then value of y_n is

(a) $\frac{(-1)^{n-1}(n-3)!}{x^{n-2}}$ (b) $\frac{(-1)^{n-1}(n-3)!}{x^{n-2}}.2$ (c) $\frac{(-1)^{n-1}(n-2)!}{x^{n-2}}$ (d) None of these