AD 271 383

THE JOHNS HOPKINS UNIVERSITY
APPLIED PHYSICS LABORATORY
8621 Georgia Avenue, Silver Spring, Maryland

Operating under Contract NOrd 7386 with the Bureau of Naval Weapons, Department of the Navy

CM-996

Copy No. 12

MAGNETIC AND GRAVITY ATTITUDE STABILIZATION OF EARTH SATELLITES

by

Robert E. Fischell

May 1961

CM-996
May 1961

Magnetic and Gravity Attitude Stabilization of Earth Satellites

by

Robert E. Fischell

THE JOHNS HOPKINS UNIVERSITY
APPLIED PHYSICS LABORATORY
8621 GEORGIA AVENUE SILVER SPRING, MARYLAND

ABSTRACT

An orbiting artificial earth satellite whose attitude in space is controlled, possesses a unique capability for acquiring scientific data. Satellite attitude stabilization also provides many important advantages in the transmission of telemeter and other data from the satellite to ground receiving stations as well as improved transmission from a ground transmitting station to the satellite.

One system for attitude stabilization is based on aligning a satellite axis along the direction of the earth's magnetic field. A second system for attitude control is based on aligning a satellite axis along the direction of the gradient of the earth's gravitational field.

The Transit 1B satellite (1960 $\gamma 2$) was the first space vehicle to be magnetically oriented. The Transit 2A satellite (1960 $\eta 2$), launched approximately two months later, also had its symmetry axis aligned along the direction of the earth's magnetic field. Data received from each of these satellites has indicated that by being magnetically stabilized, correlation could be made between solar cell performance and angle of the satellite axis with respect to the earth-sun line. Furthermore, the magnetic attitude stabilization caused the satellite's antenna to be directed downward so as to provide a favorable orientation for the receipt of radio transmission.

A research satellite currently being designed under the direction of Prof. James A. Van Allen, of the State University of Iowa, will employ magnetic attitude stabilization to

determine the directional properties of geomagnetically trapped radiation. It is felt that data acquired by this magnetically oriented satellite will significantly increase our basic knowledge concerning the origin of the auroras.

Gravity orientation of a satellite provides the opportunity to employ a directional antenna to improve signal-to-noise ratio of radio transmission to and from the satellite. The acquisition of data from a satellite containing a brilliant, flashing light can be greatly facilitated if the satellite is magnetically or gravity oriented with respect to an astronomical observatory on the earth's surface.

A description is presented of methods by which magnetic and gravity attitude stabilization systems can be established. This paper also discusses how attitude control can provide better functioning of solar power systems and can also be used to improve the satellite's thermal balance.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS AND TABLES

Figure

MAGNETIC AND GRAVITY ATTITUDE STABILIZATION OF EARTH SATELLITES

I. INTRODUCTION

Attitude control of an orbiting earth satellite can be defined as a system whereby at least one of three orthogonal axes defined in the satellite is positioned with respect to some object or a force field. The principal effort on satellite attitude control has been concerned with three particular systems: magnetic, gravity gradient, and solar attitude stabilization. Magnetic attitude stabilization is defined as the alignment of one axis of the satellite along the local direction of the earth's magnetic field. Gravity gradient attitude stabilization is characterized by having one axis of the satellite always directed toward the center of mass of the earth. Solar attitude stabilization is characterized by having one particular face of the satellite always directed toward the sun.

Controlled orientation of a satellite axis along the local direction of the earth's magnetic field provides several capabilities that are not achievable with a randomly oriented space vehicle. These advantages are as follows:

1. Control of the electrical power generating capability of solar cells mounted on the satellite.

2. Control of the satellite's thermal balance.

3. Directional control of satellite receiving or transmitting antennas.

4. The capability of directing a narrow beam, brilliant, flashing light toward an observer on the ground for the purpose of optical tracking of the satellite.

5. The ability to perform scientific experiments to determine the directional properties of charged particles and other radiation which is present in the magnetic and gravitational fields of the earth.

6. The ability to use rockets fired in orbit along an accurately known direction to alter the orbit of the satellite.

Gravity gradient attitude stabilization offers all the advantages described in items 1 through 6 above. In some cases, such as the direction of antennas and flashing lights toward ground receiving stations, gravity oriented satellites offer some advantages that are not as easily accomplished with magnetic attitude control. A magnetically oriented satellite offers a unique advantage for determining the directional properties of particles that are geomagnetically trapped in the earth's Van Allen radiation belts.

Solar attitude stabilization offers the obvious advantage of providing optimum use of solar cells for the generation of electrical power. A solar oriented satellite will produce from four to five times as much power per solar cell as compared with a randomly oriented satellite whose solar cells are equally distributed over a spherical surface. Only magnetic and gravity gradient attitude stabilization are discussed in this paper.

II. THE MAGNETIC AND GRAVITATIONAL FIELDS OF THE EARTH

In order to analyze the motions of satellites oriented in the earth's magnetic or gravitational fields it is necessary to understand the magnitude and direction of these fields at orbital altitudes. From a point of view of attitude control, the earth's magnetic field is by far the more complex both in magnitude and direction.

To a first approximation, the magnetic field of the earth is distributed as if there were a short (compared to the earth's diameter) bar magnet located at its center.

When considering magnetic attitude control of an orbiting satellite, a simplified dipole representation of the earth's magnetic field is sufficiently accurate. For this representation, the local horizontal, (H_x) and local vertical, (H_z) magnetic field components are given by:

$$H_x = \frac{Me \cos \varphi}{r_e^3} \quad \text{(oersteds)} , \tag{1}$$

and

$$H_Z = \frac{2Me \sin \varphi}{r_e^3} \quad \text{(oersteds)} , \tag{2}$$

where M_e = Strength of the earth's magnetic dipole (pole-cm),

φ = Magnetic latitude (degrees) ,

and r_e = Distance from the center of the earth (cm).

In the study of satellite attitude control, H_x and H_z individually are of little use. What is needed is the resultant (H_t) of H_x and H_z and the angle (β) between H_t and the earth's spin axis. The relations between H_x, H_z, H_t, and β are given by:

$$H_t = \sqrt{H_x^2 + H_z^2} \qquad \text{(oersteds)}\ , \tag{3}$$

and

$$\beta = \tan^{-1}\left[\frac{H_z}{H_x}\right] + \varphi \qquad \text{(degrees)}\ . \tag{4}$$

Use of these equations is based upon the simplifying assumption that the magnetic poles are coincident with the geographic poles. Figure 1 illustrates the total intensity of the earth's magnetic field as a function of altitude at the equator and at the geographic pole. Figure 2 shows the total magnetic field intensity as a function of latitude at an orbital altitude of 450 nautical miles. From Figs. 1 and 2 it can be seen that the earth's magnetic field is somewhat greater in the polar region than it is at the equator. There is sufficient field strength to obtain a significant torque when a strong magnetic dipole is contained in the satellite. Also, since modern magnetic alloys can reach maximum permeabilities at field intensities as low as 0.005 oersted, it is possible to obtain significant magnetic damping effects at altitudes even higher than 4,000 nautical miles (which is the highest altitude shown in Fig. 1.

The angle β that the satellite makes with the earth's spin axis is shown in Fig. 3 as a function of magnetic latitude. From this figure we see that there are very large

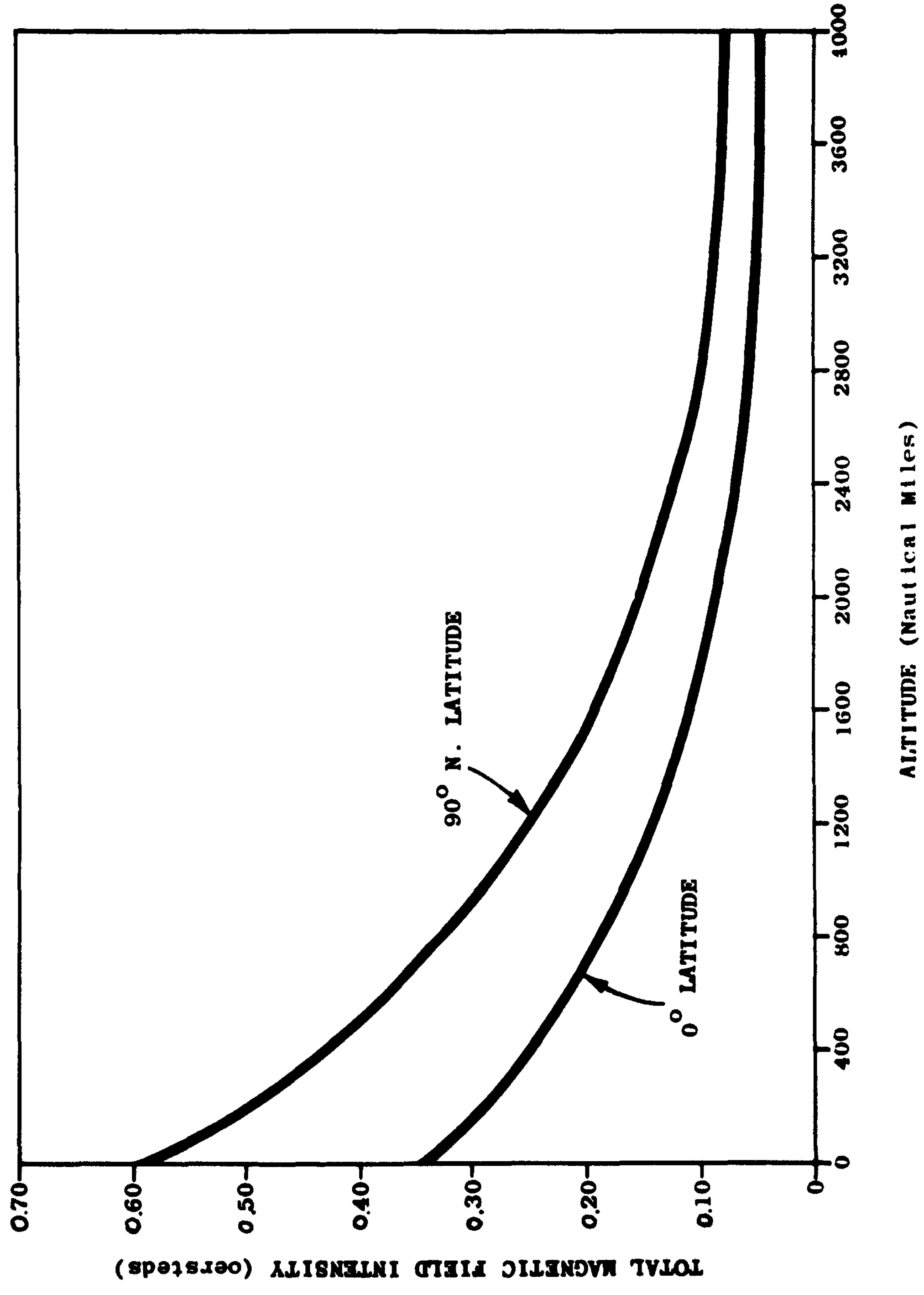

Fig. 1 THE EARTH'S TOTAL MAGNETIC FIELD INTENSITY AS A FUNCTION OF ALTITUDE AT THE EQUATOR AND THE GEOGRAPHIC NORTH POLE

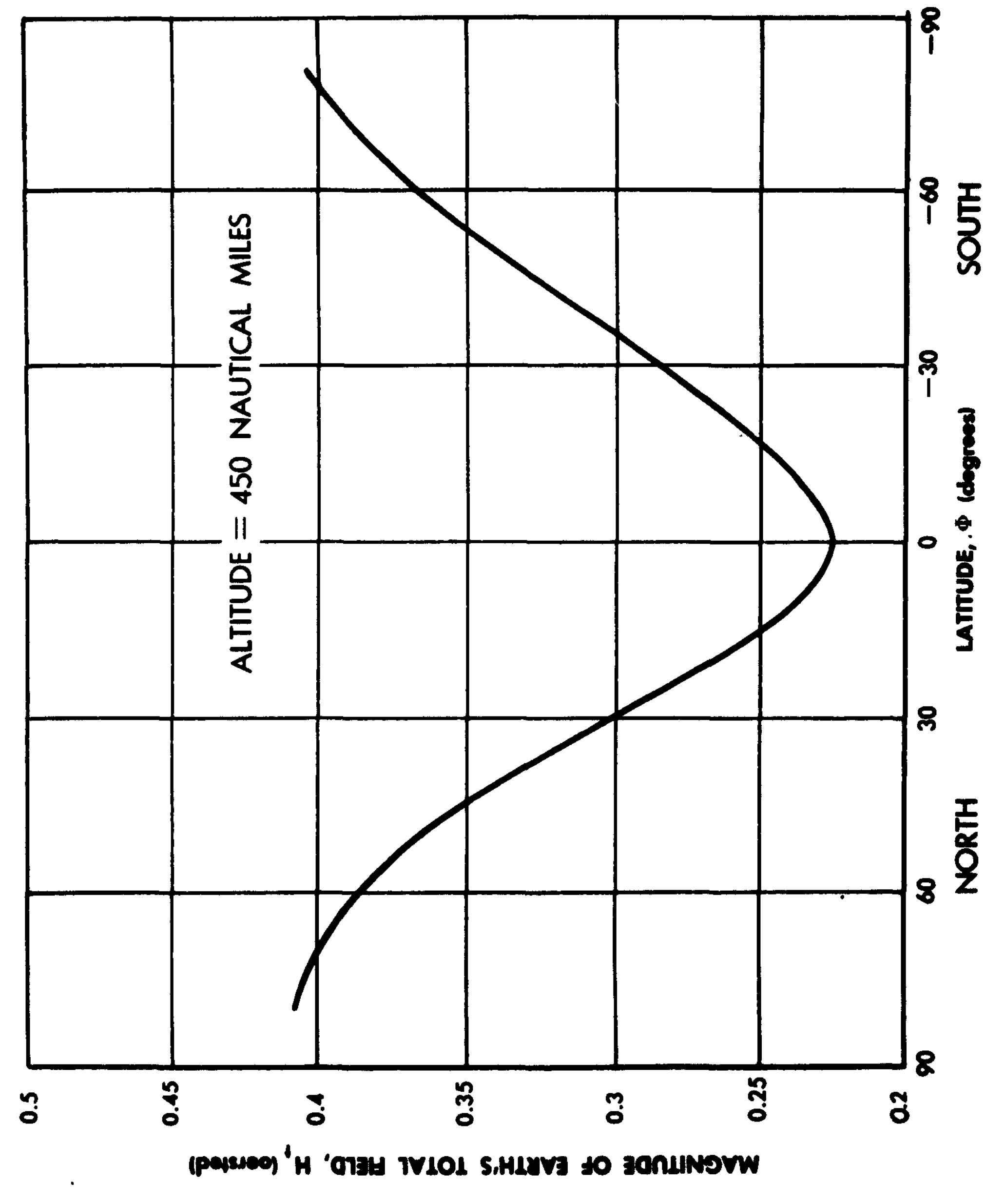

Fig. 2 THE EARTH'S TOTAL MAGNETIC FIELD INTENSITY AS A FUNCTION OF LATITUDE

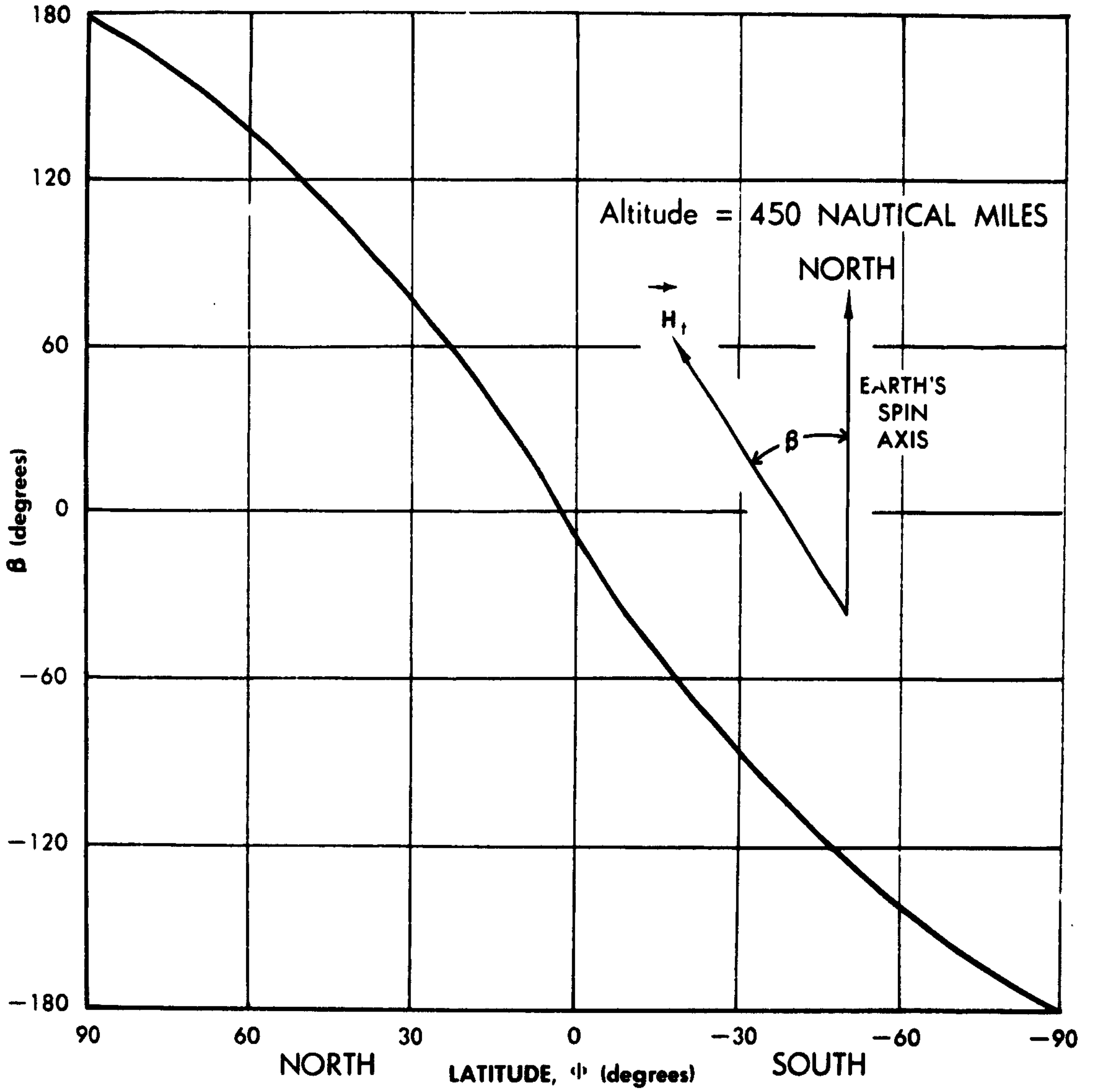

Fig. 3 ANGLE BETWEEN THE EARTH'S MAGNETIC FIELD VECTOR AND THE EARTH'S SPIN AXIS AS A FUNCTION OF LATITUDE

deviations of the angle β as the satellite moves from one magnetic pole to the other.

To analyze the angular motions of a magnetically oriented satellite in the earth's magnetic field we will use the reasonably accurate assumption that at the altitude of an orbiting satellite, the center of the magnetic dipole field is coincident with the center of mass of the earth. Using this model it will be shown that particular designs for magnetically oriented satellites will result in aligning the satellite along the local direction of the earth's magnetic field. Once the dynamics of the satellite motion are analyzed, a more accurate representation of the earth's magnetic field will be employed to determine the position of the satellite relative to the earth and to the sun. This more sophisticated model of the earth's magnetic field is a result of the work of F. T. Heuring (Ref. 1)* and is based on a magnetic dipole that lies along the line connecting the earth's north and south magnetic poles. The results obtained in using this model to determine the attitude of the satellite relative to the earth and relative to the sun are presented in another section of this report.

For the purposes of attitude control the gravitational field of the earth can be expressed in a very much simpler manner than is the case for the earth's magnetic field. The direction of gravity gradient is always well within one degree of the radius vector from satellite toward the center of mass of the earth and the magnitude of gradient (per unit mass) is expressed by:

$$\nabla\varphi = \frac{Gm}{r_e^2} , \qquad (5)$$

*All references begin on page 73.

where ϖ = earth's gravitational potential (gm cm^2 sec^{-2}),

G = gravitational constant (cm^3 gm^{-1} sec^{-2}),

m = mass of the earth (gms).

III. THE ORIENTATION OF SATELLITES

Damping of Spin and Oscillatory Motions

Before a satellite can be oriented along either the earth's magnetic or gravitational field, the spin of the satellite that is usually imparted during the launching operation must be stopped. This is a result of the fact that a spinning satellite acts like a gyroscope and will therefore be spin stabilized.

The removal of the spin energy of the satellite can be accomplished by several means. One method that has been used successfully in the Transit satellite program is a set of weights attached to the satellite by means of long cables wrapped around the equator of the satellite (Ref. 2). After a prescribed time in orbit a wire connecting two weights which are located diametrically opposite is cut and the weights are allowed to fly off, unwinding the cables as they go. When the cable is completely unwound, the weights and cable fly off from the satellite. By proper selection of the mass of the weights and the cable length the spinning of the satellite can be decreased to a low rate.

Figure 4 is a photograph of the Transit 1B satellite showing the despin weight mounted on the satellite's equator.

Another method for spin removal is based on the use of highly permeable rods of specially prepared magnetic materials (Ref. 3). This means of spin removal depends on the fact that a permeable rod when spinning in the earth's magnetic field will develop eddy current and hysteresis energy losses and by this method remove the spin energy of the satellite. In Fig. 5 is shown the spin rate of the Transit 1B satellite as

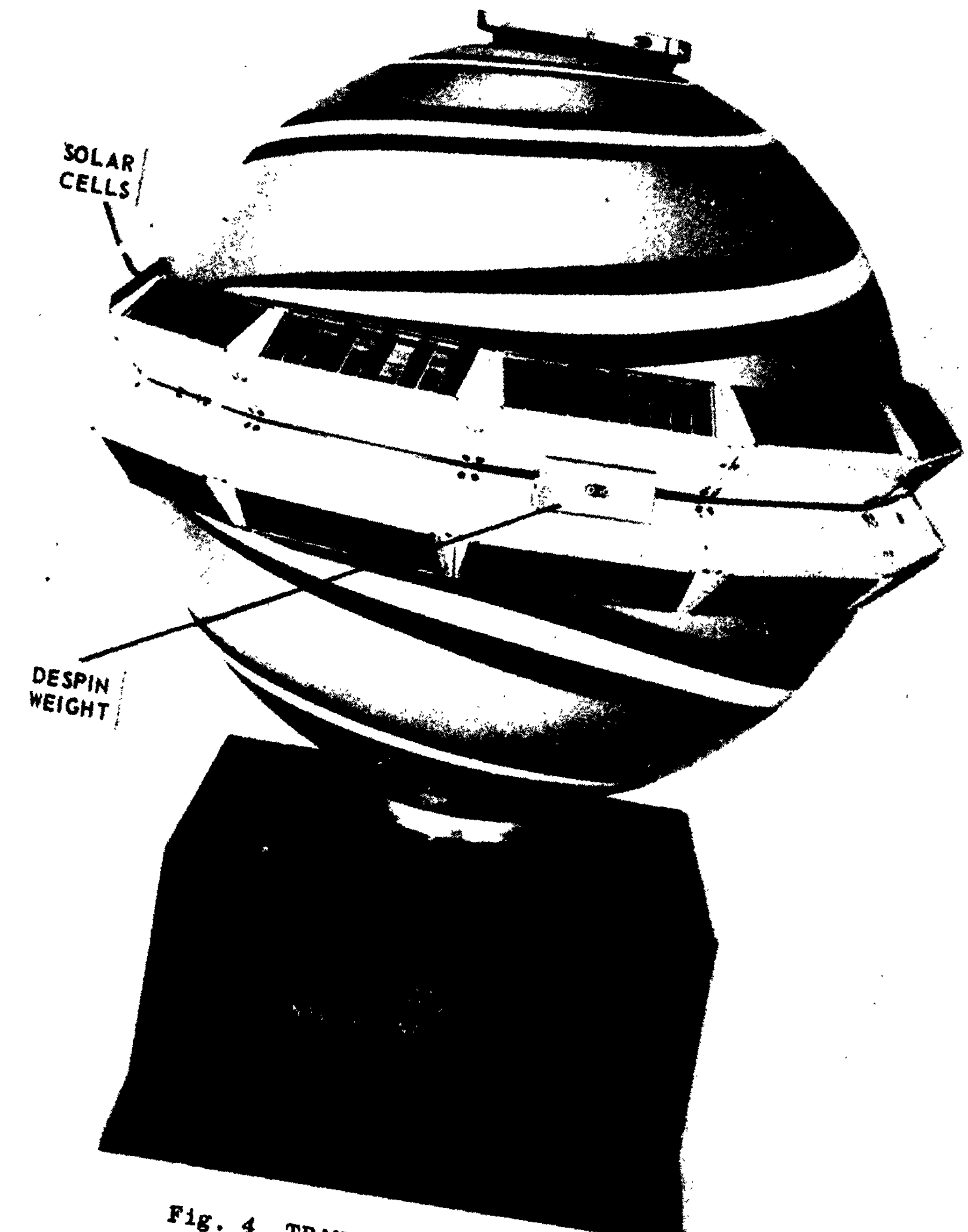

Fig. 4 TRANSIT 1B SATELLITE

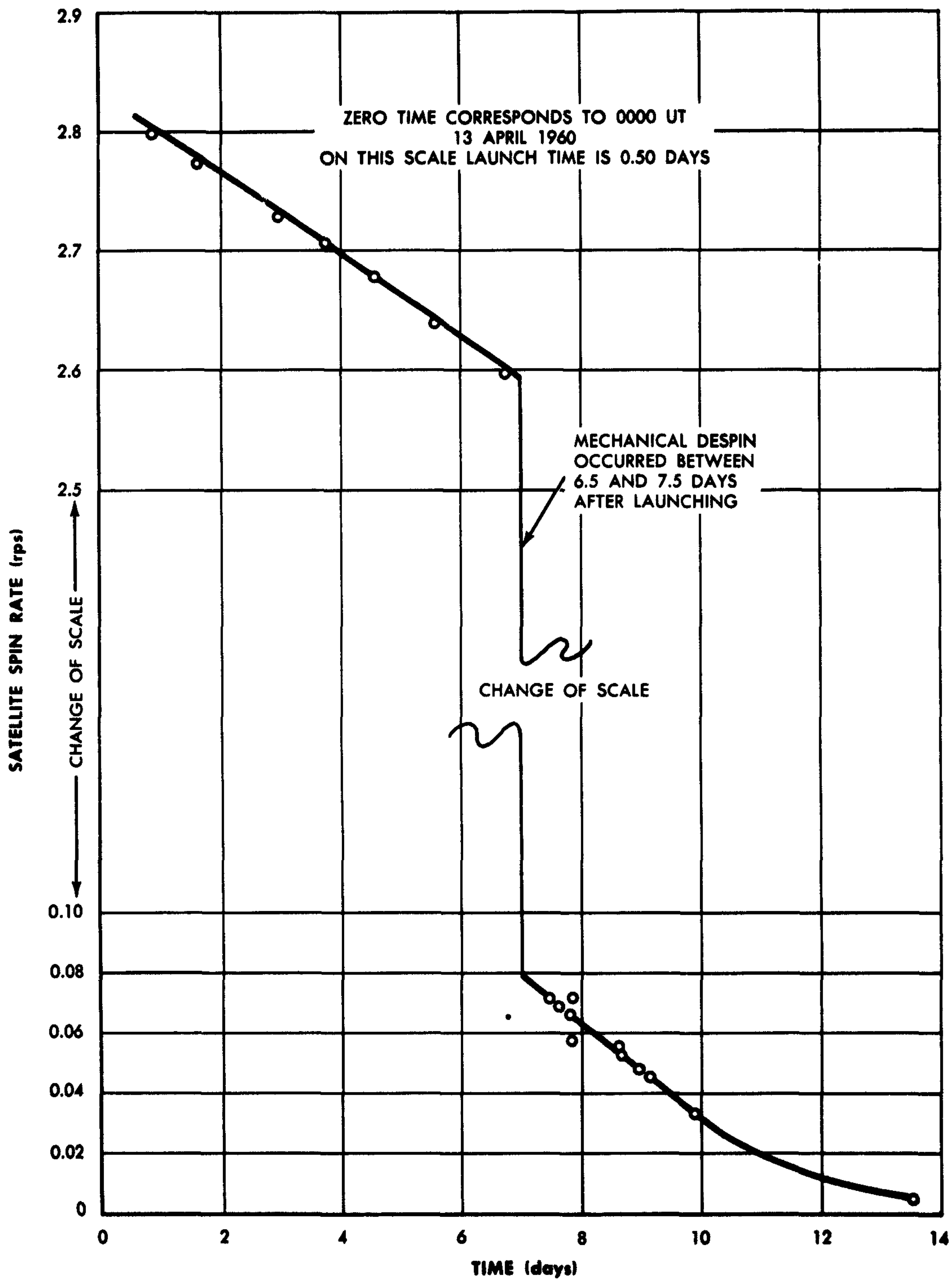

Fig. 5 SPIN RATE AS A FUNCTION OF TIME FOR THE TRANSIT 1B SATELLITE

a function of time after launch. For the first seven days the satellite spin rate was reduced by means of the despin rods which are shown in Fig. 6. On the seventh day a timer caused the despin weights to fly out from the satellite and thereby reduced the spin rate from 2.59 rps to 0.08 rps. The residual spin rate was then removed by the permeable rods. Figure 7 illustrates the spin rate decay of the Transit 2A satellite. In this case the entire spin removal was accomplished by means of highly permeable magnetic rods.

After the satellite's spin has decreased to the point where gyroscopic torques are negligible, magnetic or gravity gradient attitude control can be put into effect. One remaining problem, however, is to damp out the oscillations which will surely be present for either attitude control system.

For magnetic attitude stabilization the damping of oscillatory motions can be accomplished by the same permeable rods that are used to remove the spin of the satellite. A description of this method of damping of oscillatory motions is discussed in the sub-section, Magnetic Attitude Stabilization of an Earth Satellite.

A method for damping the oscillatory motions of a gravity gradient attitude stabilized satellite is discussed in the sub-section Gravity Gradient Attitude Stabilization.

Magnetic Attitude Stabilization of an Earth Satellite

A satellite which has a large magnetic dipole moment will, like a simple compass, tend to align itself along the local direction of the earth's magnetic field. Throughout

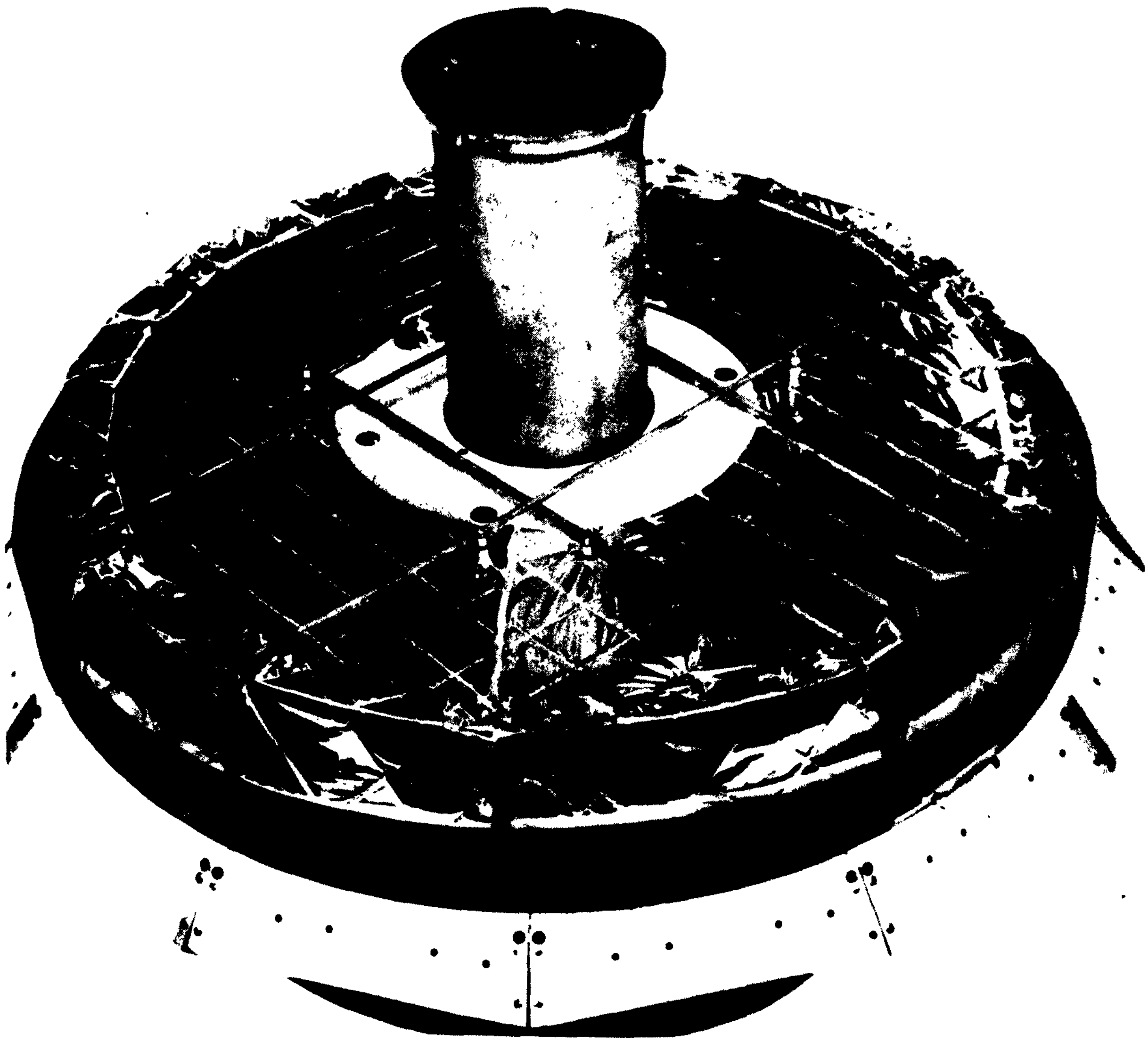

Fig. 6 DESPIN RODS IN TRANSIT 1B SATELLITE

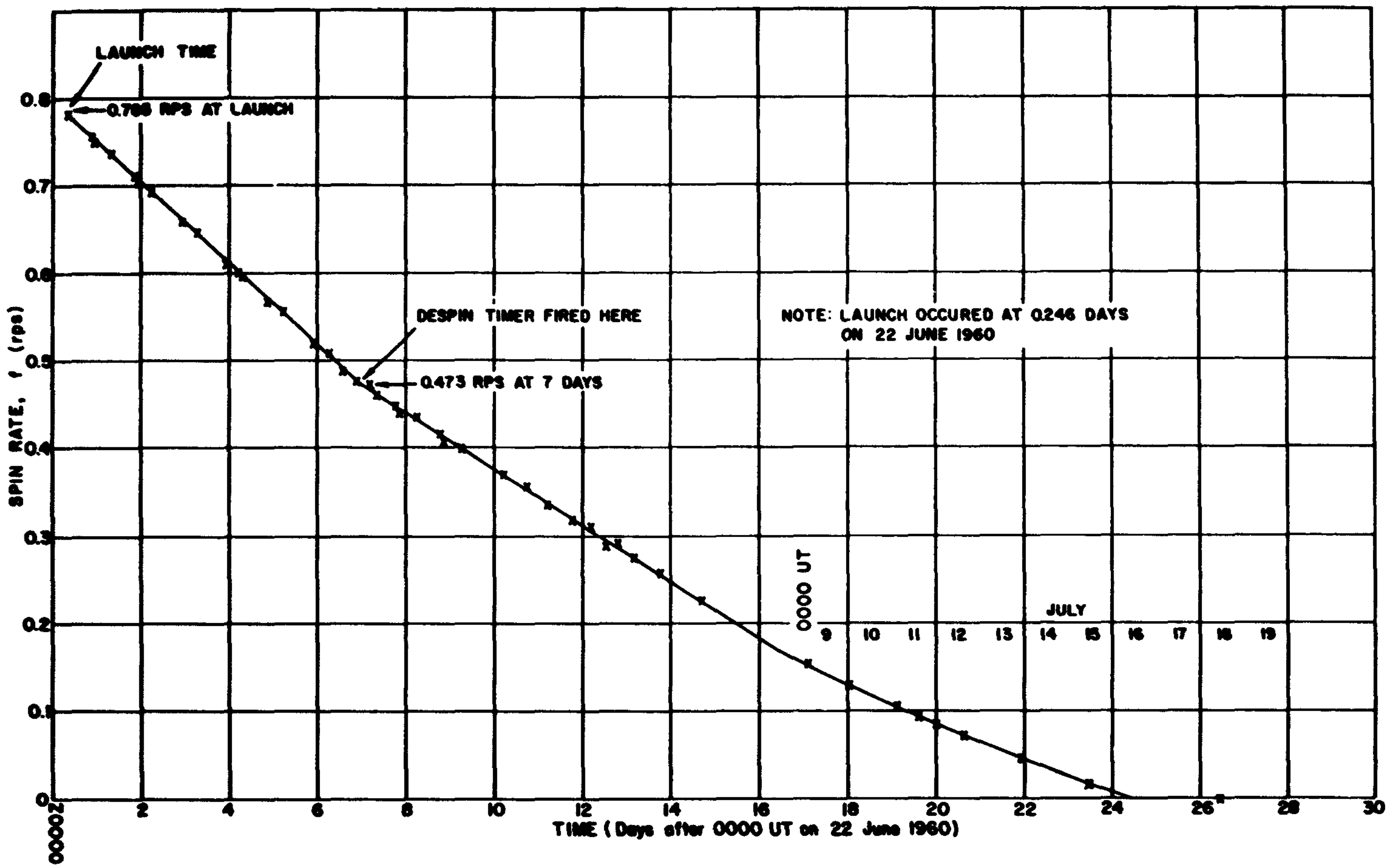

Fig. 7 SPIN RATE AS A FUNCTION OF TIME FOR THE TRANSIT 2A SATELLITE

this and subsequent discussions we shall consider that the satellite has cylindrical symmetry about the axis of its magnetic dipole moment. For the discussion of magnetic attitude stabilization we will make the quite accurate assumption that, at the altitude of an orbiting satellite, the center of the earth's magnetic dipole field is coincident with the center of mass of the earth. Based on this assumption it follows that a satellite will orbit about the earth's magnetic dipole axis in a manner analogous to its motion about the earth's spin axis. The orbital plane will have a certain inclination with respect to the earth's magnetic equator, which is analogous to (but different from) the inclination with respect to the earth's geographical equator. The major difference between the orbit's inclination with respect to the magnetic equator and with respect to the geographic equator is that in the former case the inclination changes as the dipole axis revolves about the earth's spin axis, while in the latter case, the inclination angle is virtually constant over extended periods of time. This condition results from the fact that the inclination of the satellite's orbital plane and the inclination of the earth's equatorial plane remain constant with respect to the ecliptic plane; however, the inclination of the plane of the earth's magnetic equator with respect to the ecliptic plane varies with a 24 hour period. Since the orbital period for near earth satellites is short compared to a day (approximately 100 minutes for near earth satellites as compared with 1440 minutes in a day) it is a good approximation to assume that the inclination of the satellite's orbit with respect to the plane of the earth's magnetic equator will remain essentially constant <u>during a single revolution</u>. The validity of this assumption can be analyzed in a semi-quantitative manner. Since the earth's magnetic dipole axis makes an angle of approximately 17 degrees with the earth's spin axis, there is a daily variation of ±17 degrees in the inclination of the satellite's orbital plane with respect to the plane of the magnetic equator.

Thus, take

i_g = inclination of the orbital plane with respect to the geographical equator (degrees),

and i_m = inclination of the orbital plane with respect to the magnetic equator (degrees),

Then the relation between i_m and i_g is given by

$$i_m = i_g + 17 \sin \left(\frac{\pi}{12}\right) t. \tag{6}$$

Curves of i_m as a function of time for three ranges of value of i_g are shown in Fig. 8. The maximum variation in i_m occurs at integer multiples of a half day. Next, take

T = period of the orbiting satellite (hours),

then at 12 hours $-1/2\ T$

$$i_m(12 - 1/2\ T) = i_g + 17 \sin \frac{\pi}{12} \left(12 - \frac{T}{2}\right), \tag{7}$$

at $t + 1/2\ T$

$$i_m(12 + 1/2\ T) = i_g + 17 \sin \frac{\pi}{12} \left(12 + \frac{T}{2}\right), \tag{8}$$

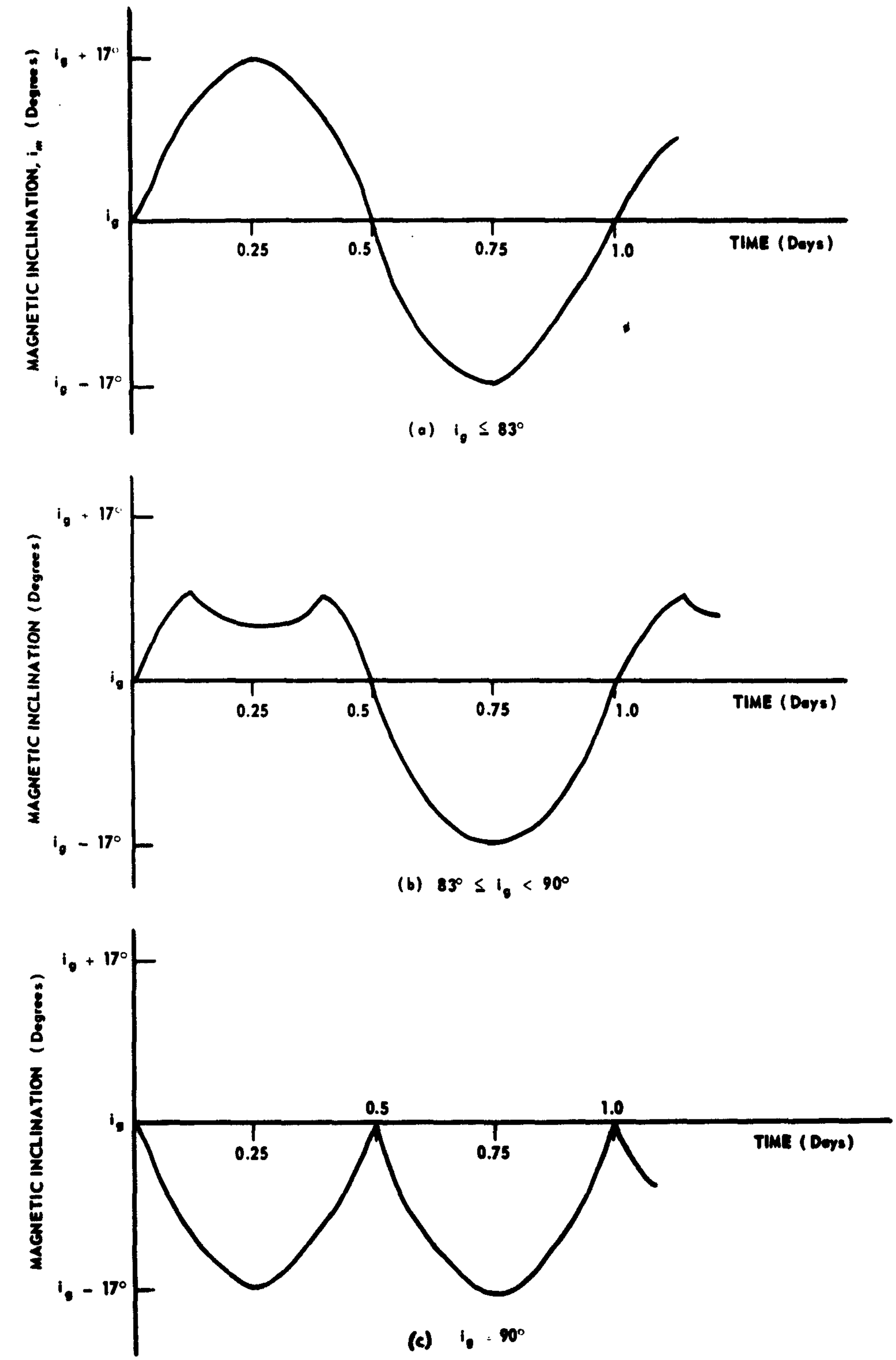

Fig. 8 ORBITAL INCLINATION WITH RESPECT TO THE PLANE OF THE EARTH'S MAGNETIC EQUATOR AS A FUNCTION OF TIME

and assuming at 100 minute period orbit yields

$$i_m(12 - 1/2\ T) = i_g + 17 \sin \frac{\pi}{12} \left[12 - \frac{100}{(2)(60)}\right]$$

$$= i_g + 17 \sin 2.925^\circ$$

$$i_m(12 - 1/2\ T) = i_g + 3.62^\circ \tag{9}$$

similarly

$$i_m(12 + 1/2\ T) = i_g - 3.62^\circ\ . \tag{10}$$

Therefore, the maximum variation of the inclination of the orbital plane during a single revolution is $\pm 3.62^\circ$.

We will now restate the assumptions that we shall use concerning the direction of the earth's magnetic field as observed at the attitude of an orbiting satellite. These are:

1. The earth's field is that of a magnetic dipole,
2. The center of the dipole is at the center of mass of the earth, and
3. The inclination angle of the orbital plane with respect to the plane of the magnetic equator does not vary appreciably during a revolution.

Based on these assumptions it is possible to analyze the motion of a satellite having a strong magnetic dipole moment.

First consider the case of a satellite traversing an orbit inclined at 0 degree with respect to the plane of the magnetic equator; i.e., $i_m = 0$ degree. For this case the satellite dipole axis will tend to align itself parallel to the direction of the earth's magnetic dipole. If the satellite is initially displaced it will oscillate about this axis. If a mechanism for damping is present within the satellite this oscillatory motion can be stopped. The permeable magnetic rods which are used for spin removal also provide a mechanism for damping the oscillations of the satellite.

Figure 9 is an illustration of a satellite with magnetic dipole moment $\vec{M}$ displaced by the angle θ from the direction of the earth's magnetic field.

This satellite also contains two magnetic damping bars which are oriented perpendicular to each other and perpendicular to the direction of the magnetic dipole.

If the first consideration presupposes that the damping is negligible then the torque on the satellite is given by

$$\vec{\tau} = \vec{M} \times \vec{H}_o \ , \qquad (11)$$

and considering only the magnitude of the torque,

$$\tau = MH_o \sin\theta \ \text{(dyne-cm)} \ , \qquad (12)$$

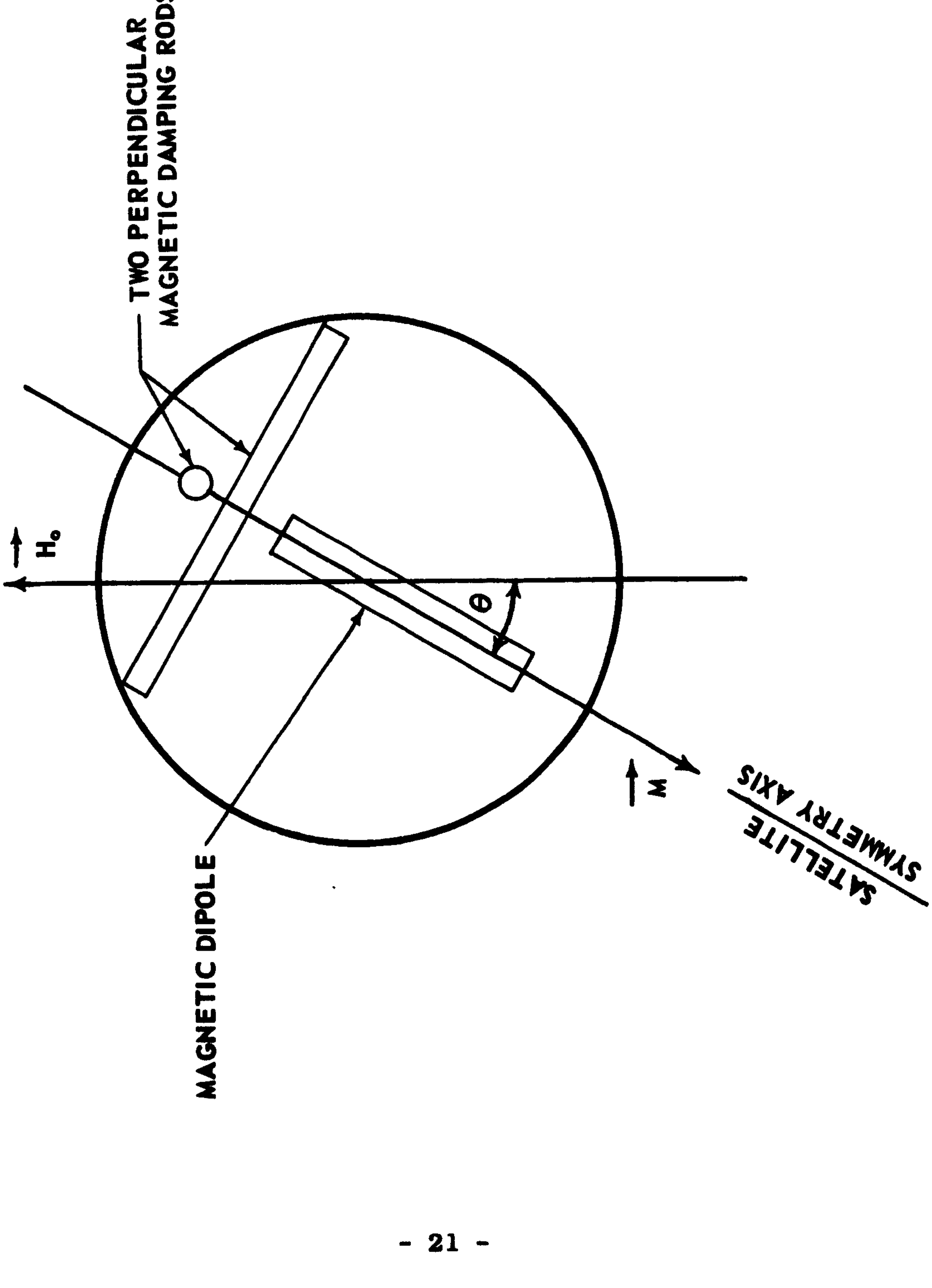

Fig. 9 SATELLITE WITH MAGNETIC DIPOLE MOMENT AND MAGNETIC DAMPING RODS

where

M = Magnitude of the satellite's magnetic dipole moment (unit-pole cm) ,

and

H_o = Magnetic field intensity (oersted) .

Let

I_x = Moment of inertia of the satellite about an axis through the center of gravity of the satellite and perpendicular to $\vec{M}$ (gm-cm^2),

and

I_z = Moment of inertia about the symmetry axis (gm-cm^2) .

We then obtain the equation of motion

$$I_x \frac{d^2\theta}{dt^2} + MH_o \sin \theta = 0 \quad . \tag{13}$$

This equation can be solved for small values of θ and yields the familiar result

$$\theta_n = \theta_o \cos 2\pi ft \quad , \tag{14}$$

where the natural frequency f is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{MH_o}{I_x}} \quad (\mathrm{sec}^{-1}) \quad . \tag{15}$$

Even though the solution of Equation (13) will not hold exactly for large values of θ, Equation (15) is a good expression for the natural period.

The potential energy of the magnetic dipole when displaced by an angle θ from the local direction of the earth's magnetic field is found by integrating the torque over the angular displacement, i.e.,

$$E(\theta) = \int_{0}^{\theta} MH_o \sin \theta d\theta \quad \text{(ergs)} , \tag{16}$$

which yields

$$E(\theta) = MH_o (1 - \cos \theta) \quad \text{(ergs)} . \tag{17}$$

In the presence of damping, a solution for θ is obtained in the form

$$\theta(t) = \theta_m(t) \, \theta_n(t) \tag{18}$$

where $\theta_m(t)$ is a damping function that approaches zero as $t \rightarrow \infty$, and $\theta_n(t)$ is a periodic function having a frequency defined by Equation (15) and which approaches $\sin 2\pi ft$ for small values of θ. This damped angular motion is illustrated in Fig. 10.

Due to the demagnetization effect (Ref. 4), only the component of the earth's magnetic field along the length of the rod is effective in magnetizing the rod. This component is given by

$$H = H_o \sin \theta \quad \text{(oersted)} \tag{19}$$

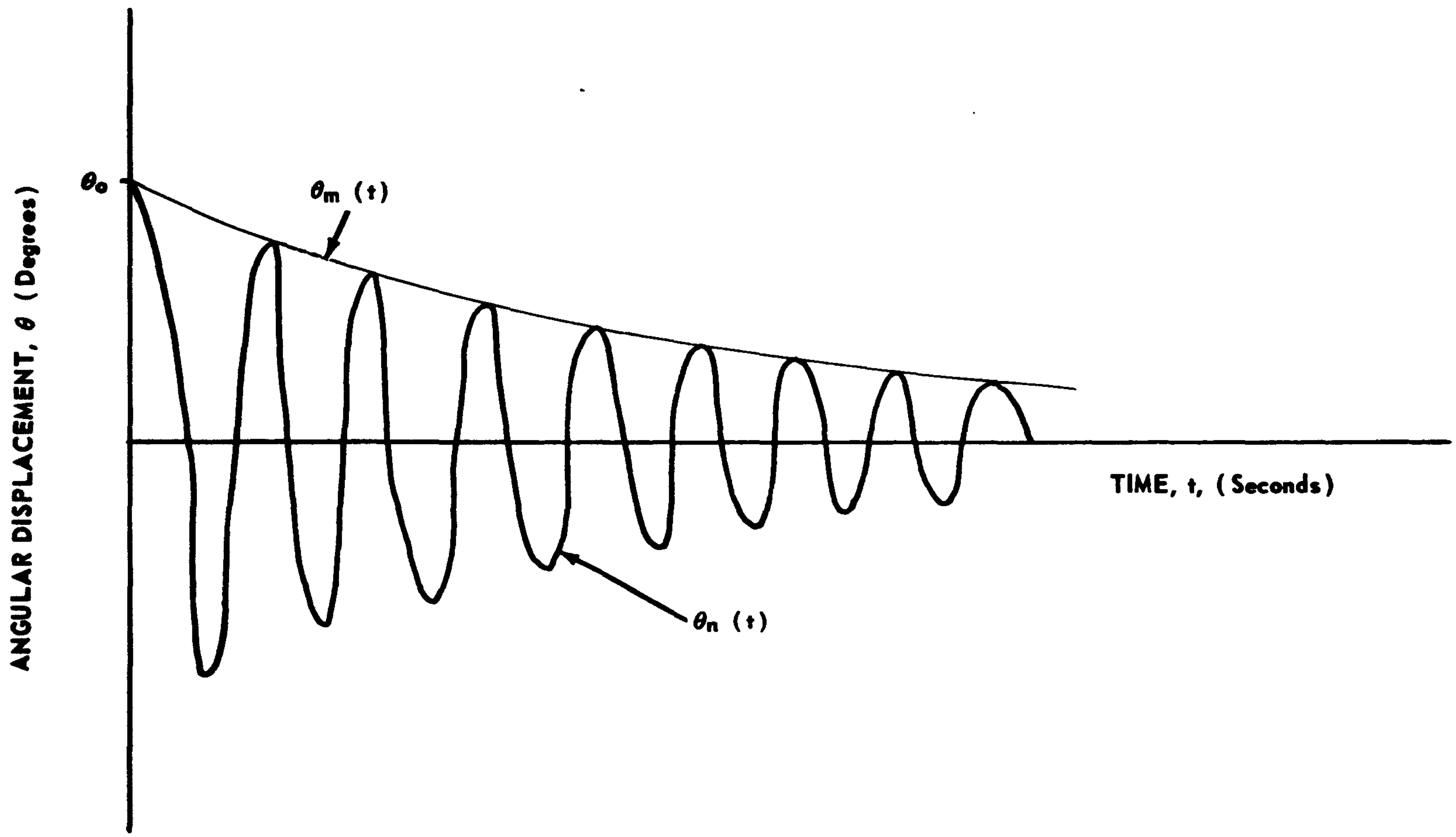

Fig. 10 MOTION OF A MAGNETICALLY ORIENTED SATELLITE WITH MAGNETIC DAMPING RODS

for bars oriented in a plane perpendicular to $\vec{M}$, and by

$$H = H_o \cos \theta \quad \text{(oersted)} \tag{20}$$

for rods oriented parallel to $\vec{M}$.

Therefore magnetic damping rods oriented perpendicular to $\vec{M}$ will undergo oscillations between positive and negative values of H as given by Equation (19). The rods oriented parallel to H will have the magnetic field intensity vary from H_o to $H_o \cos \theta$ at twice the oscillatory rate of the satellite. It is now advantageous to examine the relative effectiveness of rods oriented perpendicular and rods oriented parallel to $\vec{M}$. Assuming a value $H_o = 0.3$ oersted (which would be typical of a 500 nautical mile altitude) and an oscillation with a maximum angular displacement $\theta = 20$ degrees then the variation in H for rods perpendicular to $\vec{M}$ is given by

$$H = 0.3 \sin 20^\circ = 0.105 \quad \text{oersted}, \tag{21}$$

that is, the field intensity will vary from +0.105 oersted to -0.105 oersted for a total variation of 0.210 oersted.

For rods parallel to $\vec{M}$

$$H = 0.3 \cos 20^\circ \approx 0.282 \quad \text{oersted}, \tag{22}$$

that is, the field intensity will vary from 0.3 to 0.282 for a total variation of 0.018 oersted.

Even though the rods parallel to $\vec{M}$ will undergo a variation in field intensity at twice the oscillatory frequency, the magnitude of the change in H is so much smaller

(0.018 compared with 0.210) that the energy loss per cycle will be very much less than for the rods perpendicular to $\vec{M}$.

It can be easily shown that the energy loss per cycle due to hysteresis is proportional to the area of the traversed hysteresis loop (Ref. 3). For a 20 degree oscillation, the rods perpendicular to $\vec{M}$ will follow a major (but unsaturated) hysteresis loop with the peak value of H varying from +0.105 to -0.105 oersted. The rods parallel to $\vec{M}$ will follow a minor hysteresis loop with the field intensity varying between 0.3 and 0.282 oersted. Measurements have been made for permalloy rods having a length-to-diameter radio (L/D) of 248 under each of the two conditions described above. As a result, for an oscillation amplitude of 20 degrees, the area of the major hysteresis for the perpendicular rods was 27.9 gauss-oersteds compared to the minor hysteresis loop area of 0.08 gauss-oersteds for the parallel rods. Even though the minor hysteresis loop is traversed at twice the oscillatory frequency, the hysteresis loop area is still only 0.16 gauss-oersted per oscillation period which is still only 1/2 of 1 per cent as great as the major hysteresis loop area. Therefore, for damping of oscillations of a magnetically oriented satellite we would only employ rods that are perpendicular to $\vec{M}$.

For a single permeable rod perpendicular to $\vec{M}$, the hysteresis loss per cycle of satellite oscillation is given by (Ref. 3).

$$E = \frac{V}{4\pi} \oint H dB \quad \text{(ergs/cycle)} , \tag{23}$$

where

V = Volume of the rod (cm^3) ,

and

$\oint HdB$ = Area of the hysteresis loop (gauss-oersted).

If there are N bars (each perpendicular to $\vec{M}$ and perpendicular to the axis about which the satellite is oscillating) then the energy loss per second is given by

$$\frac{dE}{dt} = \frac{-NV}{4\pi} f \oint HdB \quad \text{(ergs/sec)} , \tag{24}$$

where the negative sign is taken because the energy is decreasing. Substituting the value of f from Equation (15) gives

$$\frac{dE}{dt} = \frac{NV}{8\pi^2} \sqrt{\frac{MH_o}{I_x}} \oint HdB \quad \text{(ergs/sec)} . \tag{25}$$

Although there will be energy loss due to eddy currents in the rods, it can be shown that for the comparatively low oscillatory rates achievable with a magnetically oriented satellite, the eddy current loss can be neglected in comparison to the hysteresis loss (Ref. 3).

Experimental measurements have been made on the hysteresis loop area as a function of the peak magnetizing field H_m. These results are shown in Fig. 11 for a permalloy rod (47.5 per nickel, 52.5 per cent iron) having an L/D ratio of 248. From the experimental curve it has been deduced that for this material and L/D ratio the value of $\oint HdB$ is very

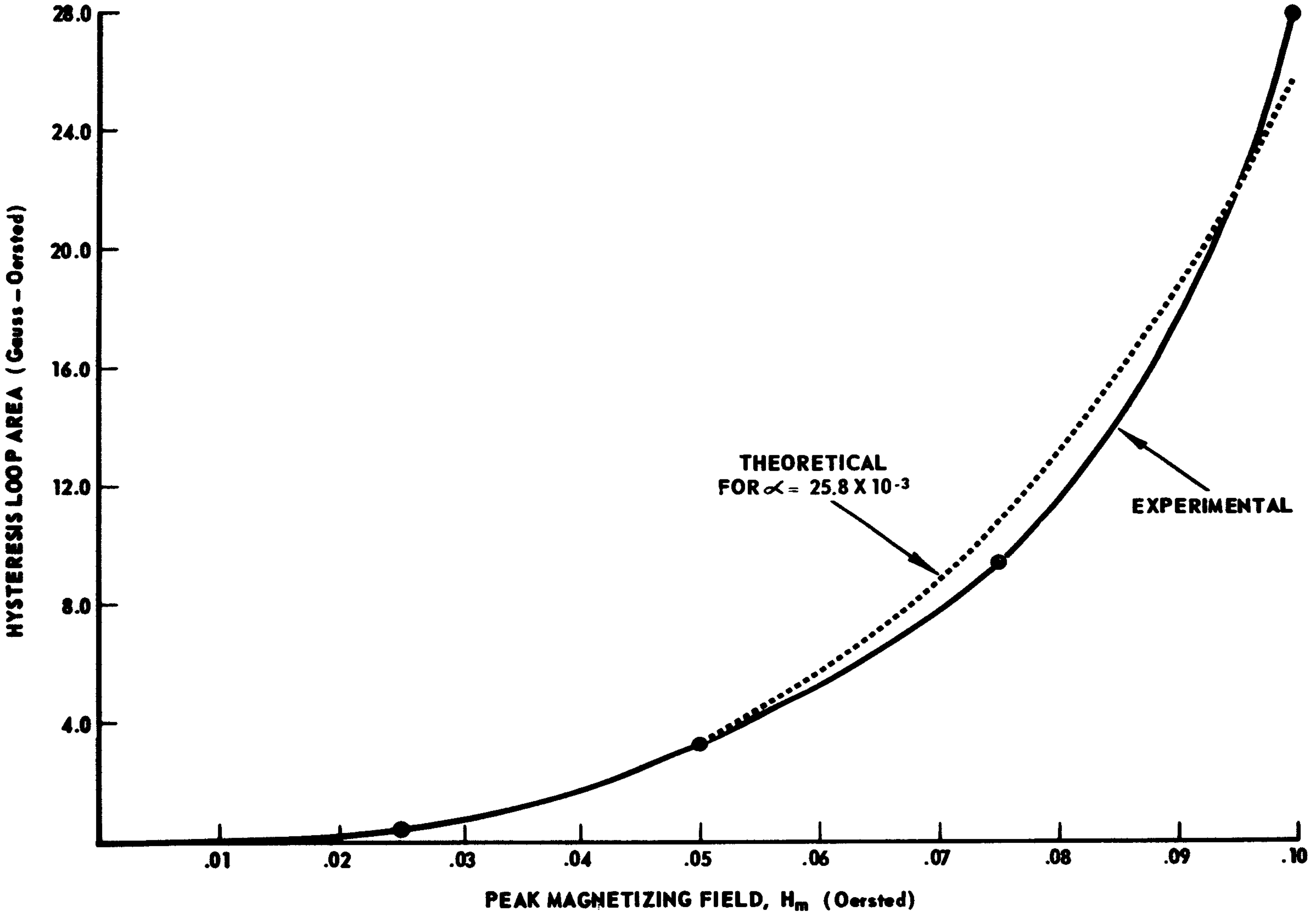

Fig. 11 EXPERIMENTAL AND THEORETICAL VALUES FOR HYSTERESIS LOOP AREA AS A FUNCTION OF PEAK MAGNETIZING FIELD

nearly proportional to the cube of the peak magnetizing field, that is,

$$\oint HdB = \alpha H_m^3 \text{ (gauss-oersted)} , \tag{26}$$

where

α = constant of proportionally having the dimensions gauss/(oersted)^2.

The rod characteristics illustrated in Fig. 10 can be closely approximated by setting $\alpha = 25.8 \times 10^3$ gauss/(oersted)^2.

We can write an expression for the peak magnetizing field along the permeable rods as

$$H_m = H_o \sin \theta_m \quad \text{(oersted)} , \tag{27}$$

where

θ_m = the maximum angular displacement from the equilibrium position (degrees) ,

then

$$\oint HdB = 25.8 \ (10^3) \ H_o^3 \sin^3 \theta_m \text{ (gauss-oersted)} , \tag{28}$$

and we can write the rate of energy loss as

$$\frac{dE}{dt} = \frac{-3.23 \ NV}{\pi 2} \sqrt{\frac{M}{I_x}} \ H_o^{7/2} \ (10^3) \sin^3 \theta_m \text{ (ergs/secs)}, \tag{29}$$

setting

$$k = \frac{3.23\ NV}{\pi^2} \sqrt{\frac{M}{I_x}}\ H_o^{7/2}\ (10^3)\ , \tag{30}$$

then

$$\frac{dE}{dt} = -k \sin^3 \theta_m \quad (\text{ergs/sec})\ . \tag{31}$$

If we rewrite equation (17) in terms of the maximum angular displacement θ_m we obtain

$$E\ (\theta_m) = MH_o\ (1 - \cos \theta_m) \quad (\text{ergs})\ . \tag{32}$$

But

$$\frac{d}{dt}\left[E\ (\theta_m)\right] = MH_o \sin \theta_m \frac{d\theta_m}{dt} \quad (\text{ergs/sec})\ . \tag{33}$$

Equating equations (31) and (33) yields

$$MH_o \sin \theta_m \frac{d\theta_m}{dt} = -k \sin^3 \theta_m\ , \tag{34}$$

simplifying and separating the variables yields

$$-\frac{d\theta_m}{\sin^2\theta_m} = \frac{k}{MH_o}\ dt\ , \tag{35}$$

and integrating becomes

$$- \int \csc^2 \theta_m \, d\theta_m = \frac{k}{MH_o} \int dt \, . \tag{36}$$

Therefore,

$$\cot \theta_m = \frac{k}{MH_o} (t) + c_1 \, . \tag{37}$$

If we take the initial condition that the satellite is displaced an angle θ_o at time $t = o$, then

$$c_1 = \cot \theta_o \, ,$$

and

$$\theta_m(t) = \text{arc cot} \left(\frac{k}{MH_o} t + \cot \theta_o \right) \, . \tag{38}$$

For reasonably small values of θ, it is possible then to write the complete solution as

$$\theta(t) = \left[\text{arc cot} \left(\frac{k}{MH_o} t + \cot \theta_o \right) \right] \cos 2\pi f t \, . \tag{39}$$

The most significant portion of this expression is the damping term which is quite accurate even for large angular displacements of the satellite from its equilibrium position.

Although this result obtained is based on the assumption of a simple cubic relation of $\oint HdB$ with H_m, the differential equation can also be solved for a general polynomial of order n that best fits the experimental curve.

To find the time for a satellite to damp down from an initial peak angular displacement of θ_o to a final displacement θ_f, solve Equation (38) for t which yields

$$t = \frac{MH_o}{k} (\cot \theta_f - \cot \theta_o) \quad \text{(seconds)} \quad . \tag{40}$$

For the Transit 3B satellite we had the following constants:

$N = 8$ (effective bars for oscillatory damping)

$\alpha = 25.8 \times 10^3$ gauss/(oersted)^2

$V = 6.2$ cm^3

$M = 7 \times 10^4$ pole-cm

$I_x = 1.1 \times 10^8$ gm-cm^2

Therefore, from Equation (30) taking $H_o = 0.3$ oersted we get $k = 6.09$. We can then use Equation (40) to determine the time required for the satellite to damp down from an initial displacement angle of $\theta_o = 90$ degrees to a final angle of $\theta_f = 2$ degrees. This gives the result of $t = 98{,}500$ seconds or roughly 1.1 days.

Therefore, for $i_m = 0$ degrees the satellite will align itself along the direction of the earth's magnetic dipole and the oscillations will be damped out in a reasonably short period of time.

Next, consider the case of a satellite traversing a magnetically polar orbit, that is, $i_m = 90$ degrees. For this case W. E. Allen (Ref. 5) has derived expressions for the angular displacement, angular rate, and angular acceleration of a magnetically oriented satellite relative to the direction of the earth's magnetic dipole axis. These three functions are defined by

$$\beta = \arc\tan\left[\frac{3 \tan \omega t}{1-2\tan^2 \omega t}\right] \quad \text{(degrees)} , \tag{41}$$

$$\frac{d\beta}{dt} = \omega \left[1 + \frac{2}{1 + 3 \sin^2 \omega t}\right] \quad \text{(radians/sec)} , \tag{42}$$

and

$$\frac{d^2\beta}{dt^2} = - \omega^2 \left[\frac{6 \sin 2\omega t}{1+3 \sin^2 \omega t}\right] \quad \text{(radians/sec}^2\text{)} , \tag{43}$$

where for the polar orbit the magnetic latitude φ is given by

$$\varphi = 57.3 \, \omega t \qquad \text{(degrees)} , \tag{44}$$

where

ω = orbital angular rate of the satellite (radians/sec.) .

These three equations are plotted in Figs. 12, 13, and 14 respectively for 1/4 of an orbital revolution; i.e., for a satellite moving from the ascending node to the north magnetic pole. These results show that a magnetically oriented satellite in a polar orbit will undergo, on the average, two revolutions per orbit as illustrated in Fig. 15. These results also show that the instantaneous angular rate of the satellite is three (3) revolutions per orbit (rpo) at the equator and 1.5 revolutions per orbit (rpo) at the pole. However, the average rate over an entire orbital period is 2.0 rpo. Another result of this analysis is that the angular acceleration is 0 at the equator and at the poles and has a maximum value of $3\omega^2$ at a magnetic latitude of 26 degrees.

For a maximum acceleration equal to $3\omega^2$ a rough estimate can be made of how well a magnetically oriented satellite will follow the dipole field for a polar orbit. To accomplish this it is necessary to equate the torque acting on the satellite due to an angular displacement from the local direction of the field to the product of the maximum acceleration that the satellite must be able to follow times I_x; i.e.

$$I_x \alpha_{max} = MH_o \sin \theta \text{ (dyne-cm)} , \tag{45}$$

where θ = approximate angle by which the satellite will be displaced from the local direction of the magnetic field.

Since $\alpha_{max} = 3\omega^2$, we have

$$3 I_x\omega^2 = MH_o \sin \theta \text{ (dyne-cm)} . \tag{46}$$

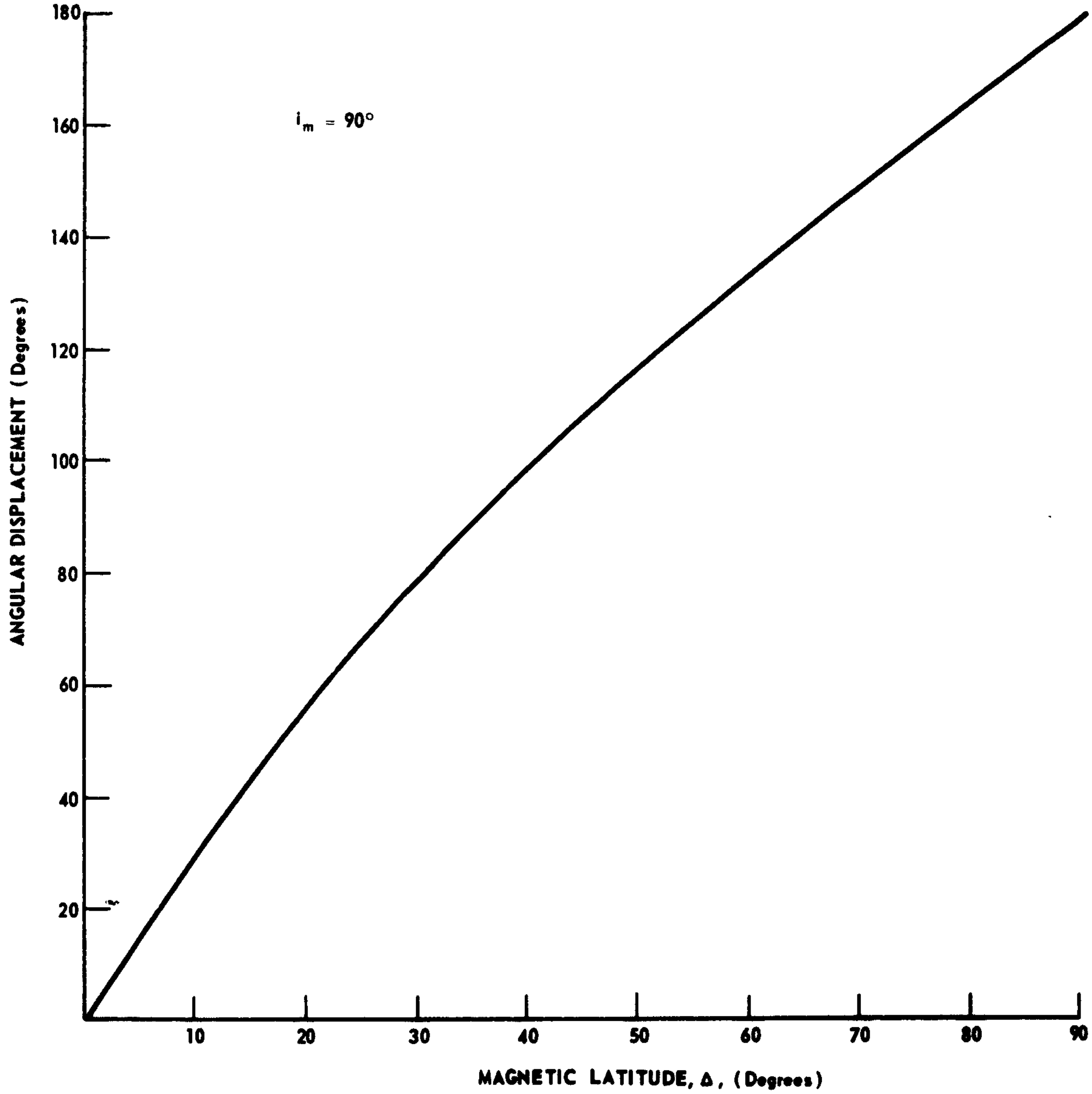

Fig. 12 ANGLE BETWEEN THE DIPOLE MOMENT OF THE EARTH AND THE MAGNETIC AXIS OF THE SATELLITE AS A FUNCTION OF MAGNETIC LATITUDE

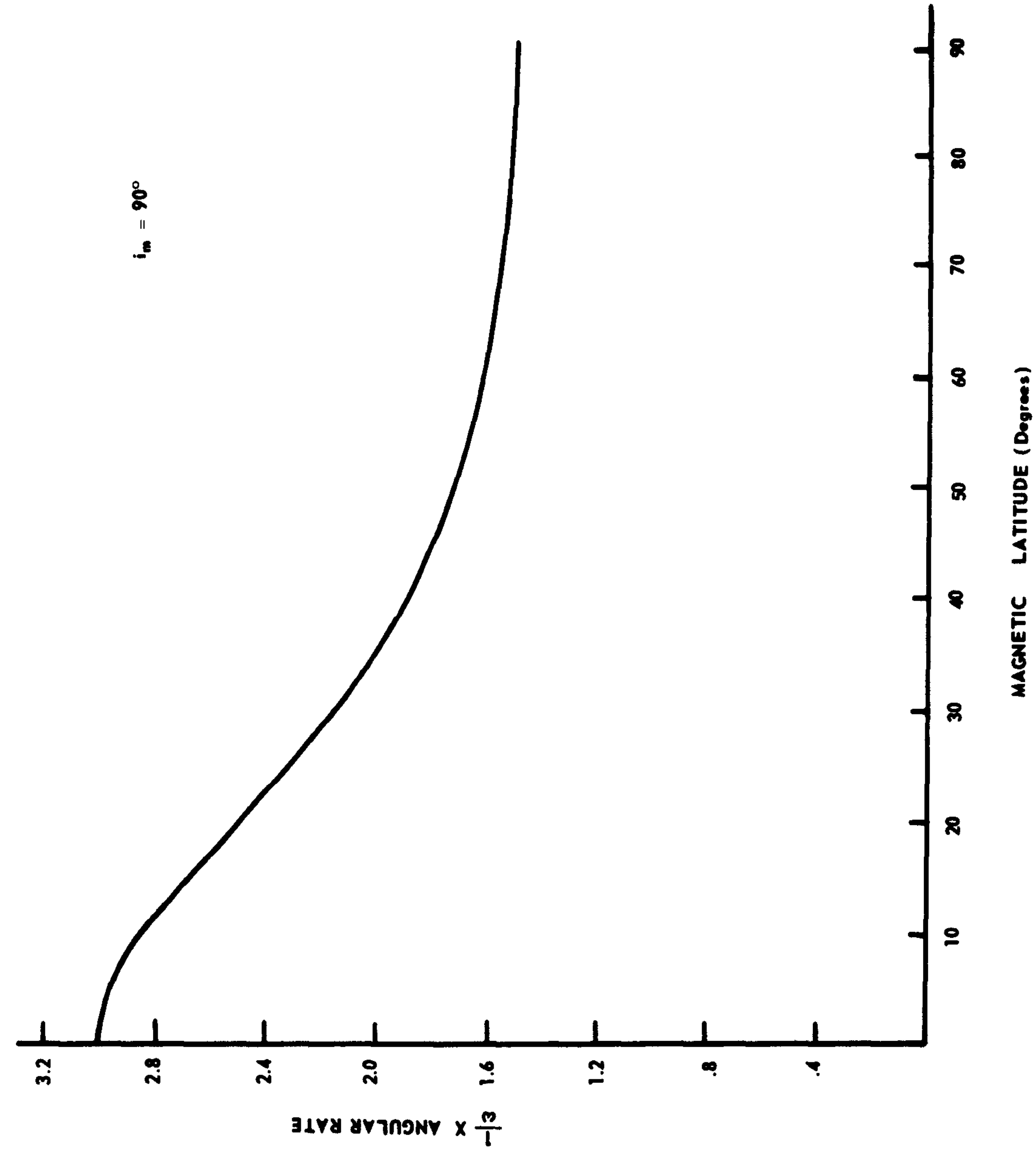

Fig. 13 ANGULAR RATE OF A MAGNETICALLY ORIENTED SATELLITE

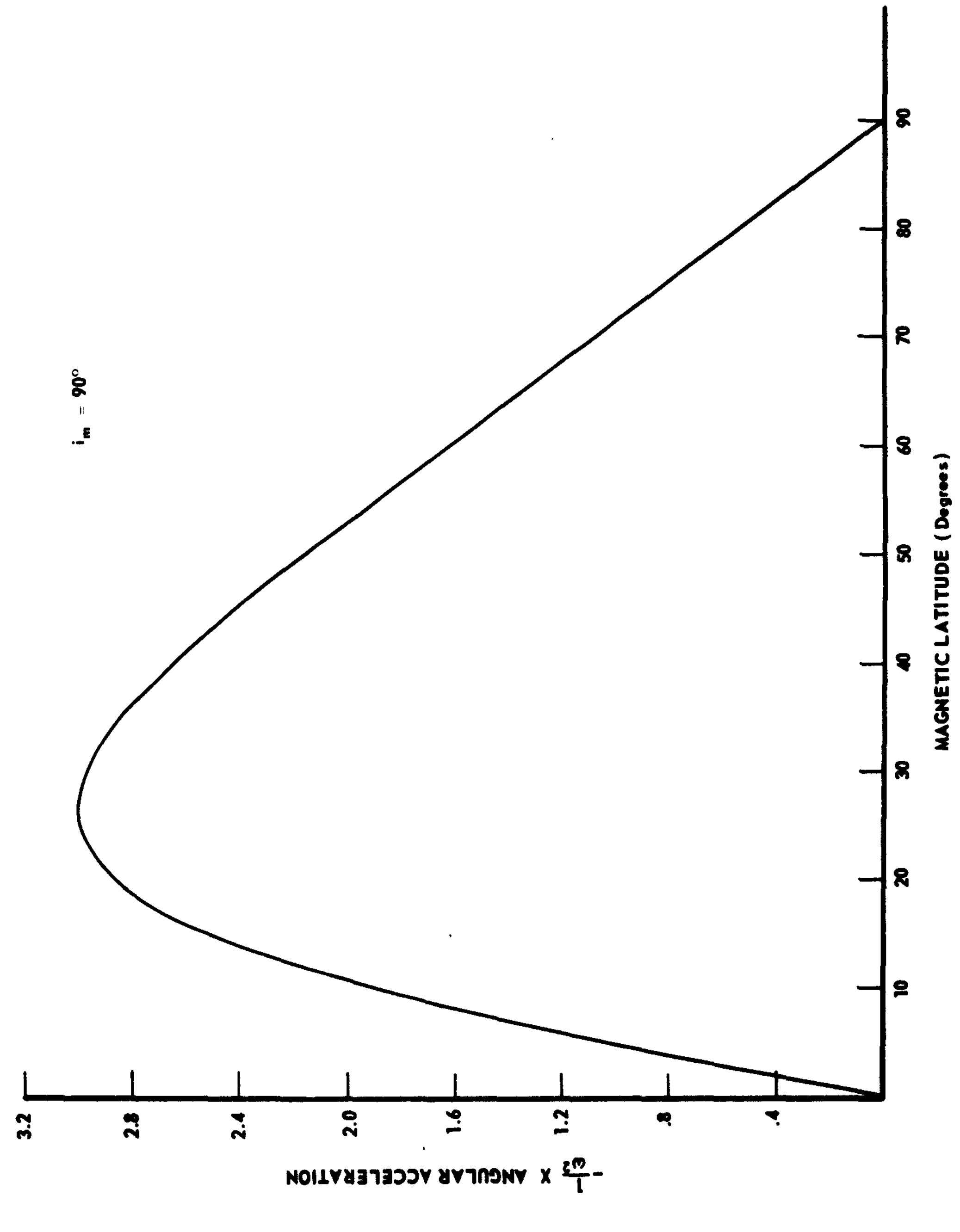

Fig. 14 ANGULAR ACCELERATION OF A MAGNETICALLY ORIENTED SATELLITE

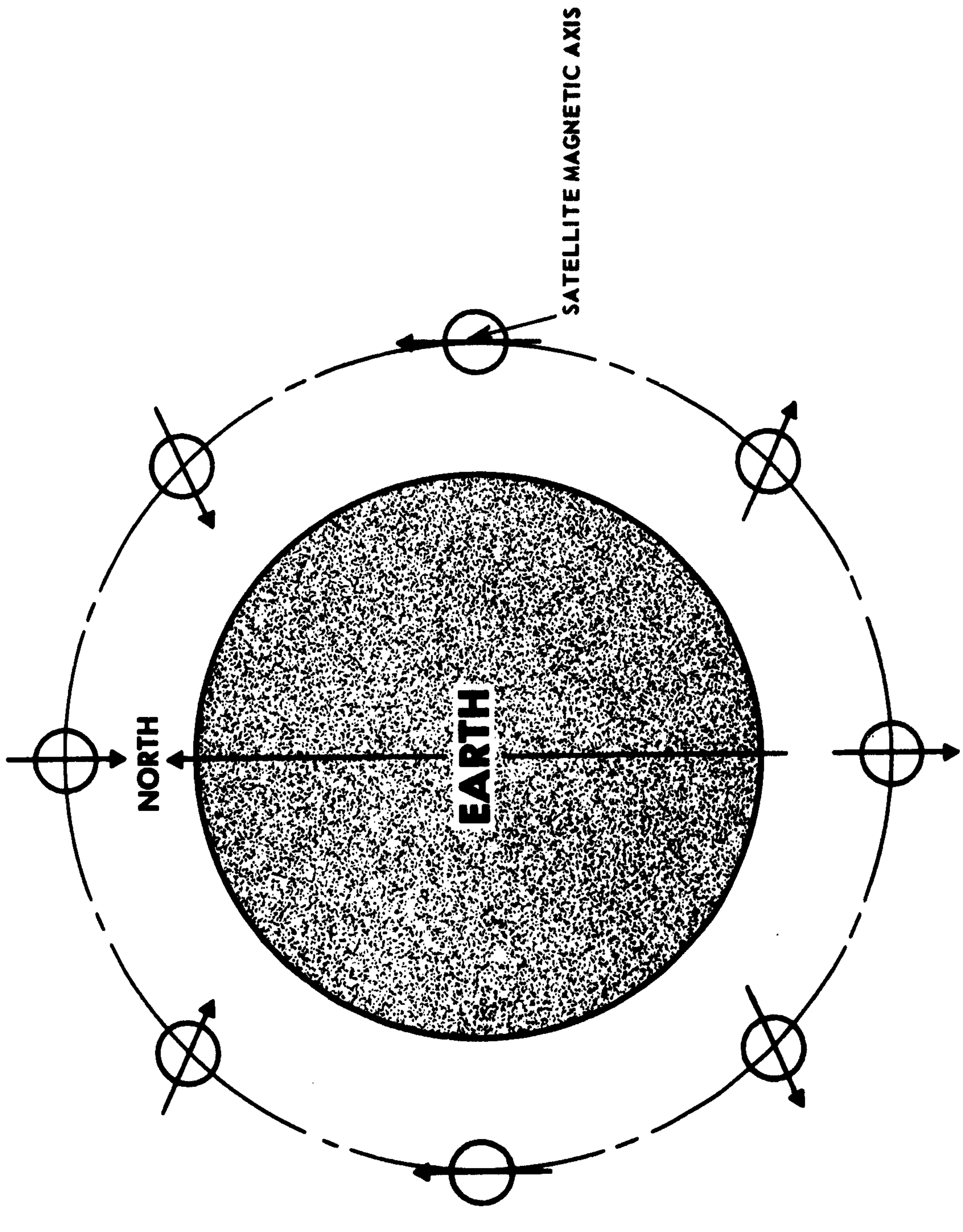

Fig. 15 MOTION OF A MAGNETICALLY ORIENTED SATELLITE IN A POLAR ORBIT

For a 450 nautical mile altitude orbit, again take $H_o = 0.3$ oersted (at $\varphi = 26^o$) and $\omega = 10^{-3}$ radians/sec. For the radiation research satellite currently being designed by Prof. James A. Van Allen and his staff from the State University of Iowa, $I_x = 4.4 \times 10^6$ gm-cm^2 and $M = 2.5 \times 10^3$ unit pole -cm.

Solving Equation (46) for θ yields

$$\theta = \text{arc sin} \frac{3 I_x \omega^2}{M H_o} \quad \text{(degrees)} \qquad (47)$$

and putting in the values from above gives

$$\theta = 1.0 \text{ degree}$$

This simple analysis gives a fairly good maximum value of the angular deviation from the local direction of the earth's magnetic dipole field if the natural period of oscillation of the magnetically oriented satellite is short compared to half the orbital period and if the satellite has some magnetic damping. The natural period of the satellite is given by the reciprocal of f from Equation (15), i.e.,

$$T = 2\pi \sqrt{\frac{I_x}{M H_o}} \quad \text{(seconds)} \quad . \qquad (48)$$

For the radiation research satellite this yields a natural period of T = 480 secs. which is quite short compared to a half orbital period of approximately 3000 seconds. The magnetic damping of permeable rods in the radiation satellite is equivalent to a value of approximately $\alpha = 2 \times 10^3$ for

Equation (26). This is sufficient to quite rapidly damp out oscillations of the satellite about the local direction of the earth's magnetic dipole field.

F. F. Mobley, of the Applied Physics Laboratory, has devised a means for obtaining a computer solution for the attitude of a magnetically oriented satellite in a magnetic dipole field (Ref. 6). For this purpose a mathematical model of the hysteresis loop of the magnetic damping rods was employed. An IBM 7090 computer was used to obtain a steady state solution for the angle that the magnetically stabilized satellite deviates from the local direction of the magnetic dipole field of the earth. Figure 16 illustrates the steady state motion of the radiation research satellite for a magnetic inclination of 90 degrees. From this curve it can be seen that the maximum value for the angular deviation of the satellite axis from the direction of the magnetic field is somewhat below the 1.0 degree that was obtained from the simplified analysis of Equation (47). Nevertheless the results of Equation (47) and the computer solution are in very good agreement.

For orbital inclinations between 0 degree and 90 degrees the motion of the satellite magnetic dipole axis is considerably more complex. The motion of the satellite for magnetic inclinations of 0 degree, 22-1/2 degrees, 50 degrees, 67-1/2 degrees and 90 degrees is shown in Fig. 17. As seen from this figure, the motion of the magnetically oriented satellite axis generally describes a conical surface.

For the radiation research satellite Mr. Mobley has prepared a computer solution for the case of an orbital inclination of 50 degrees. This result is shown in Fig. 18 and agrees very closely with the results obtained for the polar

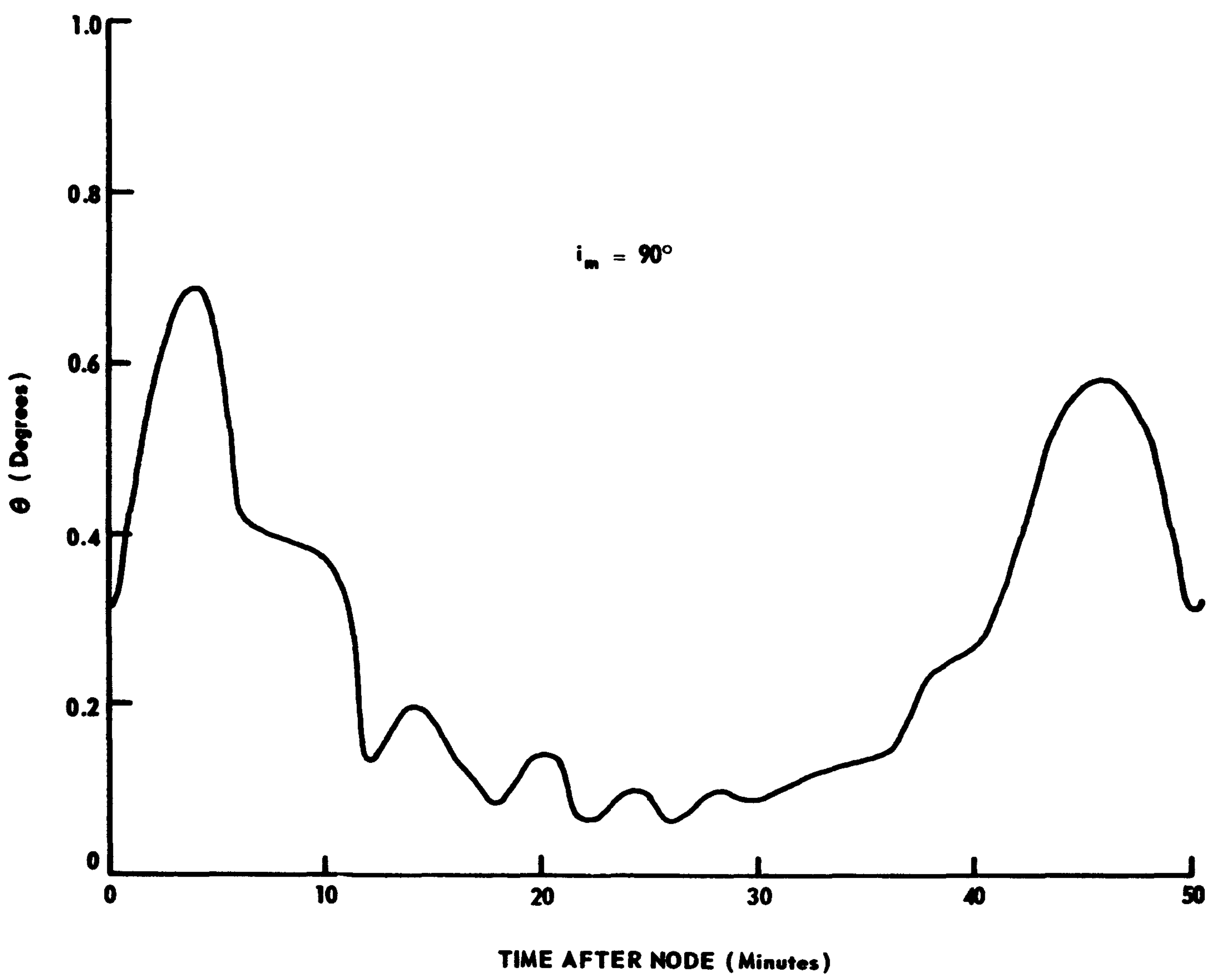

Fig. 16 EQUILIBRIUM ATTITUDE STATE FOR RADIATION RESEARCH SATELLITE, θ = ANGLE BETWEEN SATELLITE MAGNET AND MAGNETIC FIELD i_m = 90 DEGREES

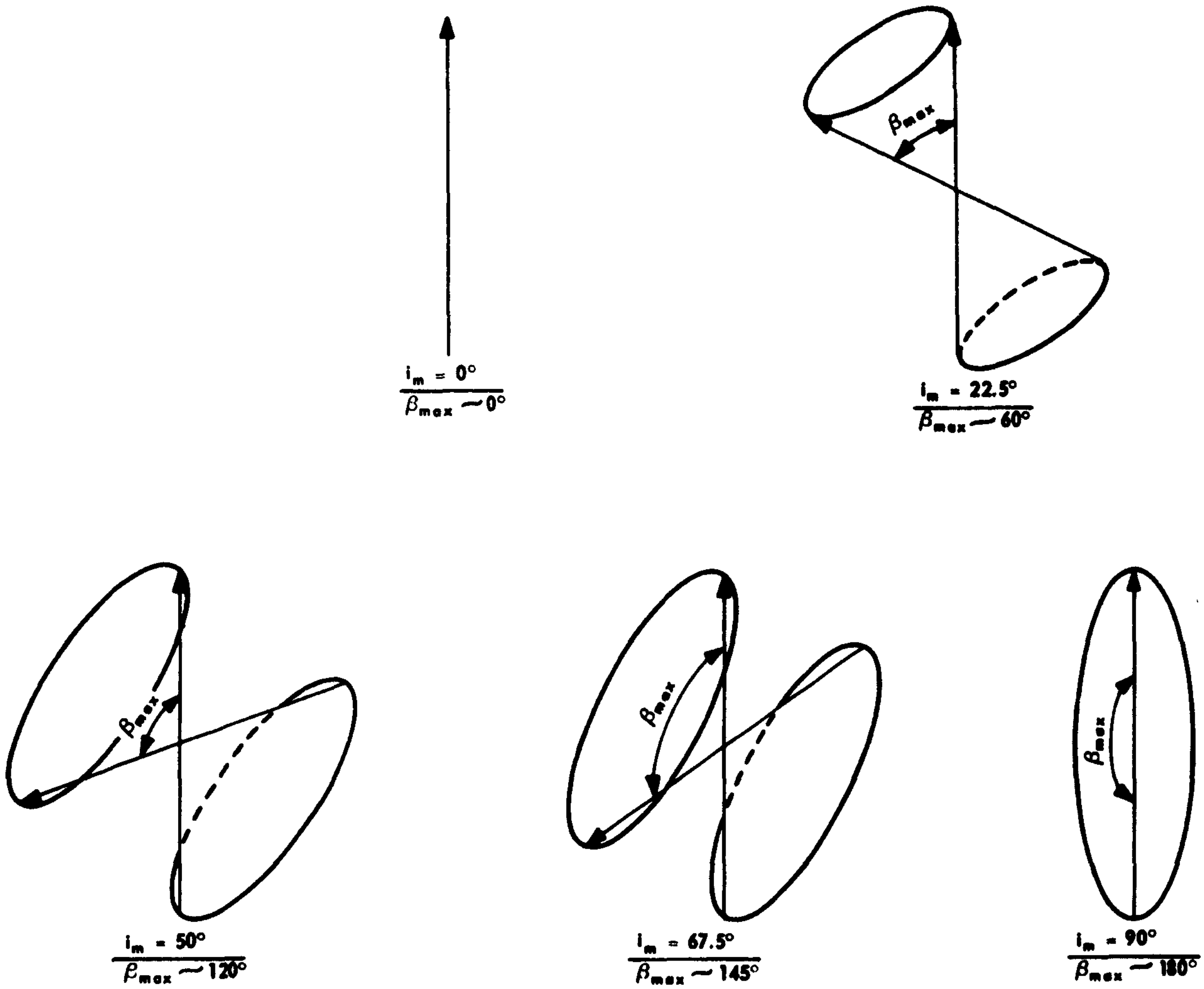

Fig. 17 MOTION OF A MAGNETICALLY ORIENTED SATELLITE FOR SEVERAL ORBITAL INCLINATIONS

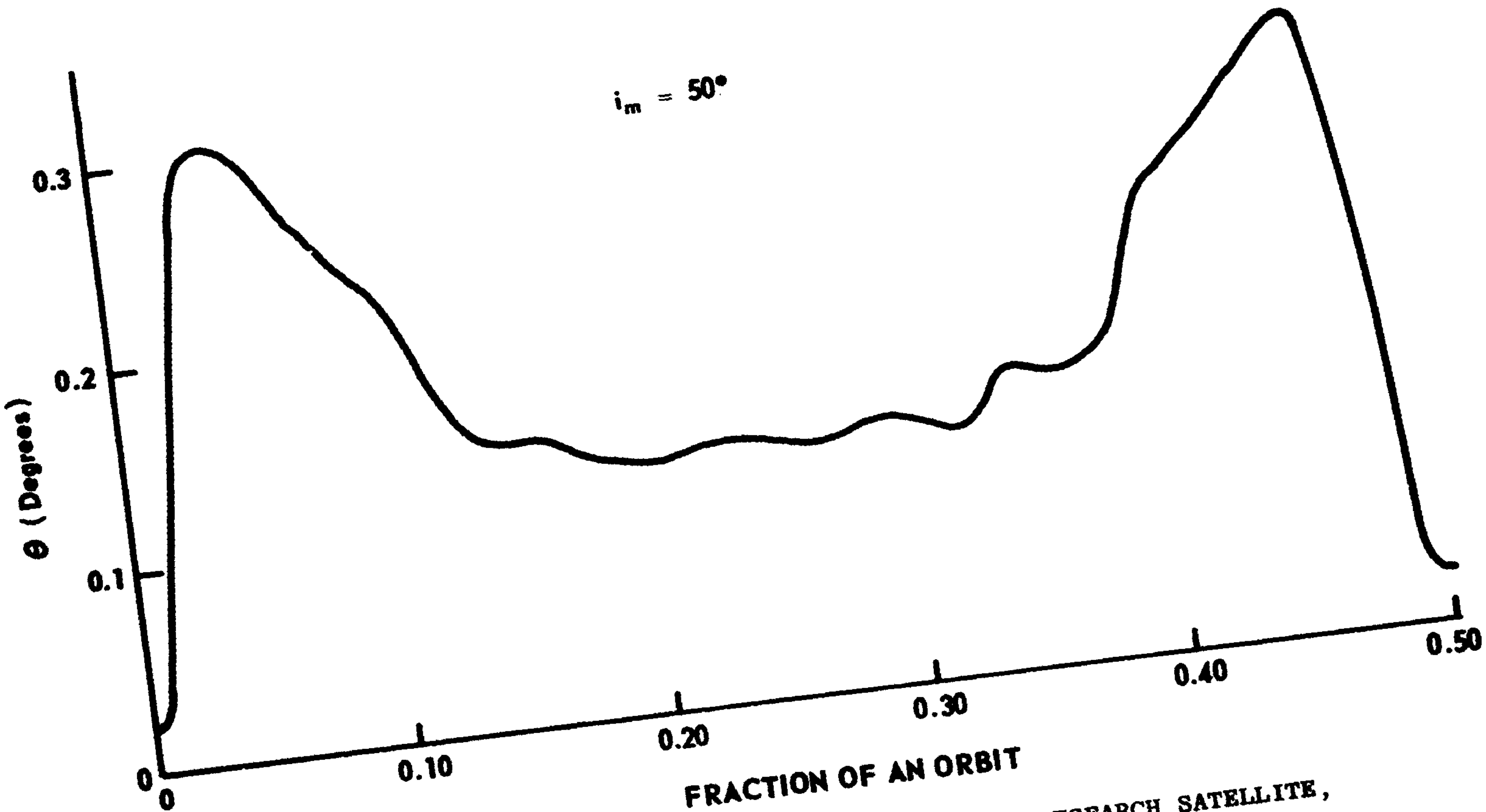

Fig. 18 EQUILIBRIUM ATTITUDE STATE FOR RADIATION RESEARCH SATELLITE,
θ = ANGLE BETWEEN SATELLITE MAGNET AND MAGNETIC FIELD
i_m = 50 DEGREES

orbit solution given in Fig. 16, but does yield a smaller maximum deviation in θ. This result might be expected since the angular accelerations encountered by the orbiting satellite is a maximum for a polar orbit and goes to zero for an equatorial orbit.

Another interesting result that was obtained as a computer solution for the radiation research satellite is an indication of the time required to damp out initial angular displacements of the radiation research satellite from its equilibrium position. In Fig. 19 is plotted the sum of the two nodal peaks (which are apparent in Fig. 16 and 18) as a function of revolution number. For the case of an 84 degree inclination the initial displacement was 45 degree and for the case of the 50 degrees inclination the initial displacement was approximately 3 degrees. In both cases the oscillatory motion of the satellite was damped out in relatively few revolutions of the satellite about the earth.

An important consideration in the design of a magnetically oriented satellite is to have the residual magnetic dipole moment of the damping rods small compared to the permanent magnetic dipole moment of the satellite. The satellite will align itself so that its total magnetic dipole vector is aligned along the earth's magnetic field. Since the total magnetic dipole moment of the satellite is the vector sum of the dipole moments of the damping rods and the permanent magnet, an accurate alignment of the permanent magnet axis along the earth's field requires that the effect of the permeable rods be very small.

Another factor which must be considered in producing a satellite whose magnetic dipole axis is accurately aligned along its symmetry axis is that electric currents within the satellites may produce undesirable magnetic dipole moments.

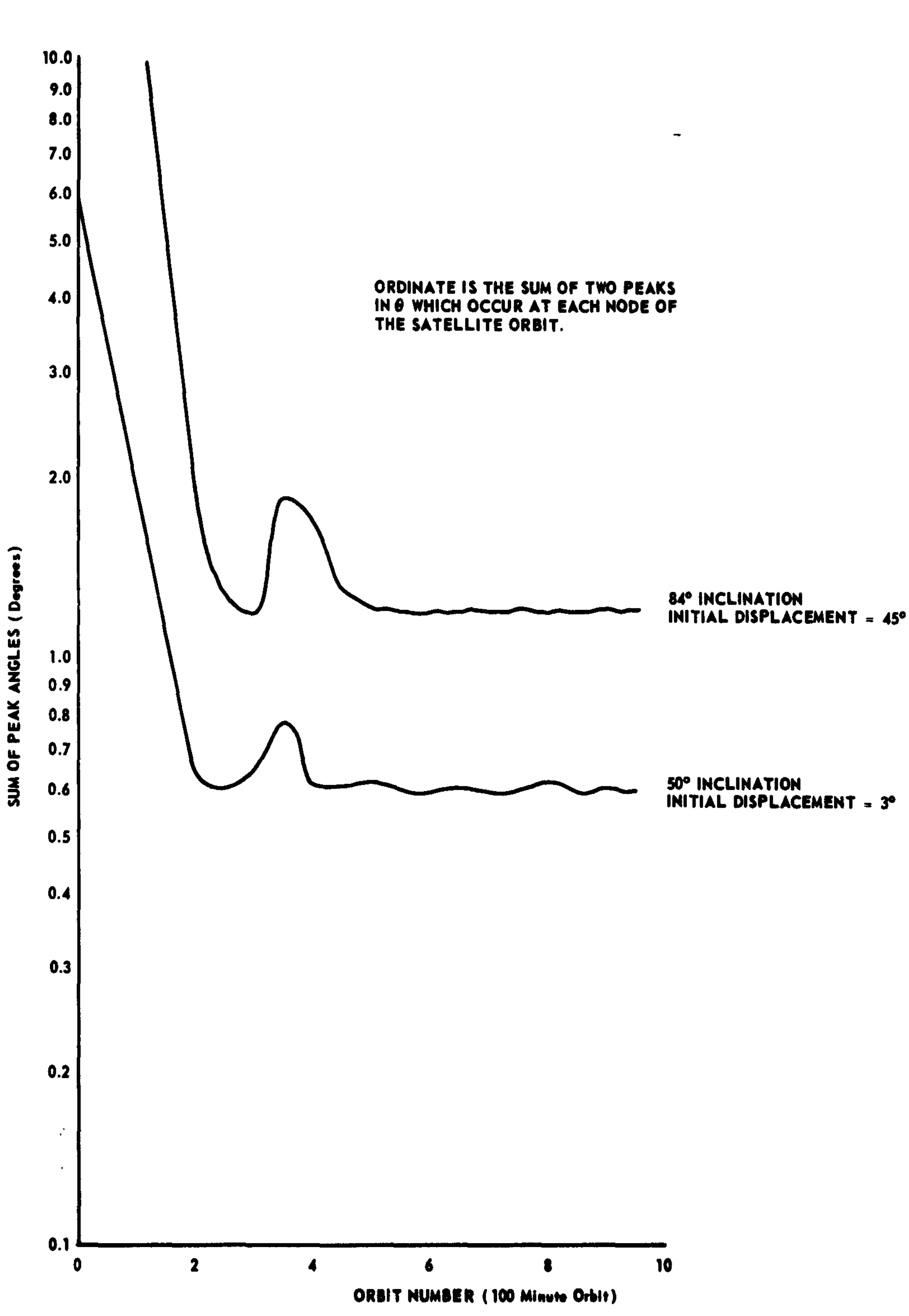

Fig. 19 RADIATION RESEARCH SATELLITE APPROACH OF ATTITUDE TO EQUILIBRIUM

Figure 20 is a photograph of the Transit 1B satellite having its magnetic field mapped in order to determine if the interval electrical currents and the magnetic damping rods would cause appreciable discrepancies in the alignment of the satellite's magnetic dipole axis. This work was accomplished at the U. S. Naval Ordnance Laboratory, White Oak, Maryland, in the Magnetic Ship Models Facility. The results of these tests showed that for the Transit 1B satellite the effect of the permeable rods and the internal electric currents were completely negligible compared to the magnetic dipole moment of a permanent magnet located within the satellite. For this satellite the permanent magnet was Alnico V, cylindrical in shape with a diameter of 1.0 inch and a length of 4.0 inches, thereby producing a dipole moment of 4.0×10^4 unit pole-cm.

Gravity Gradient Attitude Stabilization of an Earth Satellite

It is well known that a satellite with dissimilar moments of inertia about its principal axis will tend to align itself along the direction of the gradient of the earth's gravitational field so as to direct its axis at minimum moment of inertia toward the center of mass of the earth.

As was the case for magnetic orientation, we shall only consider those satellites possessing cylindrical symmetry about the axis that is to be aligned along the direction of the gradient of the earth's gravitational field. Therefore for this case the satellite will have a smaller moment of inertia about the symmetry axis (I_z) as compared with the moment of inertia about the other two orthogonal axes (where $I_x = I_y$).

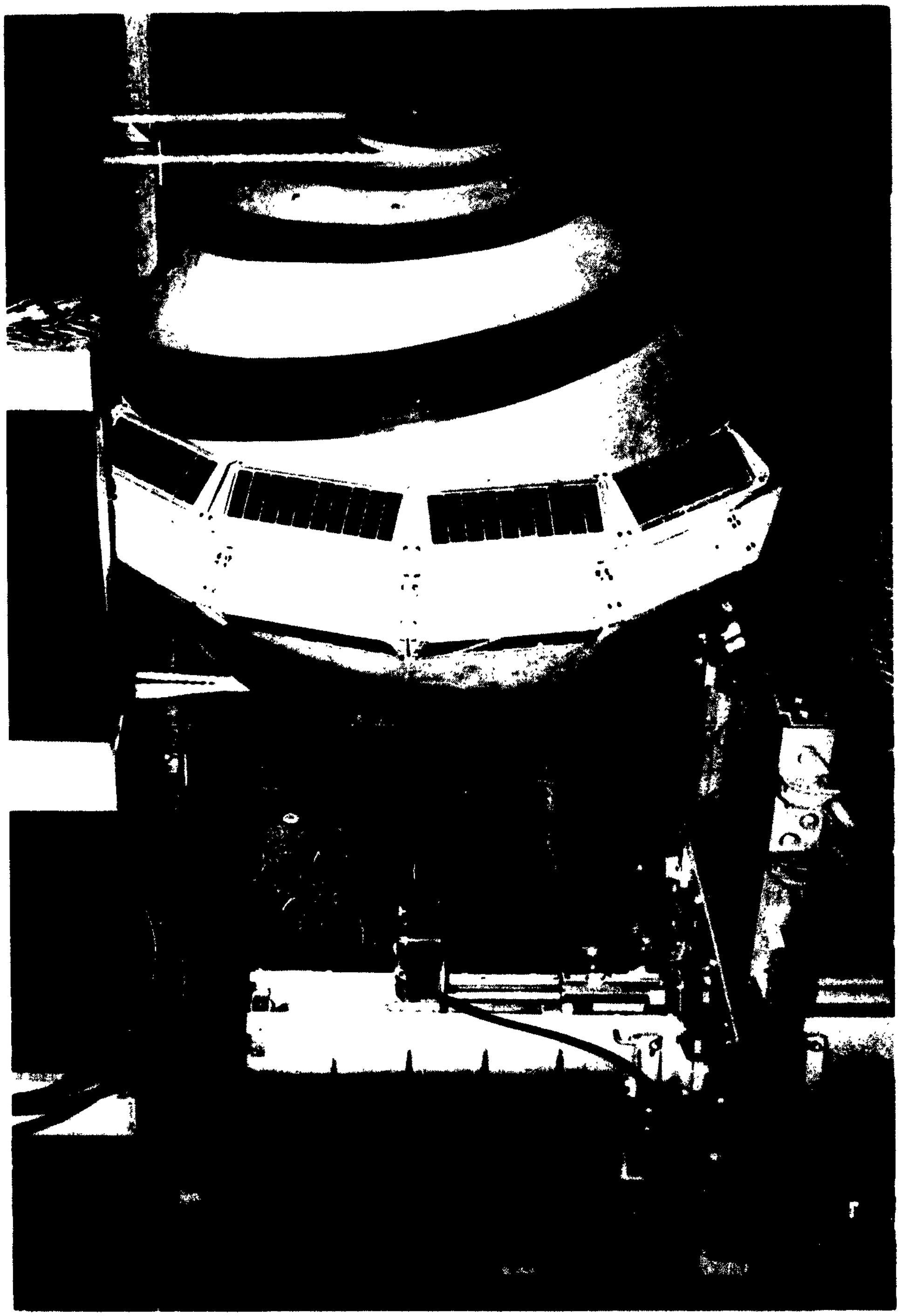

Fig. 20 SATELLITE MOUNTED TO MAP ITS MAGNETIC FIELD

For a satellite having this distribution of mass the gravity gradient torque is given by Refs. 7 and 8.

$$\gamma = 3/2\omega^2 \ (I_x - I_z) \sin 2\lambda \ \text{(dyne-cm)} \ , \qquad (49)$$

where λ = angle between the axis of minimum moment of inertia and the direction of the radius vector from the center of mass of the earth.

Although its a simple matter to obtain comparatively large torques by making the value of $I_x - I_z$ very large, it is nevertheless difficult to solve two problems that are associated with gravity gradient attitude stabilization.

One problem results from the fact that there is no preferential direction for the alignment of the axis of minimum moment of inertia along the earth's gravitational field. That is, either side of the satellite may be directed toward the earth.

The second problem is the damping of the oscillatory motion (libration) of the satellite about its equilibrium position.

Both of these problems can be solved by employing magnetic devices. A system for obtaining gravity gradient attitude stabilization has been devised which relies on initial magnetic orientation of the satellite. This method for obtaining gravity orientation of a satellite requires a sufficiently high orbital inclination so that at some time in the satellite's orbit it will pass close to the magnetic pole of the earth. Let us consider a satellite orbit that has a magnetic orbital inclination of 90 degrees, that is, it passes over the earth's magnetic pole. The procedure for obtaining gravity gradient attitude stabilization is as follows:

1. After the satellite is launched into orbit the magnetic permeable rods are used to damp out the spinning motion of the satellite.

2. After the satellite's spin is essentially stopped a boom with a mass at its end is extended from the satellite so as to yield $I_x \cong 4 \times 10^8$ gm-cm^2 and $I_z \cong 5 \times 10^7$ gm-cm^2.

3. A large solenoid within the satellite is then caused to have current flow in it so as to create a magnetic dipole moment of approximately 2×10^5 dyne-cm.

4. This large dipole moment when used in conjunction with magnetic damping rods will cause the satellite to be magnetically oriented as described in this paper under the sub-section entitled, Magnetic Attitude Stabilization of an Earth Satellite. This causes a particular side of the satellite to be directed toward the center of the earth as a satellite passes over the earth's magnetic pole.

5. As seen from Fig. 12, the angular rate of satellite as it passes over the magnetic pole is 1.5 rpo about the axis of maximum moment of inertia, I_x. As the satellite passes over the pole, the satellite will be commanded to stop the flow of electricity in the solenoid. Simultaneously the boom

with the mass at the end will be extended so as to increase I_x by a factor of 1.5. This will cause the angular rate of the satellite about the I_x axis to decrease from 1.5 rpo to 1.0 rpo which is the rate required for a gravity gradient attitude stabilized satellite. When the boom is fully extended we will have $I_x \cong 6 \times 10^8$ gm-cm^2.

If the operation described in item (5) above is performed perfectly, the satellite will be aligned along the earth's gravitational field with a predetermined axis directed downward, and with the correct spin rate (1.0 rpo) about I_x that will assure gravity gradient orientation. Oscillations about the equilibrium position will result if any of the following conditions are present:

1. The spin rate of the satellite deviates from 1.5 rpo before the final erection of the boom.

2. I_x after final erection of the boom may be slightly different from 1.5 times the value of I_x before the erection of the boom; or if,

3. The statellite is not aimed directly toward the center of mass of the earth when the final boom erection is made.

From an engineering point of view, it is believed that the total effect of these three sources of error can be controlled so as to keep the initial peak value of the angular displacement to be less than 5 degrees. To reduce this angular displacement a device to damp the libration motion must be utilized. The permeable damping rods described in the sub-section on Magnetic Attitude Stabilization of an Earth Satellite cannot be used because they damp out motions relative to the earth's magnetic field and will not be effective for damping the librational motions of a gravity oriented satellite.

One method for damping libration has been proposed that employs a viscous fluid that is caused to move by the opposing motion of a long rod mounted on a flexure joint perpendicular to the extended boom (Ref. 11). This system may cause some difficulty since the forces on the fluid may be so slight as to be below the threshold which is necessary for any fluid motion.

Another system for damping libration oscillations is based on the use of an eddy current damped cylinder mounted to the satellite by a wire having a very small torsional constant (Ref. 12). An illustration of this mechanism is shown in Fig. 21. The axis of the wire is oriented parallel to the axis about which the oscillatory motion is to be damped. Two such devices with their axes orthogonal would be mounted in the satellite. The cylinder will tend to remain fixed in inertial space and the satellite oscillating about it will, by means of a permanent magnet rigidly fastened to the satellite, cause eddy currents to be created in the cylinder. These eddy currents create a damping action (with no threshold for action) that tends to damp out the libration oscillations of the satellite. To be most effective the permanent magnet should have shaped pole pieces so as to maximize the gradient of the magnetic field intensity experienced by the cylinder. The torsional constant of the wire, the strength of the permanent magnet, and the shape, moment of inertia, and material of the cylinder must be considered in determining the optimum damping for the satellite.

We will now calculate the maximum torque acting on a satellite under the following conditions: $I_x = 6 \times 10^8$ gm-cm^2, $I_z = 5 \times 10^7$ gm-cm^2, $\omega = 10^{-3}$ radians/sec., and $\theta = 45$ degrees. From Equation (49) we see that $\tau_{max} = 825$ dyne-cm. The torque for an angular deviation of 1.0 degree from equilibrium is found by taking $\theta = 1.0$ degree in Equation (49). This gives

$$\tau = 28.8 \text{ dyne-cm/degree displacement from } \theta = 0 \text{ degree.}$$

This torque should be sufficient to overpower perturbing effects such as the interaction of the earth's magnetic field on the magnetic damping rods or unbalanced forces caused on the satellite by radiation pressure from the sun.

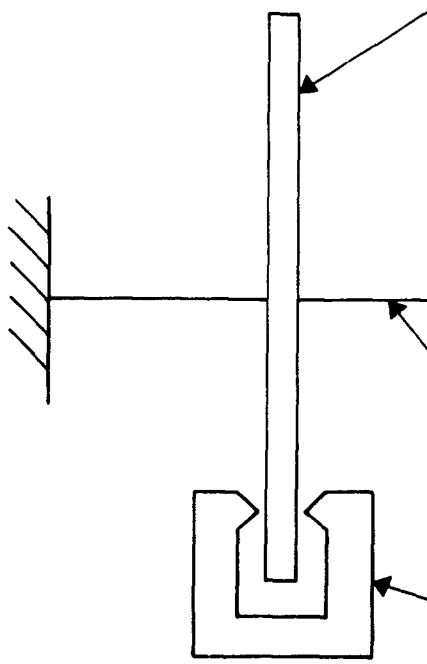

Fig. 21 MECHANISM FOR DAMPING LIBRATION OSCILLATIONS

IV. ATTITUDE CONTROL APPLICATION OF EARTH SATELLITES

The Generation of Electrical Energy from Solar Cells

As a result of magnetic or gravity attitude control a satellite will undergo predictable angular deviations of the satellite symmetry axis relative to the earth-sun line. In Fig. 22 is shown the angle between earth-sun line (sun vector) and the symmetry axis of a magnetically oriented satellite. The case shown in for a 500 nautical mile altitude, circular orbit with i_g = 67.5 degrees. In Fig. 22 two particular cases are shown. In one case the orbital plane of the satellite is essentially perpendicular to the earth-sun line. For this case we have a condition where the satellite is illuminated 100 per cent of the time. It can also be noted from this curve that the sun is shining predominantly on the equatorial region of the satellite.

The other curve in Fig. 22 is for the case when the orbital plane contains the sun vector. This is the case of minimum per cent sunlight for the satellite. At an altitude of 500 nautical miles this condition results in the satellite being illuminated 66.2 per cent of the time. From this second curve it can be seen that, for minimum per cent illumination, the symmetry axis of the satellite undergoes large variations in the angle that it makes with the sun vector.

The different attitudes with respect to the sun that result from magnetic (or gravity gradient) attitude control can be utilized to provide a more satisfactory system for electrical power generation from solar cells.

Figure 23 illustrates the Transit 1B satellite showing the arrangement of solar cells and also showing an angle α, between the satellite's symmetry axis and the earth-sun line.

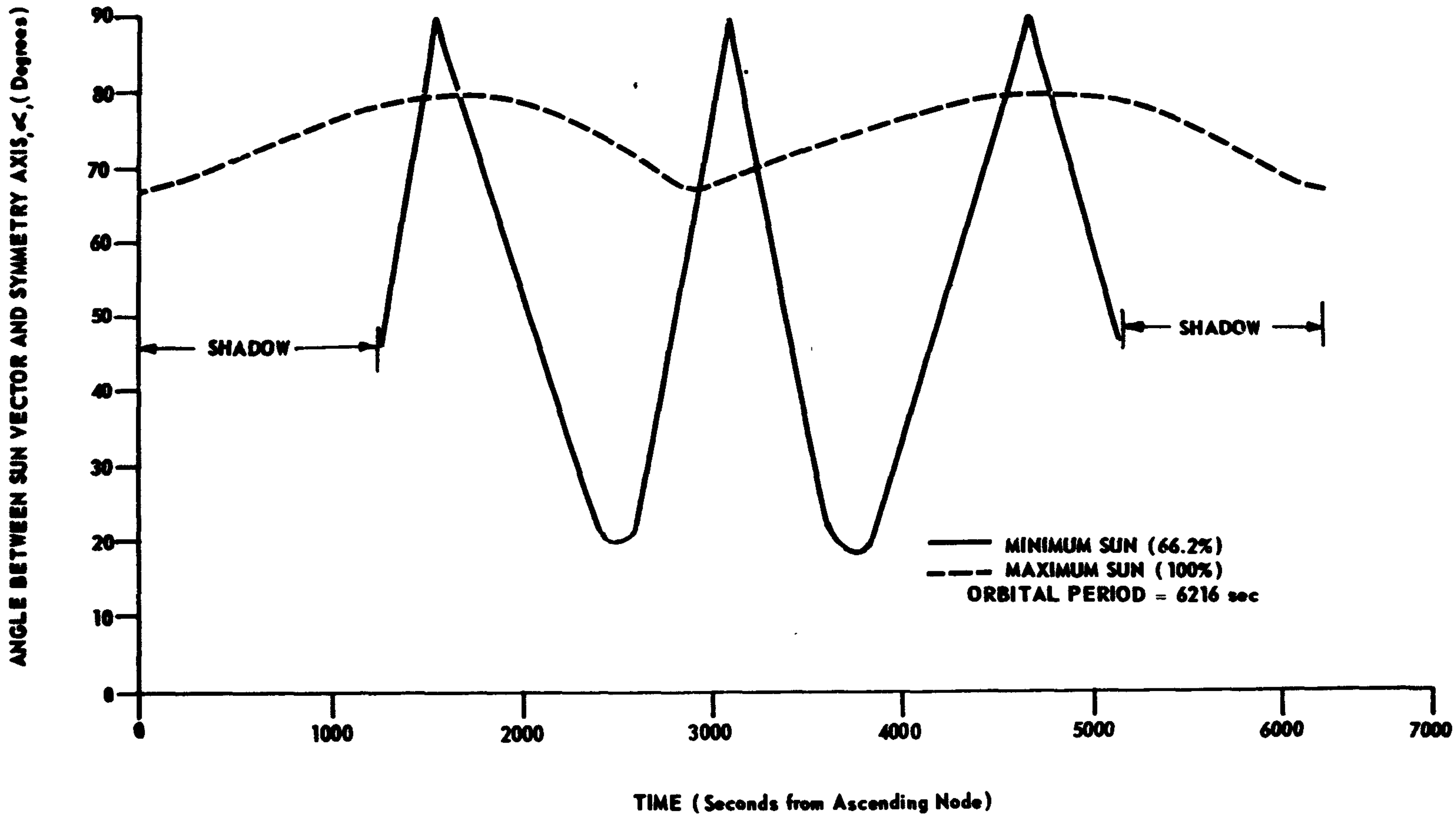

Fig. 22 ANGLE BETWEEN SUN VECTOR AND SYMMETRY AXIS AS A FUNCTION OF TIME FOR A 500 NM CIRCULAR ORBIT - INCLINATION 67.5 DEGREES

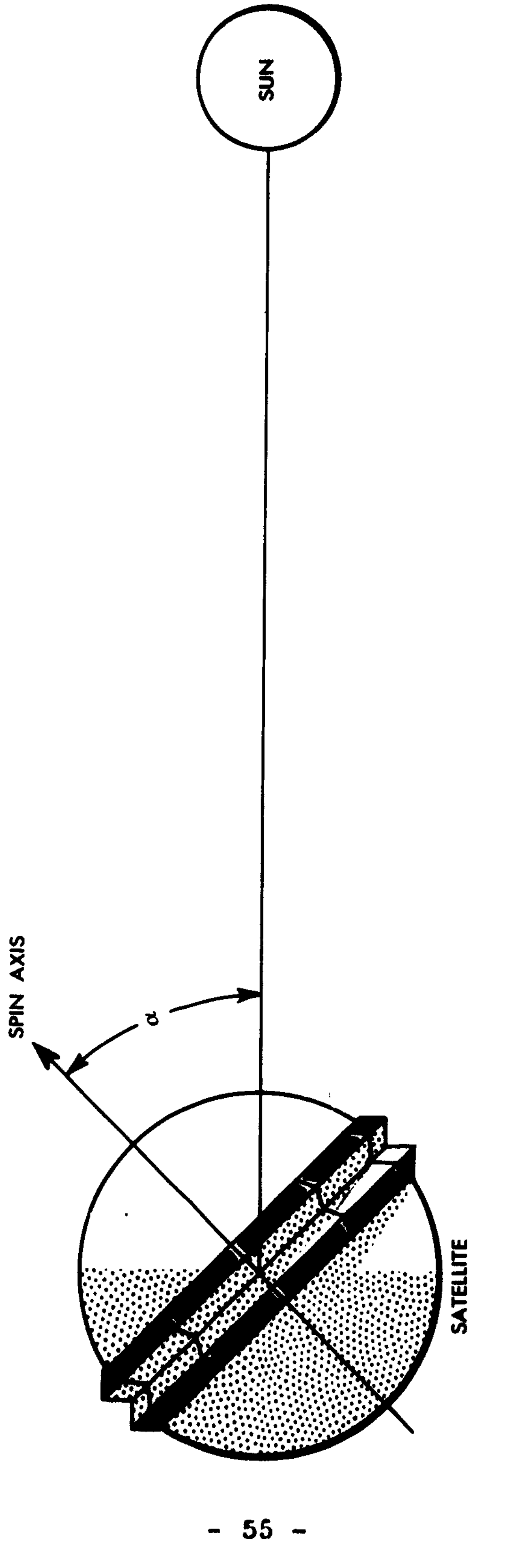

Fig. 23 TRANSIT 1B SATELLITE SHOWING SOLAR CELL ARRANGEMENT

For the Transit 1B satellite, as is the case with most solar powered satellites, the power generated by the solar cells is a function of the angle α that the satellite's symmetry axis makes with the sun vector. A curve of the output power as a function of α for a design such as the Transit 1B satellite is shown in Fig. 24.

A non-spinning satellite without attitude control might dwell in a position such that the angle α results in a minimum power output from the satellite. One immediate advantage of magnetic or gravity gradient attitude control is that, especially for the minimum sunlit orbits, the angle α is forced to undergo large excursions. This results in a "guaranteed" power output for the minimum sunlit orbits which is based on averaging the power indicated in Fig. 24.

Another advantage that can be obtained by attitude control results from the fact that for 100 per cent sunlit orbits, the satellite is being illuminated with the angle α close to 90 degrees. When the solar cells are mounted on the satellite in such a way as to minimize the output for the angle α in the region of 90 degrees, the problem of overcharging the satellite batteries can be eased and with careful design the problem can even be completely eliminated.

Figure 25 illustrates the solar cell electrical power output as a function of time for a magnetically oriented satellite undergoing the excursions shown in Fig. 22. For this case the satellite solar cell configuration was such as to provide an output as a function of α as shown in Fig. 24. The net power generated during an orbital period is a result of integrating each of the two curves of Fig. 25, considering in each case the per cent sunlight of the satellite. The curve of Fig. 24 can be shaped so that the electrical power

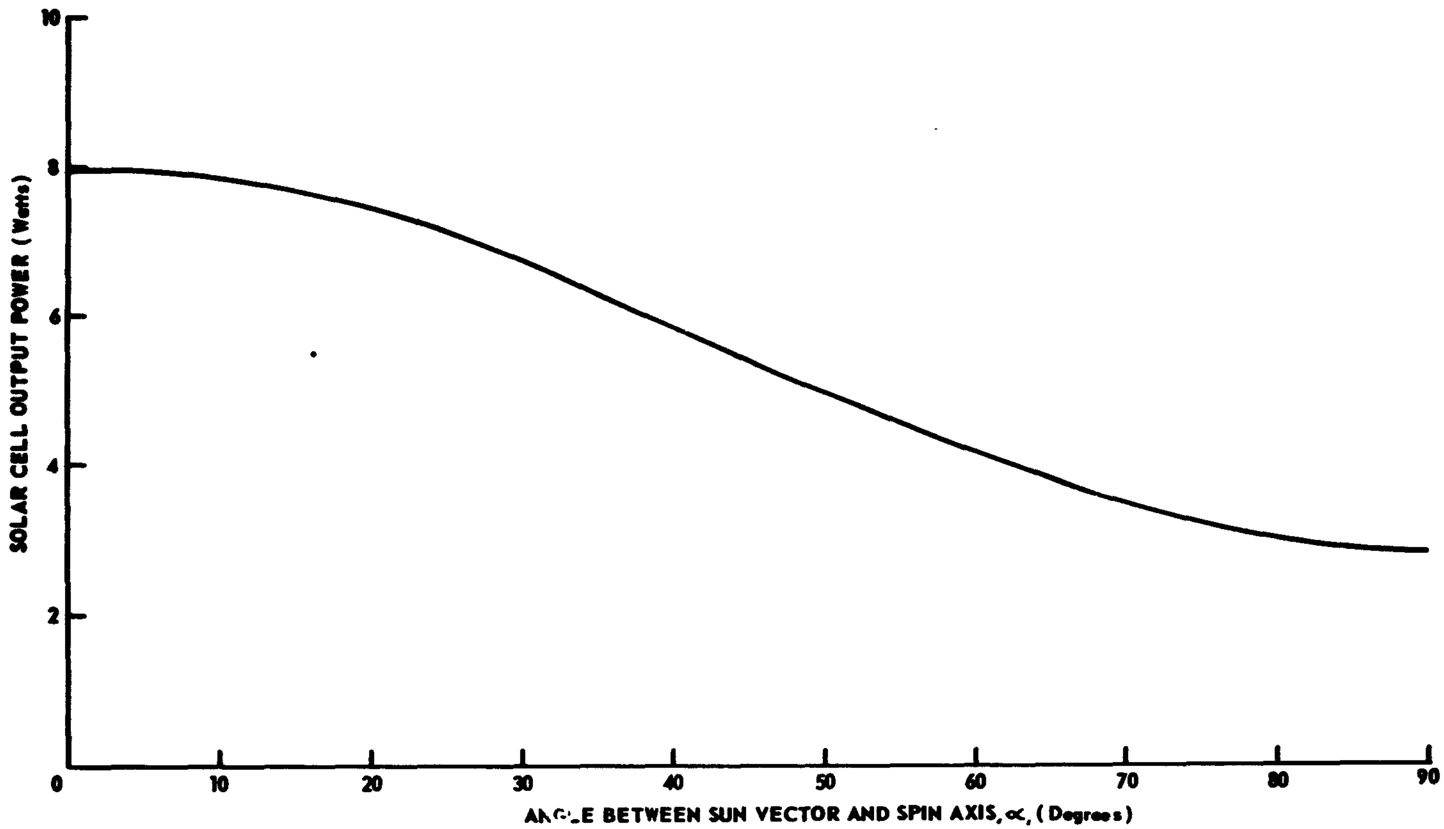

Fig. 24 SOLAR CELL OUTPUT AS A FUNCTION OF ANGLE BETWEEN SUN VECTOR AND SYMMETRY AXIS

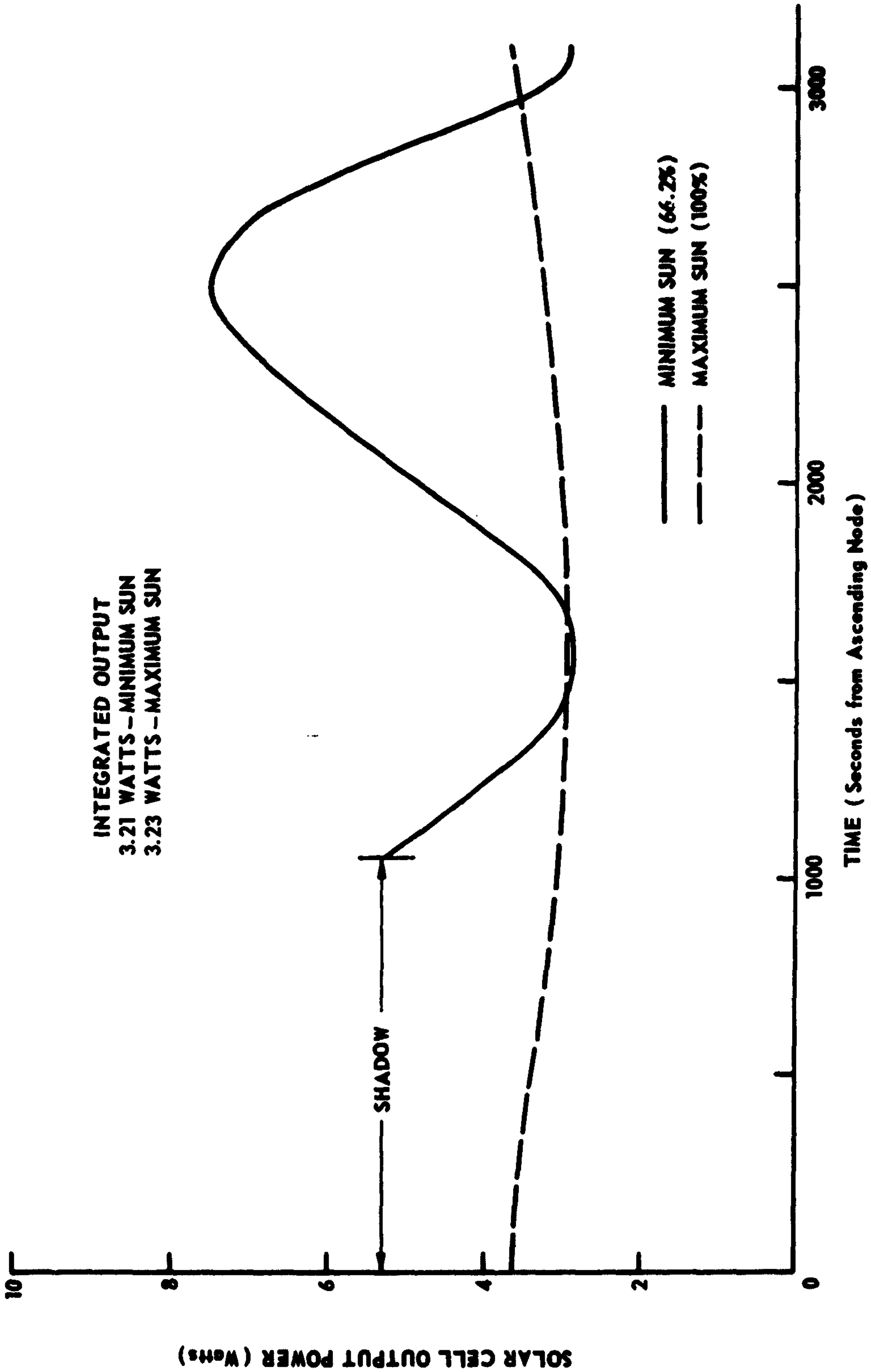

Fig. 25 SOLAR CELL OUTPUT AS A FUNCTION OF TIME FOR ONE HALF ORBITAL PERIOD, FOR A 500 NM CIRCULAR ORBIT, INCLINATION = 67.5 DEGREES

generated by the satellite is no greater in a 100 per cent sunlit orbit than it is in the darkest orbit (66.2 per cent illumination).

This method of providing electrical power for the satellite, so as to be relatively insensitive to the per cent sunlight, can be accomplished easily as well with gravity gradient attitude control. For the case of gravity gradient attitude control, it is reasonably simple to calculate the attitude of the satellite symmetry axis with respect to the sun; for magnetic attitude control the calculation is considerably more complex, but fortunately lends itself to solution by a digital computer.

F. T. Heuring and W. E. Allen have programmed the IBM 7090 computer to provide attitude and power information for a magnetically oriented satellite. The result of a typical calculation is shown in Fig. 26. The computer provides information as to the satellite symmetry axis attitude with respect to the sun (the column marked "PSI"), and with respect to the line of sight from a particular ground receiving station (the column marked "ALPHA"). By programming in the solar cell power output of the satellite as a function of the angle "PSI", the computer is capable of integrating the power output of the satellite over an orbital period. The end result of the calculation is the integrated power output of the satellite over an orbital period. This computer solution makes it possible to design magnetically oriented satellites so as to raise the minimum output power for any satellite orbit to as high a value as possible. This computation can also be used to predict the electrical power output of a satellite that has already been placed in orbit.

Telemeter results on the Transit 1B satellite have indicated that the solar cell output power is in agreement with that which is calculated for a magnetically oriented satellite.

SATELLITE SPIN-AXIS ORIENTATION TO SUN EARTH-LINE AND OBSERVER LINE OF SIGHT, MAGNETIC DIP AND SOLAR CELL OUTPUT

INPUTS- ASC NODE FROM GRNWH = 106.70 INCLINATION = 67.50 SM AXIS = 7303.2 ECC = 0.0001 ARG OF PER = 237.20
DAY NO. = 165 YEAR = 1961 OBSERVER COORDINATES- GEOC LAT = 39.16 GEOC LONG = -76.90

TIME (SEC UT)	LAT (DEG)	LONG (DEG)	ALT (NM)	DIP (DEG)	ALPHA (DEG)	AZIM (DEG)	ELEV (DEG)	PSI (DEG)	CURRENT (MA)	SUM MA DEL T
30840.00	-50.96	-42.59	504.52	55.83	SATELLITE BELOW HORIZON			77.72	955.25	955.25
30900.00	-53.68	-39.26	504.52	58.24	SATELLITE BELOW HORIZON			78.52	952.18	1907.43
30960.00	-56.28	-35.43	504.52	60.45	SATELLITE BELOW HORIZON			79.62	948.32	2855.75
31020.00	-58.70	-31.10	504.52	62.45	SATELLITE BELOW HORIZON			80.96	943.63	3799.38
31080.00	-60.97	-26.04	504.53	64.31	SATELLITE BELOW HORIZON			82.55	938.61	4737.99
31140.00	-62.96	-20.30	504.53	66.00	SATELLITE BELOW HORIZON			84.32	934.36	5672.35
31200.00	-64.69	-13.64	504.54	67.56	SATELLITE BELOW HORIZON			86.28	930.86	6603.21
31260.00	-66.04	-6.27	504.55	68.96	SATELLITE BELOW HORIZON			88.37	929.81	7533.02
31320.00	-66.99	1.95	504.56	70.24	SATELLITE BELOW HORIZON			89.38	929.31	8462.34
31380.00	-67.46	10.67	504.57	71.37	SATELLITE BELOW HORIZON			87.01	930.50	9392.83
31440.00	-67.42	19.42	504.59	72.34	SATELLITE BELOW HORIZON			84.58	933.85	10326.68
31500.00	-66.87	28.07	504.60	73.16	SATELLITE BELOW HORIZON			82.02	939.95	11266.63
31560.00	-65.87	36.07	504.62	73.80	SATELLITE BELOW HORIZON			79.43	949.01	12215.63
31620.00	-64.44	43.43	504.63	74.27	SATELLITE BELOW HORIZON			76.73	959.71	13175.35
31680.00	-62.69	49.86	504.65	74.53	SATELLITE BELOW HORIZON			74.02	976.87	14152.22
31740.00	-60.62	55.57	504.67	74.59	SATELLITE BELOW HORIZON			71.22	1001.76	15153.98
31800.00	-58.36	60.45	504.69	74.44	SATELLITE BELOW HORIZON			68.43	1033.56	16187.56
31860.00	-55.88	64.75	504.71	74.06	SATELLITE BELOW HORIZON			65.58	1067.68	17255.25
31920.00	-53.25	68.50	504.73	73.45	SATELLITE BELOW HORIZON			62.71	1106.39	18361.64
31980.00	-50.55	71.72	504.75	72.64	SATELLITE BELOW HORIZON			59.88	1141.44	19503.07
32040.00	-47.71	74.60	504.77	71.61	SATELLITE BELOW HORIZON			57.03	1176.11	20679.18
32100.00	-44.84	77.11	504.80	70.38	SATELLITE BELOW HORIZON			54.26	1204.55	21883.74
32160.00	-41.86	79.40	504.82	68.93	SATELLITE BELOW HORIZON			51.50	1229.74	23113.48
32220.00	-38.88	81.42	504.84	67.29	SATELLITE BELOW HORIZON			48.87	1253.01	24366.49
32280.00	-35.81	83.29	504.87	65.42	SATELLITE BELOW HORIZON			46.34	1271.62	25638.12
32340.00	-32.75	84.98	504.89	63.36	SATELLITE BELOW HORIZON			44.02	1288.83	26926.94
32400.00	-29.62	86.56	504.91	61.04	SATELLITE BELOW HORIZON			41.93	1300.41	28227.35
32460.00	-26.47	88.03	504.94	58.47	SATELLITE BELOW HORIZON			40.17	1310.08	29537.43
32520.00	-23.34	89.38	504.96	55.67	SATELLITE BELOW HORIZON			38.87	1316.10	30853.54
32580.00	-20.15	90.69	504.99	52.53	SATELLITE BELOW HORIZON			38.10	1319.56	32173.10
32640.00	-17.00	91.91	505.01	49.13	SATELLITE BELOW HORIZON			38.00	1320.00	33493.10
32700.00	-13.78	93.10	505.03	45.32	SATELLITE BELOW HORIZON			38.67	1316.99	34810.09
32760.00	-10.61	94.23	505.05	41.19	SATELLITE BELOW HORIZON			40.15	1310.17	36120.26
32820.00	-7.38	95.35	505.07	36.60	SATELLITE BELOW HORIZON			42.51	1297.17	37417.43
32880.00	-4.20	96.43	505.10	31.65	SATELLITE BELOW HORIZON			45.67	1276.49	38693.92
32940.00	-0.97	97.53	505.12	26.21	SATELLITE BELOW HORIZON			49.64	1246.92	39940.84
33000.00	2.27	98.61	505.14	20.36	SATELLITE BELOW HORIZON			54.27	1204.42	41145.26
33060.00	5.45	99.69	505.16	14.24	SATELLITE BELOW HORIZON			59.36	1147.95	42293.21
33120.00	8.68	100.80	505.18	7.77	SATELLITE BELOW HORIZON			64.90	1076.89	43370.10
33180.00	11.86	101.91	505.19	1.22	SATELLITE BELOW HORIZON			70.53	1008.68	44378.78
33240.00	15.08	103.08	505.21	-5.45	SATELLITE BELOW HORIZON			76.24	961.93	45340.71
33300.00	18.24	104.27	505.22	-11.94	SATELLITE BELOW HORIZON			81.67	941.17	46281.88
33360.00	21.44	105.53	505.24	-18.31	SATELLITE BELOW HORIZON			86.80	930.60	47212.48
33420.00	24.57	106.84	505.25	-24.27	SATELLITE BELOW HORIZON			88.62	929.69	48142.17
33480.00	27.74	108.25	505.26	-29.96	SATELLITE BELOW HORIZON			84.58	933.84	49076.01
33540.00	30.84	109.74	505.27	-35.17	SATELLITE BELOW HORIZON			81.24	942.67	50018.67
33600.00	33.96	111.37	505.28	-40.05	SATELLITE BELOW HORIZON			78.52	952.19	50970.87
33660.00	37.05	113.14	505.29	-44.54	SATELLITE BELOW HORIZON			76.46	960.93	51931.80
33720.00	40.05	115.05	505.29	-48.59	SATELLITE BELOW HORIZON			75.06	969.59	52901.39
33780.00	43.06	117.19	505.30	-52.35	SATELLITE BELOW HORIZON			74.23	975.42	53876.81
33840.00	45.97	119.54	505.30	-55.76	SATELLITE BELOW HORIZON			73.94	977.54	54854.35
33900.00	48.86	122.22	505.30	-58.93	SATELLITE BELOW HORIZON			74.13	976.12	55830.46
33960.00	50.27	123.56	505.30	-60.42	SATELLITE BELOW HORIZON			74.40	974.20	56804.67
34020.00	54.32	128.64	505.30	-64.50	SATELLITE BELOW HORIZON			75.73	964.91	57769.57
34080.00	56.86	132.53	505.30	-66.95	SATELLITE BELOW HORIZON			77.03	958.38	58727.96
34140.00	59.27	137.08	505.30	-69.23	SATELLITE BELOW HORIZON			78.62	951.83	59679.78
34200.00	61.48	142.32	505.29	-71.32	SATELLITE BELOW HORIZON			80.46	945.40	60625.18
34260.00	63.41	148.26	505.29	-73.20	SATELLITE BELOW HORIZON			82.47	938.83	61564.02
34320.00	65.06	155.11	505.28	-74.91	SATELLITE BELOW HORIZON			84.68	933.65	62497.67
34380.00	66.31	162.67	505.27	-76.37	SATELLITE BELOW HORIZON			87.00	930.50	63428.17
34440.00	67.15	171.03	505.26	-77.66	SATELLITE BELOW HORIZON			89.47	929.26	64357.43
34500.00	67.49	179.69	505.24	-78.65	SATELLITE BELOW HORIZON			87.99	930.01	65287.44
34560.00	67.33	-171.45	505.23	-79.34	SATELLITE BELOW HORIZON			85.33	932.34	66219.78
34620.00	66.68	-163.02	505.22	-79.68	SATELLITE BELOW HORIZON			82.64	938.40	67158.18
34680.00	65.56	-155.07	505.20	-79.67	SATELLITE BELOW HORIZON			79.86	947.49	68105.66
34740.00	64.07	-148.01	505.19	-79.30	SATELLITE BELOW HORIZON			77.08	958.14	69063.81
34800.00	62.22	-141.68	505.17	-78.62	SATELLITE BELOW HORIZON			74.23	975.40	70039.20
34860.00	60.09	-136.16	505.15	-77.67	SATELLITE BELOW HORIZON			71.36	1000.39	71039.59
34920.00	57.78	-131.44	505.13	-76.51	SATELLITE BELOW HORIZON			68.53	1032.38	72071.97
34980.00	55.26	-127.23	505.11	-75.15	SATELLITE BELOW HORIZON			65.66	1066.54	73138.51
35040.00	52.64	-123.71	505.09	-73.65	SATELLITE BELOW HORIZON			62.86	1104.39	74242.90
35100.00	49.88	-120.54	505.07	-71.99	SATELLITE BELOW HORIZON			60.05	1139.38	75382.28
35160.00	47.06	-117.78	505.04	-70.23	SATELLITE BELOW HORIZON			57.34	1172.61	76554.99
35220.00	44.13	-115.29	505.02	-68.30	60.85	292.50	0.31	54.66	1200.71	77755.60
35280.00	41.19	-113.10	505.00	-66.23	59.42	285.96	1.66	52.13	1223.84	78979.44
35340.00	38.15	-111.09	504.97	-64.09	57.33	278.77	2.71	49.70	1246.42	80225.85
35400.00	35.07	-109.26	504.95	-61.72	54.64	271.18	3.34	47.45	1263.84	81489.69
35460.00	32.01	-107.61	504.93	-59.23	51.45	263.50	3.50	45.47	1277.98	82767.67
35520.00	28.87	-106.06	504.90	-56.31	47.77	255.73	3.19	43.76	1290.33	84058.00
35580.00	25.76	-104.63	504.88	-53.62	43.86	248.35	2.44	42.44	1297.57	85355.56
35640.00	22.58	-103.28	504.85	-50.46	39.65	241.30	1.29	41.57	1302.38	86657.95
35700.00	19.44	-102.01	504.83	-47.08	SATELLITE BELOW HORIZON			41.24	1304.18	87762.13
35760.00	16.23	-100.78	504.81	-43.37	SATELLITE BELOW HORIZON			41.53	1302.59	89264.72
35820.00	13.06	-99.62	504.78	-39.43	SATELLITE BELOW HORIZON			42.49	1297.32	90562.03
35880.00	9.84	-98.48	504.76	-35.05	SATELLITE BELOW HORIZON			SATELLITE IN SHADOW OF EARTH		
35940.00	6.61	-97.36	504.74	-30.36	SATELLITE BELOW HORIZON			SATELLITE IN SHADOW OF EARTH		
36000.00	3.43	-96.28	504.72	-25.40	SATELLITE BELOW HORIZON			SATELLITE IN SHADOW OF EARTH		
36060.00	0.20	-95.19	504.70	-20.05	SATELLITE BELOW HORIZON			SATELLITE IN SHADOW OF EARTH		
36120.00	-2.99	-94.12	504.68	-14.50	SATELLITE BELOW HORIZON			SATELLITE IN SHADOW OF EARTH		
36180.00	-6.22	-93.02	504.66	-8.66	SATELLITE BELOW HORIZON			SATELLITE IN SHADOW OF EARTH		
36240.00	-9.40	-91.93	504.64	-2.78	SATELLITE BELOW HORIZON			SATELLITE IN SHADOW OF EARTH		
36300.00	-12.63	-90.79	504.62	3.21	SATELLITE BELOW HORIZON			SATELLITE IN SHADOW OF EARTH		
36360.00	-15.80	-89.63	504.61	9.05	SATELLITE BELOW HORIZON			SATELLITE IN SHADOW OF EARTH		
36420.00	-19.00	-88.41	504.59	14.79	SATELLITE BELOW HORIZON			SATELLITE IN SHADOW OF EARTH		
36480.00	-22.20	-87.13	504.58	20.27	SATELLITE BELOW HORIZON			SATELLITE IN SHADOW OF EARTH		
36540.00	-25.33	-85.81	504.57	25.37	SATELLITE BELOW HORIZON			SATELLITE IN SHADOW OF EARTH		
36600.00	-28.49	-84.37	504.56	30.20	SATELLITE BELOW HORIZON			89.44	929.28	91491.31
36660.00	-31.59	-82.86	504.55	34.60	SATELLITE BELOW HORIZON			86.96	930.52	92421.83
36720.00	-34.70	-81.20	504.54	38.70	SATELLITE BELOW HORIZON			84.98	933.04	93354.87
36780.00	-37.74	-79.42	504.53	42.40	SATELLITE BELOW HORIZON			83.54	936.16	94291.03
36840.00	-40.78	-77.44	504.53	45.83	SATELLITE BELOW HORIZON			82.56	938.59	95229.62
36900.00	-43.77	-75.24	504.52	48.95	SATELLITE BELOW HORIZON			82.06	939.86	96169.47
36960.00	-46.67	-72.82	504.52	51.75	SATELLITE BELOW HORIZON			81.97	940.10	97109.57
37020.00	-49.54	-70.07	504.52	54.33	SATELLITE BELOW HORIZON			82.27	939.33	98048.90
37080.00	-49.60	-70.25	504.52	54.39	SATELLITE BELOW HORIZON			82.32	939.21	98988.11

SATELLITE NUMBER AND POWER EQUATION IS 411
AVERAGE CURRENT IN MILLIAMPS FOR ONE REVOLUTION = 956.24

PERIOD = 103.518

*** END MAGNETIC ORIENTATION COMPUTATION ***

Fig. 26 COMPUTER RESULTS FOR A MAGNETICALLY ORIENTED SATELLITE

Regulating the Thermal Balance of a Satellite

A satellite which is essentially spherical in shape and which has a constant ratio of absorptivity to emissivity (α/ϵ) over its surface will run hotter in a 100 per cent sunlit orbit than it does in an orbit of minimum per cent illumination. If the satellite is cylindrical in shape, the ratio of cylinder length to cylinder diameter and the proper selection of α/ϵ ratios for the different portions of the cylinder can result in a curve of equilibrium temperature as a function of the angle α that closely simulates the power curve shown in Fig. 24. That is, the equilibrium temperature of the satellite in the sun can be made higher for small values of α and substantially lower for values of α in the vicinity of 90 degrees. A result would then be obtained which is analgous to the case of electrical power generation for the 100 per cent sunlit orbit as compared with the orbit of minimum per cent illumination. The net result can be a satellite that actually runs colder under 100 per cent illumination than it does in the darkest orbits.

Directional Satellite Antennas

A magnetically or gravity gradient attitude stabilized satellite is constrained in such a manner as to provide known angles between line of sight from a receiving station and the symmetry axis of the satellite. This result can be utilized in conjunction with directional antennas on the satellite to provide improved communication from the satellite to a ground receiving station and also from a ground transmitter to the satellite.

Let us consider a satellite in a circular orbit at a 500 nautical mile altitude with $i_g = 28$ degrees. Let us also consider the angle that the symmetry axis of a magnetically oriented satellite makes with the ground receiving station as the satellite having this orbit crosses the plane of longitude containing the receiving station. This case has been plotted for a particular location of a receiving station, and the result is illustrated in Fig. 27. From this curve it can be seen that, at this location, the satellite presents a nearly polar aspect to the ground receiving station. Furthermore the polarity of the magnet in the satellite requires that the <u>same</u> face of the satellite is always directed toward the ground receiving station. As a result, special satellite antennas can be utilized to improve the reception of radio signals to and from the satellite.

The Transit 1B and 2A satellites were designed so that the face of the satellite that is directed essentially downward at high latitudes in the northern hemisphere transmits left circular polarized radio waves and the other hemisphere produces the reverse polarity. At the Transit Development Station 01 at the Applied Physics Laboratory a test was run to determine if the Transit 2A satellite did in fact orient itself magnetically so as to provide left circular polarized radiation when it was observed at this station (Ref. 10). The experimental configuration at the Development Station is shown in Fig. 28. The results of these tests are shown in Fig. 29. The experimental data indicate that the hemisphere with the correct circular polarization always resulted in improved reception of satellite radio signals. The result of this investigation also showed that the satellite is magnetically oriented.

The fact that for some orbital inclinations the attitude of the satellite is directed so as to provide a predominantly

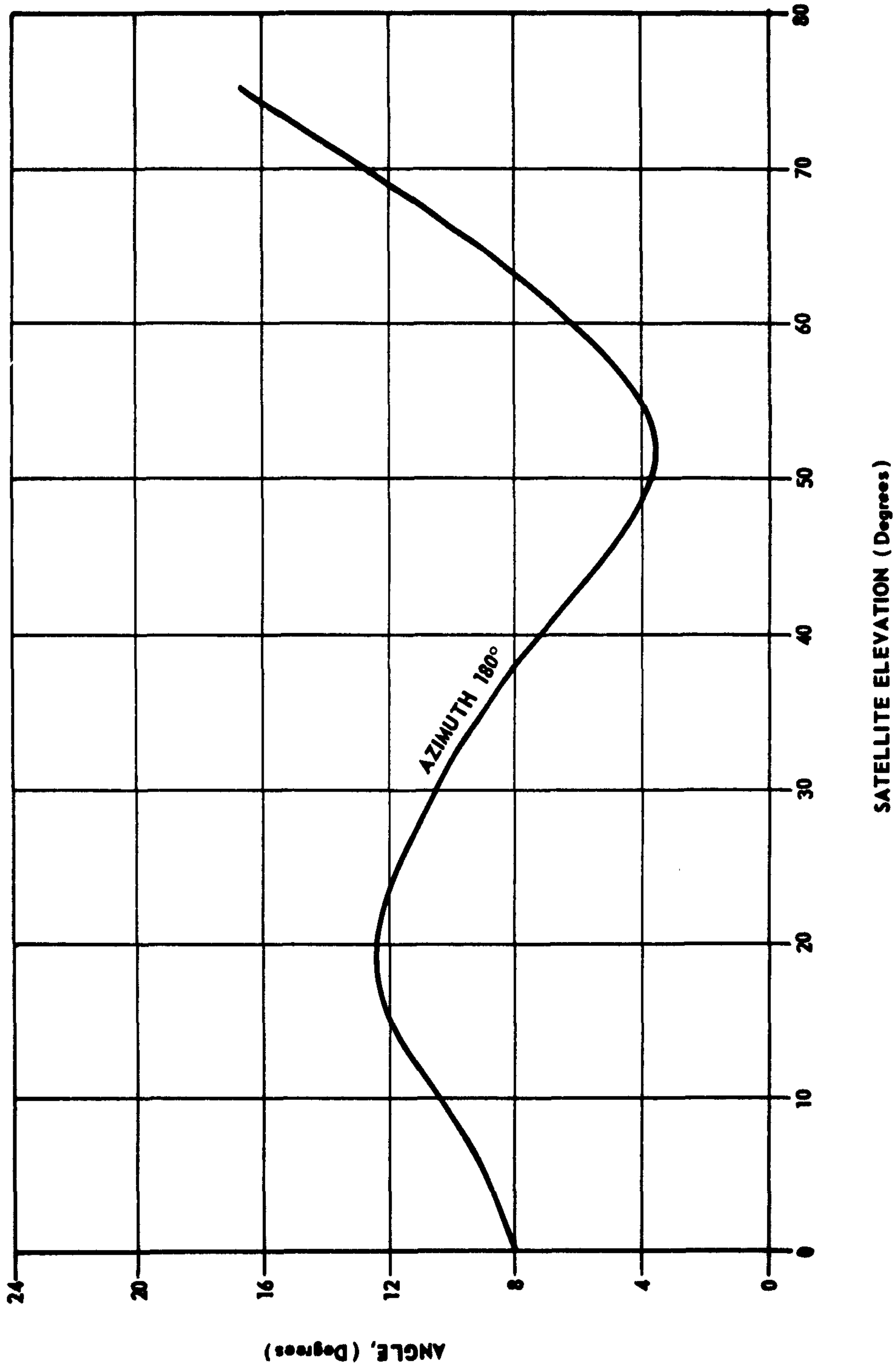

Fig. 27 ANGLE BETWEEN LINE OF SIGHT FROM LAS CRUCES, NEW MEXICO AND SATELLITE MAGNETIC AXIS AS A FUNCTION OF ELEVATION FOR 500 NM ALTITUDE CIRCULAR ORBIT

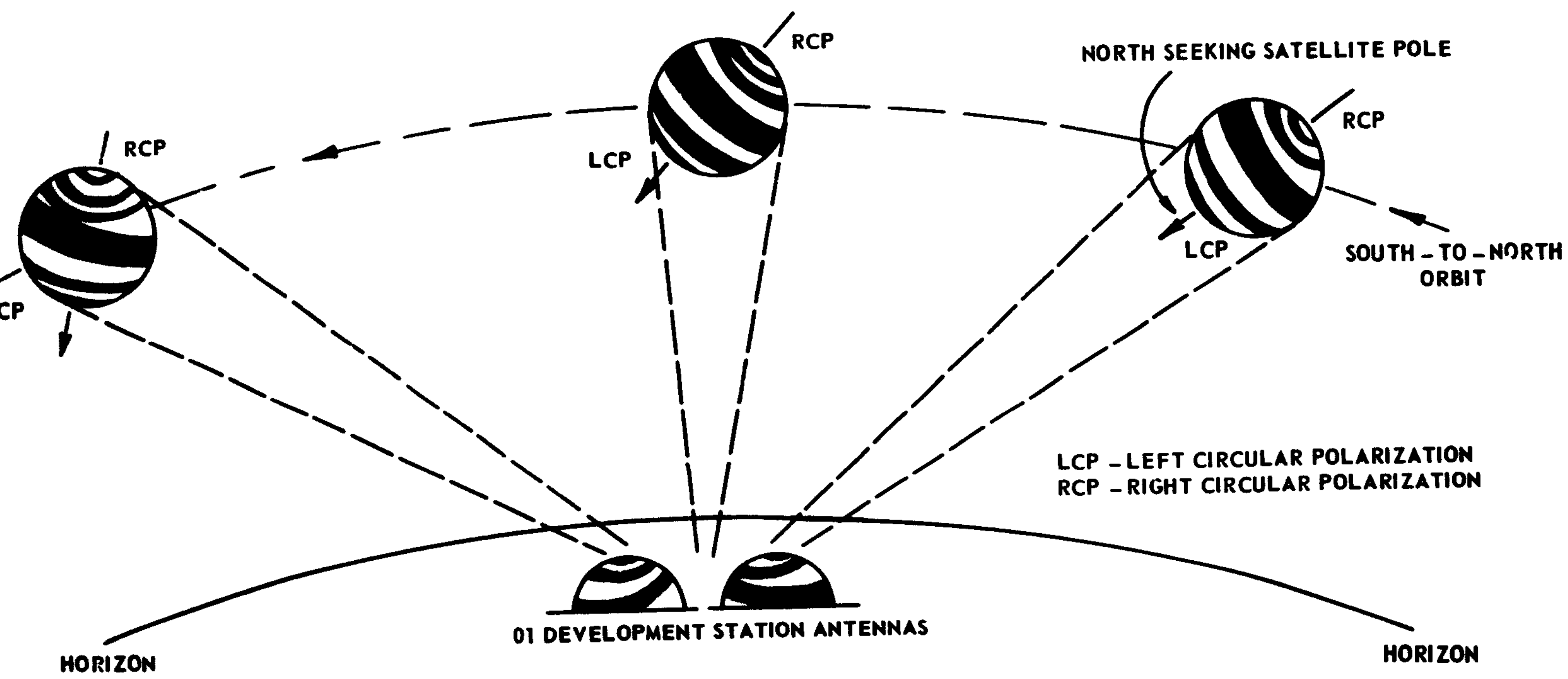

Fig. 28 SATELLITE ORBIT SHOWING CHANGE IN SATELLITE ORIENTATION WITH RESPECT TO A RECEIVING STATION

	UNIVERSAL TIME	AZIMUTH ANGLE	ELEVATION ANGLE	DOPPLER SHIFT (108 mcs)
RISE	1622	218	01	2202
PRE CA	1630	271	57	0643
POST CA	1631	318	59	-0277
SET	1638	025	02	-2244

(CA - CLOSEST APPROACH)

0 db = 1 MICROVOLT ACROSS 50 ohm ANTENNA LEADING TO CLARKE 2501 RECEIVER

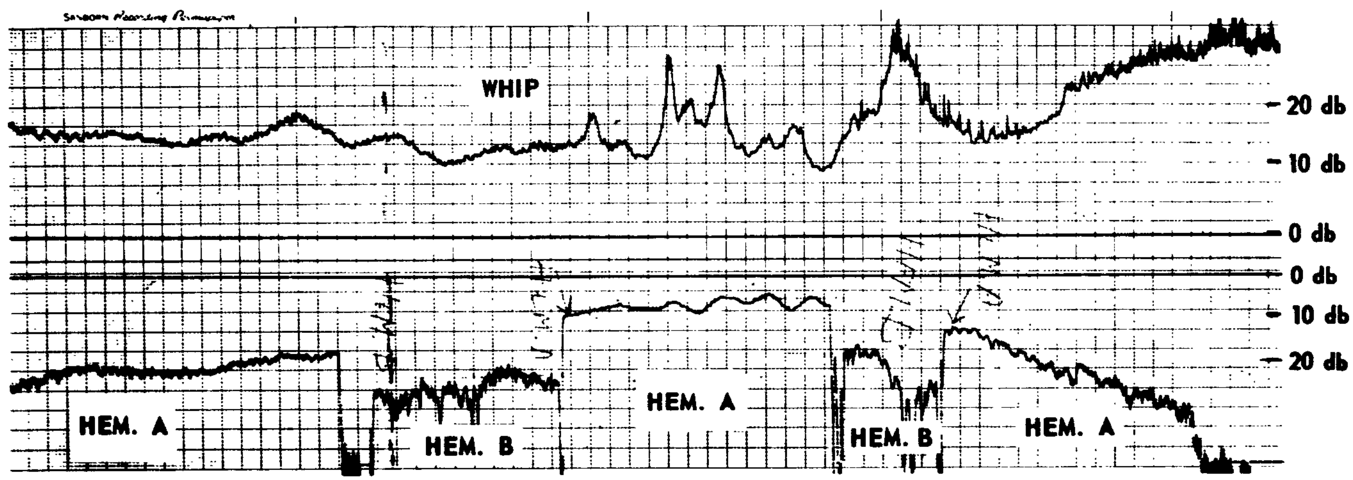

Fig. 29 SIGNAL STRENGTH DEPENDENCE ON TYPE OF RECEIVING ANTENNA

polar aspect of the satellite at some receiving stations (as illustrated in Fig. 27) suggests the use of directional antennas whose improved gain characteristics could enhance communication from the satellite to the ground and vice versa. Particularly if signals were only required to be observed in either the northern or southern hemisphere (but not both) a 3 db gain in the antenna power of transmission to the earth could be realized by radiating from only that side of the satellite that is magnetically directed downward in that hemisphere.

A gravity gradient attitude-stabilized satellite offers obvious advantages in improved radio communication from the satellite to a ground receiving station and from a ground transmitter to the satellite. If we know which side of a gravity oriented satellite is directed downward, and if we transmit all the satellite radiation in a region that covers only from horizon to horizon on the surface of the earth, we can obtain considerably increased radio signals from the satellite. In Fig. 30 is illustrated the power gain that is available at the surface of the earth for a gravity oriented satellite with a directional antenna that covers only from horizon to horizon. For this case the power received at the surface of the earth is compared with a satellite producing isotropic radiation. The power gain available in this manner is particularly attractive for satellites orbiting at altitudes greater than 500 miles. By use of this technique the reduction in signal strength that accompanies operation at a very high altitude can be almost completely compensated for by the increased gain of the directional antenna.

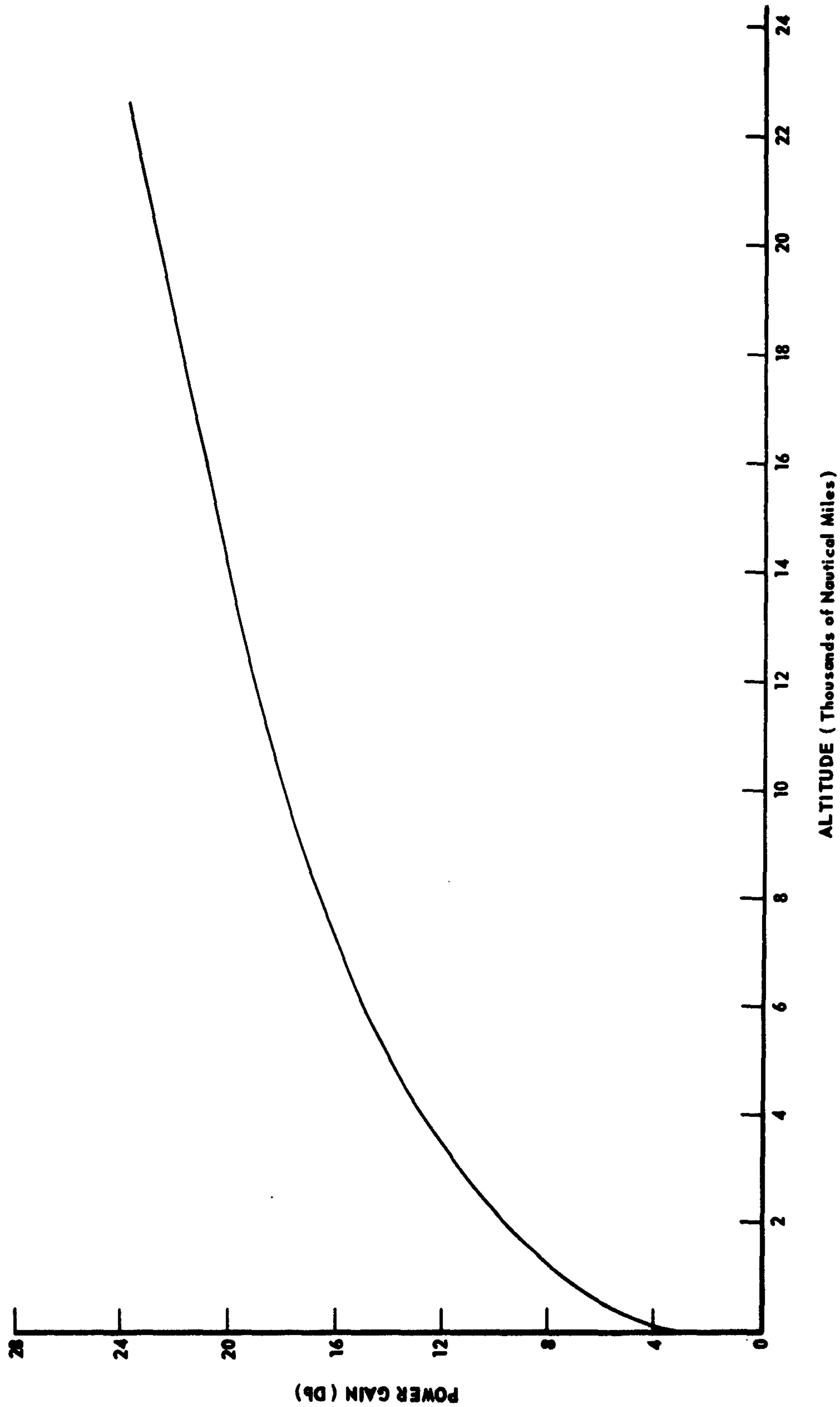

Fig. 30 POWER GAIN AT THE SURFACE OF THE EARTH FOR A GRAVITY ORIENTED SATELLITE WITH A DIRECTIONAL ANTENNA COVERING FROM HORIZON TO HORIZON

Optical Tracking of a Flashing Light Satellite

The most accurate tracking of a satellite can be accomplished by telescopic observations of a brilliant, flashing light mounted on the satellite. The advantages of magnetic and gravity gradient attitude control in directing a particular axis toward an observer on the ground applies equally well to optical observations of a source of light as it does for radio transmission. One immediate advantage of magnetic or gravity orientation is that a particular face of the satellite can be directed downward in a particular hemisphere of the earth and therefore a flashing light is required on only one side of the satellite. Furthermore, particularly when combined with gravity gradient attitude control, the beam width of the light can be narrowed so as to provide no greater than horizon-to-horizon coverage on the surface of earth. The improved intensity of the light as received on the surface of the earth provides better observations and also makes possible flashes of shorter time duration, thereby resulting in improved tracking accuracy.

The Performance of Scientific Experiments using Altitude Control

For many types of scientific experiments it is desirable to know the attitude of the satellite relative to the earth and/or relative to the sun.

A radiation research satellite currently being designed under the direction of Prof. James A. Van Allen of the State University of Iowa will employ magnetic attitude stabilization to determine the directional properties of geomagnetically trapped radiation. This type of orientation is particularly useful in studies of the Van Allen radiation belt where the particles are believed to move along the lines of the earth's magnetic force field. For this experiment, radiation counters will be employed that measure particle fluxes along particular directions with respect to the local direction of the earth's magnetic field. Table I lists the type of experiment to be performed in this satellite and the orientation of the detectors relative to the direction of the earth's magnetic field.

A gravity oriented satellite is capable of conducting experiments where the directional properties of radiation going out from the earth or going toward the earth is of interest.

As previously shown, a satellite with magnetic or gravity gradient attitude stabilization has a controlled and predictable angular aspect with respect to the sun vector. This fact can be utilized in the performance of experiments where the aspect of the satellite with respect to the sun is a necessary attribute of the experiment.

TABLE I

Studies to be Performed with the Iowa State University Radiation Research Satellite

Instrument	Detection Feature	Orientation*
CdS total energy	Protons > 2 kev Electrons > 100 ev Light	$\theta = 180^{o}$
CdS with magnetic broom	Protons > 2 kev Electrons > 250 kev Light	$\theta = 180^{o}$
CdS total energy	Protons > 2 kev Electrons > 100 ev Light	$\theta = 90^{o}$
CdS with magnetic broom	Protons > 2 kev Electrons > 250 kev Light	$\theta = 90^{o}$
CdS optical monitor	Light	$\theta = 90^{o}$
213 GM counter	Electrons > 20 kev Protons > 0.5 Mev	$\theta = 90^{o}$
Photometer	Light of 5577 Å	$\theta = 0^{o}$
Magnetic spectrometer		
GM a GM b GM c	Electrons 40-50 kev Electrons 90-110 kev Background monitor	$\theta = 90^{o}$
Solid state detector	Protons 1-10 Mev	$\theta = 90^{o}$
Solid state detector	Protons 10-30 Mev	$\theta = 90^{o}$
Solid state detector	Protons 1-10 Mev	$\theta = 180^{o}$
Solid state detector	Protons 10-30 Mev	$\theta = 180^{o}$

*Orientation is referred to the magnetic field line, with $\theta = 0^{o}$ being the downward direction in the northern hemisphere.

The Use of Rockets for Altering the Satellite's Orbit

If an orbiting vehicle has rocket power for propelling the vehicle along an axis whose attitude is controlled, it is possible to alter the shape of the orbit for any number of useful purposes. This would be accomplished by causing the rocket to be fired by ground command when the space vehicle attains a particular orientation (Ref. 13). A system of magnetic attitude control would be particularly desirable for this purpose since, for high inclination orbits, it provides virtually any angle of the satellite's magnetic axis with respect to the earth. For example, as the satellite crosses the earth's magnetic equator it will be oriented with its magnetic axis parallel to the earth's surface. At this point a rocket fired along the direction of the magnetic axis can be used to either increase or decrease the velocity of the space vehicle. By selecting the time of firing, viz. when perigee is at the earth's magnetic equator, this system can be used to increase the satellite's perigee altitude, thereby resulting in a more circular orbit.

There are many other conditions of firing rockets along the direction of motion of the satellite, or firing retro-rockets which can be used to shape the orbit of the space vehicle.

V. CONCLUSIONS

The use of a strong magnetic dipole moment in the satellite in conjunction with magnetic damping rods can provide alignment of a satellite axis along the local direction of the earth's magnetic field. A magnetically oriented satellite can be the first step toward orienting a satellite along the direction of the gradient of the earth's gravitational field. The condition of gravity attitude stabilization can then be realized by extending a boom with a mass at its end as the satellite passes over a magnetic pole of the earth. A means for damping the libration oscillation of a gravity oriented satellite can be provided.

A satellite whose attitude is controlled offers many advantages in the performance of scientific missions in space.

REFERENCES

1. F. T. Heuring, "Magnetic Orientation," The Johns Hopkins University, Applied Physics Laboratory, Memorandum to H. D. Black, November 23, 1960.

2. J. F. Smola, "An Alternate Solution to the Problem of Despinning an Earth Satellite and an Extension of the Theory to Improve the Control over Residual Angular Velocity," SSD-5-6, The Johns Hopkins University, Applied Physics Laboratory Memorandum, May 20, 1960.

3. R. E. Fischell, "Magnetic Damping of the Angular Motions of Earth Satellites," ARS Paper No. 1502-60, presented at The American Rocket Society Annual Meeting, Washington, D. C., December 5-8, 1960.

4. R. D. Bozorth and D. M. Chapin, "Demagnetizing Factors of Rods," Journal of Applied Physics, Vol. 13, pages 320-326, May 1942.

5. Private communication with W. E. Allen of The Johns Hopkins University, Applied Physics Laboratory.

6. Private communication with F. F. Mobley of The Johns Hopkins University, Applied Physics Laboratory.

7. R. R. Newton, "Method of Stabilizing an Astronomical Telescope," Journal of The American Rocket Society, p. 665-666, September 1959.

8. R. E. Fischell and R. R. Newton, "Proposal for a Series of Research Satellites, "The Johns Hopkins University, Applied Physics Laboratory Memorandum SRA-10-60, September 27, 1960.

9. W. E. Allen, "Computer Program for Magnetic Orientation," Johns Hopkins University, Applied Physics Laboratory Memorandum SSE-4-006, March 10, 1961.

10. L. Walters and R. Finneran, The Johns Hopkins University, Applied Physics Laboratory Memorandum SGS/STSS01-005, November 16, 1960.

11. L. J. Kamm, "Vertistat -- An Improved Satellite Orientation Device," ARS Paper No. 1503-60, presented at the American Rocket Society Annual Meeting, Washington, D. C., December 5-8, 1960.

12. Private communication with F. F. Mobley and R. G. Roll of The Johns Hopkins University, Applied Physics Laboratory.

13. J. Dassoulas, "Pick-a-back Satellite Techniques," The Johns Hopkins University, Applied Physics Laboratory Memorandum SDO-112, October 26, 1960.

ACKNOWLEDGMENT

The author wishes to acknowledge the invaluable assistance of Mr. Walter E. Allen for his efforts in performing calculations, deriving certain equations, and preparing many of the illustrations. Mr. F. F. Mobley and Mr. F. T. Heuring performed the computer programming that was required to solve complex problems relating to magnetic attitude stabilization. Messrs. A. B. Leak and R. G. Roll offered valuable suggestions on various matters relating to attitude control.

The author also wishes to express his appreciation to Messrs. L. Parsons, D. I. Gordon, E. Adams, H. Helms, and R. Sery of the U. S. Naval Ordnance Laboratory, White Oak, Maryland, for their invaluable assistance in providing magnetic materials, magnetic testing facilities, and technical consultation.

Finally the author wishes to acknowledge the efforts of Mrs. E. J. Lopez whose secretarial work in the preparation of the manuscript often went "beyond the call of duty."

This report is one of the CM series which consists of technical papers or extensive progress reports that are characterized by an intensive treatment of the subject matter. CM reports have received official Laboratory review to substantiate the validity of their technical content and to establish suitability for distribution outside of Section T to qualified personnel in the pertinent technical field.

Initial distribution of this document has been made in accordance with a list on file in the Technical Reports Group of the Applied Physics Laboratory, The Johns Hopkins University.