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Game Theory

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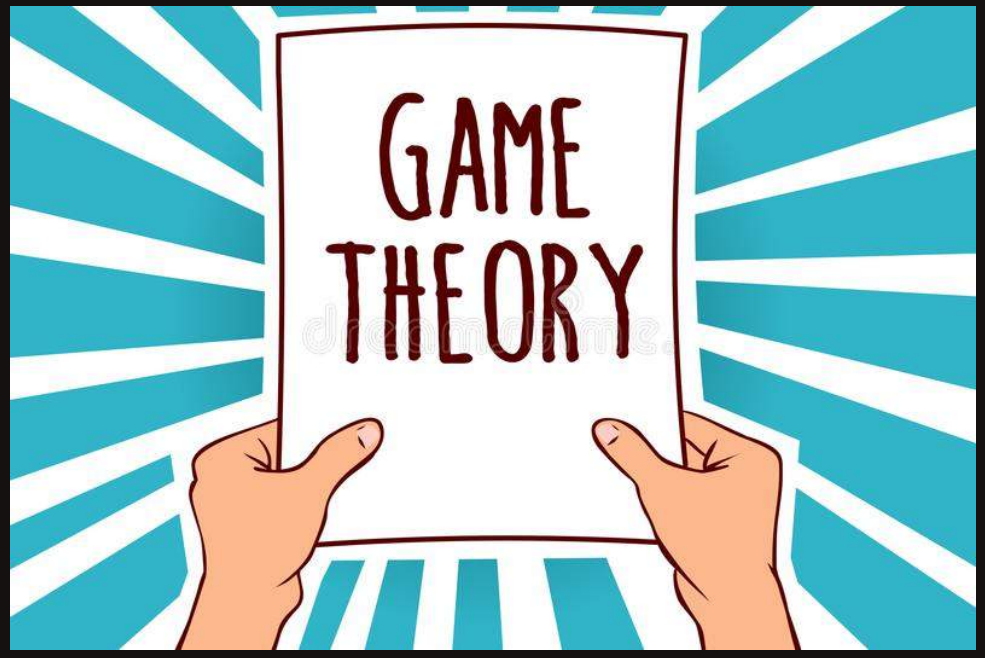
ABOUT THE PROJECT

GAME THEORY, BRANCH OF APPLIED MATHEMATICS THAT PROVIDES TOOLS FOR ANALYZING SITUATIONS IN WHICH PARTIES, CALLED PLAYERS, MAKE DECISIONS THAT ARE INTERDEPENDENT.

THIS PROJECT IINVOLVES SOME OF THE ASPECTS OF GAME THEORY.

" GAME THEORY IS ABOUT EXPLORING OF FREEDOM OF CHOICES AND THE EQUILIBRIUM WHICH COMES FROM UNDERSTANDING THE CONSEQUENCES "

-VINEET RAJ KAPOOR



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WHAT IS AN AUCTION?

AUCTION IS THE MECHANISM OF 'ALLOCATION' OF A PARTICULAR OBJECT AT A CERTAIN PRICE.

'**Allocating**' part concerns who will get the object and the price part speaks for itself. There are several ways to design this mechanism, namely there is more than one way to conduct an auction.

One needs an auction to allocate an object and to decide on the price because some players have private information relevant for the decisions of other players, and as rationality requires them, they use that private information to maximize their payoffs.

THE TYPES DISCUSSED

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1. THE FIRST-PRICE, SEALED-BID AUCTION

2. THE SECOND-PRICE, SEALED-BID (VICKREY) AUCTION

THEORY

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FIRST PRICE, SEALED-BID AUCTION

IN A SEALED BID, OR FIRST PRICE, AUCTION, BIDDERS SUBMIT SEALED BIDS B1, ..., BN. **THE BIDDERS WHO SUBMITS THE HIGHEST BID IS AWARDED THE OBJECT, AND PAYS HIS BID.**

UNDER THESE RULES, IT SHOULD BE CLEAR THAT BIDDERS WILL NOT WANT TO BID THEIR TRUE VALUES. BY DOING SO, THEY WOULD ENSURE A ZERO PROFIT. BY BIDDING SOMEWHAT BELOW THEIR VALUES, THEY CAN POTENTIALLY MAKE A PROFIT SOME OF THE TIME.

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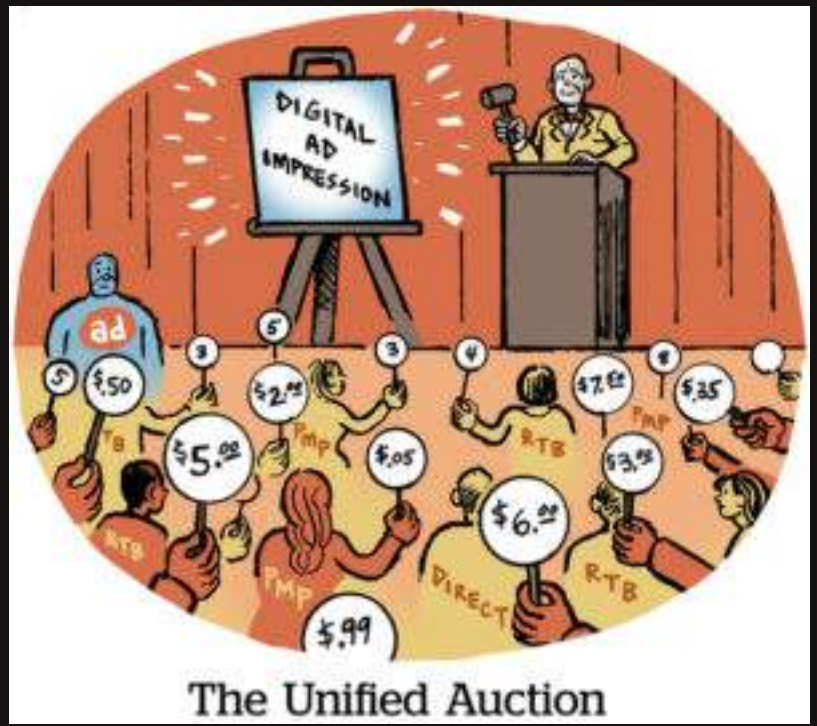
THEORY

SECOND PRICE, SEALED-BID (VICKREY) AUCTION

IN A VICKREY, OR SECOND PRICE, AUCTION, BIDDERS ARE ASKED TO SUBMIT SEALED BIDS B1, ..., BN. **THE BIDDER WHO SUBMITS THE HIGHEST BID IS AWARDED THE OBJECT, AND PAYS THE AMOUNT OF THE SECOND HIGHEST BID.**

AS IN OTHER VICKREY AUCTIONS, IT IS A DOMINANT STRATEGY IN A SECOND-PRICE AUCTION FOR A BIDDER TO BID THEIR TRUE VALUE.

IT GOT IT'S NAME IN THE HONOR OF WILLIAM VICKREY, WHO WROTE THE FIRST GAME-THEORITIC ANALYSIS OF THE AUCTION



The Unified Auction

AUCTION THEORY

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MODELLING AN AUCTION AS A GAME

WE HAVE PLAYERS WHO SELFISHLY MAXIMIZE THEIR PAYOFFS. USING ALL THE INFORMATION AVAILABLE TO THEM, THEY CHOOSE THEIR BEST RESPONSES TO OTHER PLAYER'S STRATEGIES.

ALL THE GAMES MUST HAVE SOME RULES IN IT. RULES OF THE GAME INCLUDE: **PLAYERS, STRATEGIES AVAILABLE FOR EACH PLAYER, PAYOFFS FOR ALL POSSIBLE COMBINATIONS OF STRATEGIES OF ALL PLAYERS.**

WE NEED TO ADD ONE MORE ELEMENT AS **ASYMMETRIC INFORMATION** THAT IS EACH PLAYER CAN BE OF MORE THAN ONE TYPE AND EACH PLAYER'S TYPE IS HIS PRIVATE INFORMATION.

ASSUMPTIONS WHILE ANALYSING DATA

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- EACH BUYER KNOWS FOR SURE HIS OWN VALUATION AND THE NUMBER OF BUYERS WHO PARTICIPATE IN THE AUCTION, BUT NOT THE EXACT VALUATIONS OF THE OTHER BUYERS.
- EACH BUYER KNOWS THE DISTRIBUTION OF VALUATIONS: RANGE OF ALL POSSIBLE VALUATIONS FOR THE OBJECT AND THE PROBABILITY WITH WHICH EACH VALUATION HAPPENS.
- AFTER THE GAME IS OVER, THE PLAYERS RECEIVE THEIR PAYOFFS THAT ARE DETERMINED BY THE STRATEGIES THAT ALL PLAYERS CHOSE TO PLAY.

ASSUMPTIONS WHILE ANALYSING DATA

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- AUCTIONEER HAS NO CLUE WHETHER ACTUAL VALUATIONS OF ALL BUYERS WHO CAME TO HIS AUCTION ARE HIGH, LOW, ONE HIGH AND OTHERS DOWN, TWO HIGH AND A BUNCH OF ORDINARY, BUNCH OF HIGH AND BUNCH OF JOINT. ALL VALUATIONS ARE PRECISELY AVERAGE, ETC.

- EACH BUYER ATTEMPTS TO MAXIMIZE ITS PROFIT, THUS THEY WILL CHOOSE THE STRATEGY WHICH GIVES THEM THE HIGHEST PAYOFFS, GIVEN THE EXPECTED STRATEGIES OF OTHER PLAYERS

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GAME 1:

THE CLOSEST ONE!!

GAME ON FIRST AUCTION THEORY

LET'S BEGIN

EXPLAINATION OF THE GAME

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TO CONDUCT AN EXPERIMENT ANALYSTS ASKED PEOPLE TO BID FOR A COMMODITY.

THERE WILL BE A TOTAL OF EIGHT ROUNDS AND THE BIDDER WHO WINS THE MAXIMUM NUMBER OF ROUNDS WILL BE DECLARED AS THE WINNER AND HE HAS TO PAY THE AMOUNT EQUAL TO THE MEAN OF HIS BIDS.

DETAILS OF THE GAME

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- THE NUMBER OF TIMES THE GAME WAS PLAYED - 4
- THE NUMBER OF PARTICIPANTS IN THE GAME - 4 (CAN BE ANY NUMBER)
- SAMPLE SIZE TOTAL - 16

Turn	A bid	B bid	C bid	D bid		Highest Bid		
Game 1								
1	500	450	550	525		C		
2	525	475	550	575		D		
3	550	500	575	575		RANDOM ALLOTMENT D WON		
4	625	550	600	600		A		
5	650	650	600	625		RANDOM ALLOTMENT B WON		
6	675	700	650	650		B		FINAL WINNER D
7	750	725	725	700		A		
8	800	800	800	825		D		

WE CAN CLEARLY SEE FINAL WINNER OF THE AUCTION IS D AND HE HAS TO PAY THE MEAN OF HIS BIDS.

A LOOK AT THE DATA

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MATHEMATICS INVOLVED!!

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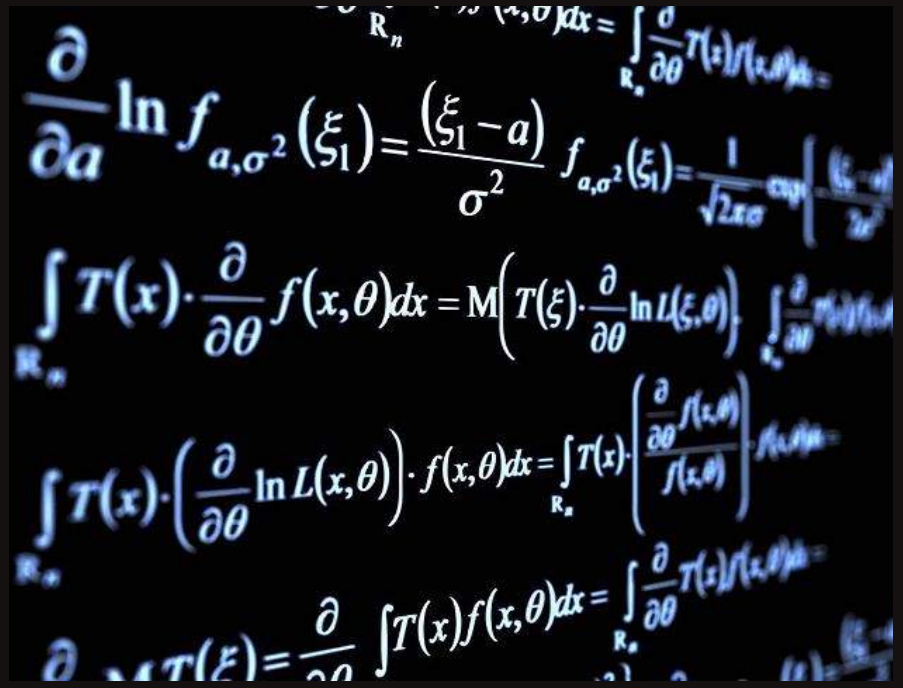

FIRST PRICE AUCTION'S MATHEMATICS

IN THIS SETTING, WE ASSUME BIDDERS HAVE A RISK-NEUTRAL, QUASI-LINEAR UTILITY FUNCTIONS OF THE FORM .

$$u_i(b_i, \mathbf{b}_{-i}) = v_i x_i(b_i, \mathbf{b}_{-i}) - p_i(b_i, \mathbf{b}_{-i}).$$

THIS MEANS, THAT THE RESULTING PAYOFF EACH BIDDER RECEIVES IS

$$u_i(b_i, \mathbf{b}_{-i}) = \begin{cases} v_i - b_i, & \text{if } i = i^* \\ 0, & \text{otherwise.} \end{cases}$$

IF ALL BIDDERS BID THEIR VALUATION (I.E., THEY ARE TRUTHFUL IN THEIR REPORTS), THEN ALL BIDDERS WILL RECEIVE ZERO PAYOFF. CONSEQUENTLY, IT IS IN EACH BIDDER'S INTEREST TO SHADE THEIR BID: EACH BIDDER WILL WANT TO REPORT A BID SO THAT IN EXPECTATION, THEY RECEIVE POSITIVE PAYOFF THAT IS, BIDDERS ARE SOLVING FOR THE FOLLOWING:

$$b_i = \max_{x \in \mathbb{R}} (v_i - x) \prod_{i \neq j \in N} \Pr(x \geq b_j).$$

MATHEMATICS INVOLVED!!

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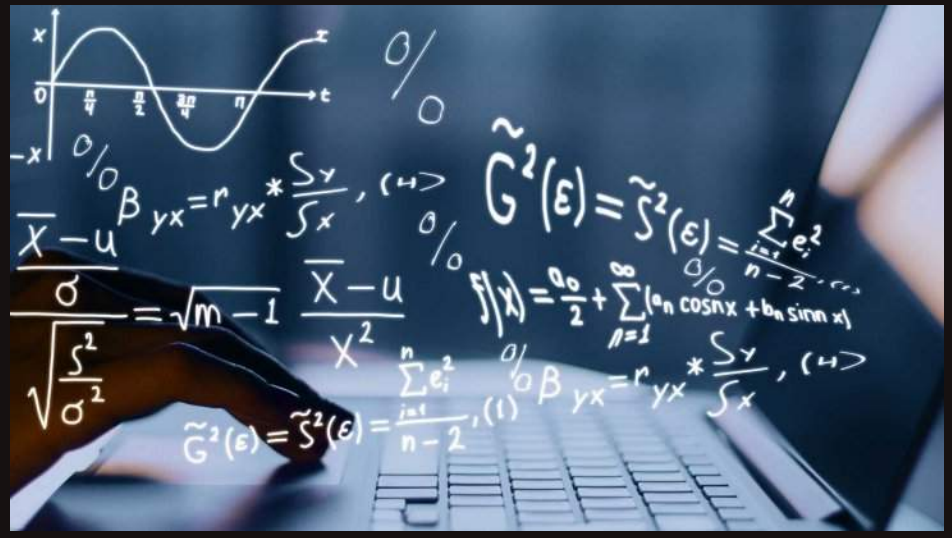
FIRST PRICE AUCTION'S MATHEMATICS



THE PAYOFF FUNCTION PI OF PLAYER I IN THE GAME ASSOCIATED WITH THE FIRST-PRICE AUCTION IS DEFINED AS FOLLOWS, WHERE B IS THE VECTOR OF THE SUBMITTED BIDS. ALSO HIS VALUE FOR THE SOLD OBJECT IS VI(ASSUMPTION FOR THE ABOVE FORMULA)

IF WE WERE TO STATE IT IN SIMPLE WORDS THEN BUYER'S EXPECTED UTILITY IS

$$EU = (Valuation - bid)(probability - to - win)$$

IMPORTANT NOTE

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NOTE THAT THE WINNER'S PROFIT CAN BE NEGATIVE. THIS HAPPENS WHEN HE WINS THE OBJECT BY OVERBIDDING, I.E., SUBMITTING A BID HIGHER THAN HIS VALUATION OF THE OBJECT BEING SOLD. SUCH A SITUATION IS CALLED THE WINNER'S CURSE.

OUTCOMES

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IN SUCH KIND OF AUCTION THE OPTIMAL BID TURNS OUT TO BE IN SOME PROPORTIONS OF THE BUYER'S ESTIMATION OF THE PERFECT INITIAL PRICE. AS ALREADY STATED IN FIRST PRICE AUCTION,THE ONE WITH THE HIGHEST BID WINS.

WE ALREADY KNOW THAT BIDS ARE PROPORTIONAL TO BUYERS' VALUATIONS AND THE HIGHER IS THE VALUATION, THE HIGHER IS THE BID. THUS THE BUYER WITH THE HIGHEST ESTIMATION OF THE COMMODITY SUBMITS THE HIGHEST BID AND WINS.

FINAL CONCLUSION - PRICE IS EQUAL TO THE HIGHEST BID AND IT IS SOME PROPORTION OF THE HIGHEST VALUATION.

ANALYSIS OF THE OUTCOMES

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IN FIRST-PRICE SEALED BID AUCTION EACH BUYER'S OPTIMAL BID IS EQUAL TO SOME FRACTION OF THE BUYER'S TRUE VALUATION (MEANS THAT OPTIMAL BID IS ALWAYS BELOW THE VALUATION). AS THE NUMBER OF BUYERS GOES UP, CHANCES OF WINNING FOR EACH BUYER GO DOWN, THEREFORE, BUYERS SHOULD BID A HIGHER PROPORTION OF THEIR VALUATION. BUYERS' BIDS DEPEND ON BUYERS BELIEFS ABOUT OTHER BUYERS' BIDS AND ALSO ON THE NUMBER OF BUYERS

EVEN THOUGH MOST OF THE PRICES WERE IN LINE WITH THE MODERN AUCTION THEORY, **THERE WERE SOME DEVIATIONS.**

MOREOVER AS THE WINNING BID IS BEING ANNOUNCED AFTER EACH ROUND THEREFORE BIDDERS ARE GETTING A CHANCE TO ADJUST THEIR BID AND **PLACE THEMSELVES IN OTHER'S SHOES** TO ESTIMATE THEIR NEXT MOVE,THUS GIVING THEM ANOTHER CHANCE TO WIN. ANOTHER REASON IS THEY HAD TO PAY THE **MEAN OF ALL THE BIDS** PLACED BY THEM.

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GAME 2:

SCOTLAND YARD

GAME ON SECOND AUCTION THEORY

LET'S BEGIN

EXPLAINATION OF THE GAME

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IN THE GAME OF SCOTLAND YARDS,TWO PLAYER EACH HAVING 50 BUS TICKETS HAVE TO REACH CLOSEST TO MR X. THEY DECIDED TO BID AND THE WINNER OF THE BID HAS TO GIVE THE AVERAGE OF OTHER PLAYERS BID AS BUS TICKETS.

NOTE THAT UPDATING EACH PLAYER' S TICKET COUNT ONLY DEPENDS ON THE DIFFERENCE BETWEEN THEIR BIDS, NOT THE BID VALUES THEMSELVES. IN CASE OF A TIE RANDOM ALLOTMENT IS BEING DONE.

THE ONE WHO REACHES CLOSEST TO MR X WILL BE DECLARED AS THE WINNER.

DETAILS OF THE GAME

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- NUMBER OF TIMES THE GAME WAS PLAYED - 6
- NUMBER OF PARTICIPANTS IN THE GAME - 2
- SAMPLE SIZE - 12

Turn	A	B	A Bid	B Bid	Score	
Game 2						
1	50	50	5	6	0--1	
2	51	49	8	10	0--2	
3	53	47	13	10	1--2	
4	50	50	20	15	2--2	
5	45	55	45	55	2--3	B WON

THE BIDDER B WINS THE OVERALL AUCTION AND HAS TO PAY THE MEAN OF THE VALUES A USED IN BIDDING.

A LOOK AT THE DATA

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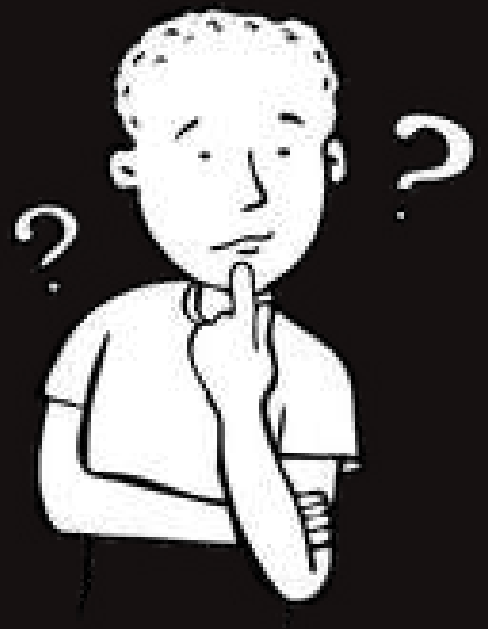
https://docs.google.com/spreadsheets/d/1ks1sbydedqhy_g7omx4mrpvwchvcndw1hvhsk2ys-ma/edit#gid=0



MATHEMATICS INVOLVED!!

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SECOND-PRICE AUCTION'S MATHEMATICS

AS BEFORE THE WINNER IS THE BIDDER WHO SUBMITTED THE HIGHEST BID, BUT NOW HE PAYS TO THE SELLER THE AMOUNT EQUAL TO THE SECOND HIGHEST BID.

SO IN THIS AUCTION IN THE ABSENCE OF TIES THE WINNER PAYS TO THE SELLER A LOWER PRICE THAN IN THE FIRST-PRICE AUCTION.

THE PAYOFF FUNCTION PI OF PLAYER I IN THE GAME ASSOCIATED WITH THE SECOND-PRICE AUCTION IS DEFINED AS FOLLOWS, WHERE B IS THE VECTOR OF THE SUBMITTED BIDS. ALSO HIS VALUE FOR THE SOLD OBJECT IS VI.

THE PAYOFFS ARE NOW DEFINED AS FOLLOWS:

$$p_i(b) := \begin{cases} v_i - \max_{j \neq i} b_j & \text{if } i = \operatorname{argsmax} b \\ 0 & \text{otherwise} \end{cases}$$

OUTCOMES

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WINNER IS THE BUYER WITH THE HIGHEST VALUATION, PRICE IS EQUAL TO THE SECOND-HIGHEST VALUATION.

WE GET TO KNOW IT BECAUSE BUYERS HAVE A DOMINANT STRATEGY, ASSUMING THE BUYERS TO BE SMART, THE BIDS THAT AUCTIONEER ACTUALLY COLLECTED WERE THE REFLECTION OF THE TRUE ACTUAL VALUATIONS OF THE BUYERS. SINCE THE HIGHEST BID IS EQUAL TO THE HIGHEST VALUATION, THEN THE OBJECT GOES TO THE PERSON WITH THE HIGHEST VALUATION.

SINCE THE PRICE IS EQUAL TO THE SECOND HIGHEST BID AND SECOND HIGHEST BID IS EQUAL TO THE SECOND HIGHEST VALUATION, BY SIMPLE LOGIC PRICE IS EQUAL TO THE ACTUAL TRUE SECOND-HIGHEST VALUATION.

ANALYSIS OF THE OUTCOMES

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MOST OF THE RESULTS WERE IN LINE WITH THE SECOND PRICE AUCTION THEORY IN A VICKREY AUCTION, THE INDIVIDUAL IS BIDDING THEIR TRUE VALUE AND ARE NOT TRYING TO ASSESS WHAT OTHER IS GOING TO BID. THEREFORE, IN A VICKREY AUCTION, THE INDIVIDUAL IS BIDDING THE MAXIMUM AMOUNT THEY ARE WILLING TO PAY AND ARE NOT DISADVANTAGED BY IT.

THERE WERE STILL SOME DEVIATIONS, ONE OF THEM WAS THE WINNER HAS TO PAY AN AMOUNT EQUALLING THE AVERAGE OF ALL THE BIDS PLACED BY THE OTHER BIDDER.

ALSO THE DEVIATIONS WERE A RESULT OF " **INCREASE IN THE NUMBER OF BIDDING ROUNDS PER AUCTION** ".

BEHAVIOURAL ANALYSIS

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AFTER ASSUMING THE AUCTIONS TO BE INDEPENDENT ,

WE CAN CLEARLY SEE THAT THE BIDDERS ARE CONCERNED ABOUT HOW MUCH THEY CAN PAY FOR A COMMODITY WITHOUT THINKING ABOUT HOW MUCH IS IT WORTH TO ANYBODY ELSE.

THIS CLEARLY PROVES THAT THE SELLER IS BEING BENIFITTED BY ORGANIZING A FIRST PRICE AUCTION RATHER THAN A SECOND PRICE AUCTION BY COMPARING THE OUTCOMES OF THE TWO GAMES ORGANIZED.

AFTERALL HE IS BEING PAID THE HIGHEST AMOUNT NOT THE SECOND HIGHEST AMOUNT.

BEHAVIOURAL ANALYSIS

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BUT HERE INFACT,

THE FIRST PRICE AUCTION WILL HAVE LOWER EXPECTED REVENUE THAN THE SECOND PRICE AUCTION
BECAUSE THE WINNER'S PAYMENT IN THE FIRST PRICE AUCTION IS BASED ONLY ON HER OWN SIGNAL, WHILE IN THE SECOND PRICE AUCTION,IT IS BASED ON HER OWN SIGNAL AND THE SECOND-HIGHEST SIGNAL.

THERE IS ONE PRINCIPLE NAMED **"LINKAGE PRINCIPLE"** WHICH BASICALLY STATES THAT IN THIS GENERAL SYMMETRIC SETTING, THE MORE INFORMATION ON WHICH THE WINNER'S PAYMENT IS BASED, THE HIGHER WILL BE THE EXPECTED REVENUE.THIS CLEARLY PROVES THE ABOVE MENTIONED STATEMENT.

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Thank You

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