Variance Threshold Strategy Based on Kalman Filter Residuals

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Contents

1 Introduction

This report presents the implementation of a trading strategy based on the volatility of residuals obtained from the Kalman filter. The strategy aims to enter or exit trades when residual volatility crosses predefined thresholds, allowing traders to respond to market dynamics.

2 Objective

The primary objective of this project is to:

- Estimate the hedge ratio using the Kalman filter.
- Compute residuals representing deviations from the expected spread.
- Use rolling volatility to determine entry and exit thresholds.
- Backtest the volatility-based strategy and compare its performance to a Z-score strategy.

3 Mathematical Formulation

3.1 Kalman Filter for Hedge Ratio Estimation

The hedge ratio β_t at time t is estimated using a Kalman filter:

$$\beta_t = \beta_{t-1} + w_t, \quad w_t \sim \mathcal{N}(0, Q) \tag{1}$$

where w_t is the process noise and Q is its variance.

The observed spread s_t is:

$$s_t = y_t + \beta_t x_t + v_t, \quad v_t \sim \mathcal{N}(0, R) \tag{2}$$

where v_t is the observation noise with variance R.

3.2 Residual Computation

The residuals are computed as:

$$\epsilon_t = s_t - \hat{s_t} \tag{3}$$

where $\hat{s_t}$ is the predicted spread from the Kalman filter.

3.3 Rolling Volatility Calculation

Rolling volatility is computed using an exponential moving average:

$$\sigma_t = \sqrt{\frac{1}{N} \sum_{i=t-N}^{t} (\epsilon_i - \bar{\epsilon})^2} \tag{4}$$

where N is the window size and $\bar{\epsilon}$ is the mean residual.

3.4 Trading Signal Generation

The entry and exit thresholds are:

$$\text{Entry threshold} = \lambda_{entry} \cdot \sigma_t \tag{5}$$

$$\text{Exit threshold} = \lambda_{exit} \cdot \sigma_t \tag{6}$$

Trades are executed when the Z-score of the spread crosses these thresholds.

4 Results and Discussion

4.1 Cumulative Profit and Loss Comparison

The cumulative PnL of both strategies is shown below:

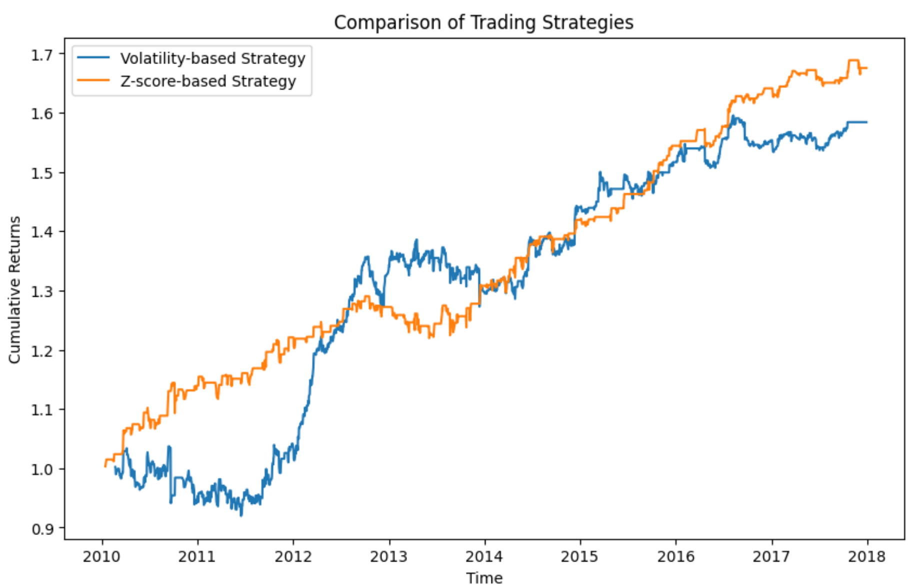

Figure 1: Cumulative Profit and Loss Comparison

4.2 Volatility-Based Strategy Signals

The buy and sell signals generated by the volatility-based strategy are displayed in the figure below:

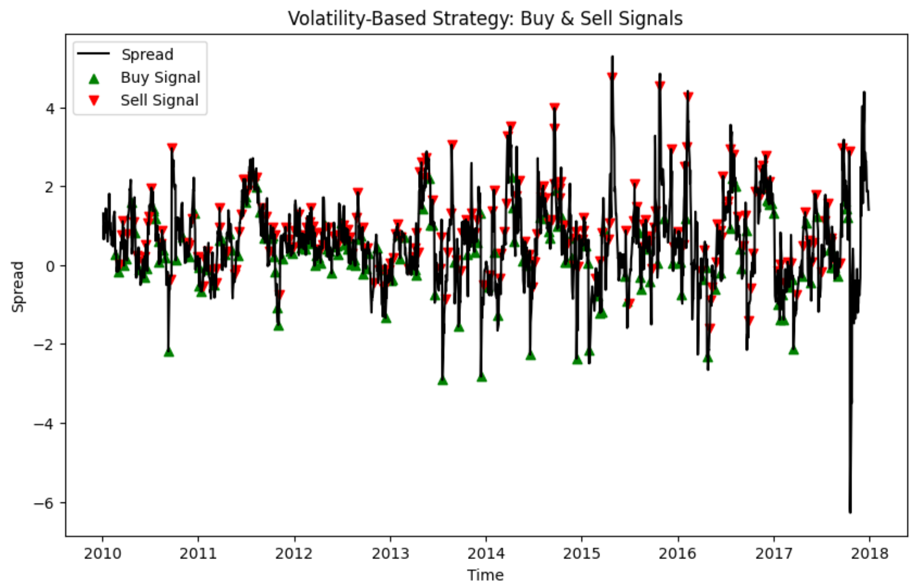

Figure 2: Buy and Sell Signals from Volatility-Based Strategy

4.3 Sharpe Ratio Comparison

The Sharpe ratio of both strategies is computed using:

$$\text{Sharpe Ratio} = \frac{E[R_t]}{\sigma[R_t]}\sqrt{252} \tag{7}$$

where $E[R_t]$ is the expected return and $\sigma[R_t]$ is the standard deviation of returns. The results indicate that the volatility-based strategy provides a lower risk-adjusted return compared to the Z-score strategy.

4.4 Final Performance Metrics

The final profit and loss (PnL) and Sharpe ratios for both strategies are summarized below:

Strategy	Final P&L	Sharpe Ratio
Volatility-Based Strategy	0.58	0.74
Z-Score Strategy	0.68	1.26

Table 1: Final Performance Metrics of Trading Strategies

4.5 Analysis of Strategy Performance

The results indicate that the Z-score strategy outperforms the volatility-based strategy in terms of final P&L and Sharpe ratio. Possible reasons for this include:

- The Z-score strategy relies on fixed standard deviation thresholds, making it more robust to varying market conditions compared to the dynamically changing volatility-based thresholds.
- The Z-score strategy is based on mean reversion principles, which have been historically effective in statistical arbitrage.
- The volatility-based strategy adjusts dynamically but may sometimes lead to delayed entries or exits, reducing its effectiveness in capturing profitable mean reversion opportunities.
- The rolling volatility calculation may introduce additional noise, leading to suboptimal trade signals compared to the more stable Z-score approach.

5 Conclusion

This project successfully implemented and tested a variance threshold trading strategy using Kalman filter residuals. The volatility-based strategy was compared against a standard Z-score strategy, providing insights into their effectiveness. The use of rolling volatility allowed for dynamic trade execution. However, results indicate that the Z-score strategy achieved a higher final P&L (0.68 vs. 0.58) and a better Sharpe ratio (1.26 vs. 0.74), suggesting that it provides superior risk-adjusted returns in this specific backtest.