Image Restoration using Wiener Filter

Pranjali Jain
16110119

Libraries Used :

- cv2
- os
- numpy
- scipy
- matplotlib

Aim : The aim is to restore an image which is blurred and noisy to its original form.

Wiener filter can both de-noise and blur a degraded image.It works in the frequency domain and minimises the impact of noise.

- The degraded image can be represented as follows :

$g(u,v) = h(u,v) * f(u,v) + n(u,v)$

$g(u,v)$ = Degraded image which is blurred and noisy
$h(u,v)$ = Point spread function or impulse response of the imaging system
$f(u,v)$ = Original image
$n(u,v)$ = Additive noise

Where $*$ represents that $h(u,v)$ is convolved with $f(u,v)$

- Also let,

$f''(u,v)$ = restored image which is an estimate of $f(u,v)$ from $g(u,v)$

Wiener filter tries to obtain the original image $f(u,v)$ from the supplied degraded image $g(u,v)$.

Modelling the wiener filter :

1. A coloured $N^2 x N^2$ image is taken and then converted to grayscale. This image acts as $f(u,v)$.
2. Blur is added to the original image using through convolving it with a linear shift invariant filter $h(u,v)$. This blurring acts as a low pass filter and attenuates higher spatial frequencies.
3. Further additive noise $n(u,v)$ is added to the blurred image.
4. This is the degraded image $g(u,v)$

$$g(u,v) = h(u,v) * f(u,v) + n(u,v)$$

In frequency domain,

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

$G(u,v)$ = Fourier Transform of $g(u,v)$
$H(u,v)$ = Fourier Transform of $h(u,v)$
$F(u,v)$ = Fourier Transform of $f(u,v)$
$N(u,v)$ = Fourier Transform of $n(u,v)$

5. The frequency response of Wiener filter is :

$$W(u,v) = \frac{H^*(u,v)}{|H(x,y)|^2 + K(u,v)}$$

Where
$H^*(u,v)$ = conjugate of $H(u,v)$
$K(u,v) = S_n(u,v)/S_f(u,v)$
$S_n(u,v) = |N(u,v)|^2$, power spectral density of $n(u,v)$
$S_f(u,v) = |F(u,v)|^2$, power spectral density of $f(u,v)$

6. The fourier transform of the restored image is given as :

$$F''(u,v) = W(u,v)G(u,v)$$

Where

$G(u,v)$ = Fourier transform of $g(u,v)$

7. The restored image $f"(u,v)$ is the inverse fourier transform of $F"(u,v)$

Choice of K :

For any degraded image, either $|F(u,v)|$ and $|N(u,v)|$ are known approximately, or K is set to a constant scalar which is determined empirically.

Also,

- If $K = 0$ then $W(u,v) = 1/H(u,v)$, that is an inverse filter
- If $K >> |H(u,v)|$ for large u, v ; then higher frequencies are attenuated

MSE and $PNSR$ Metric :

1. Wiener Filter minimises the least squared error also known as MSE

$$MSE = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (f(x,y) - f"(x,y))^2 dxdy$$

2. $PNSR$ or Peak signal-to-noise ratio is generally used to compare the quality of image produced by the image enhancement algorithm which is the wiener filter in this case.

The filter tends to minimise the value of $PNSR$

$$PNSR = 20 log_{10}\left(\frac{MAX_f}{\sqrt{MSE}}\right)$$

Training and Testing

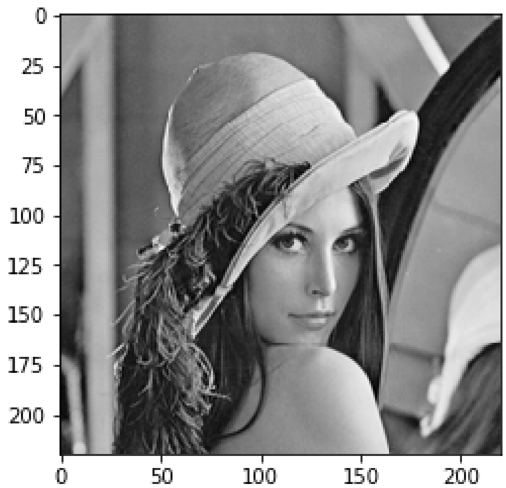
In the results that follow, the point spread function is a Gaussian filter and the noise introduced in the blurred image is also gaussian.

<u>Training :</u>

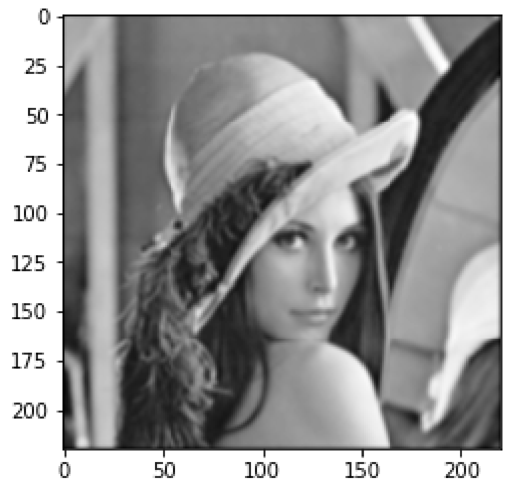
Image used : Lena image

PSF : Gaussian PSF of size 5 and variance 1

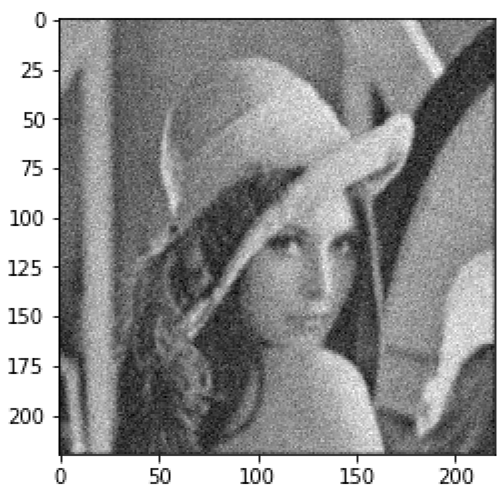
Noise : Gaussian noise with zero mean and variance 100



gray image

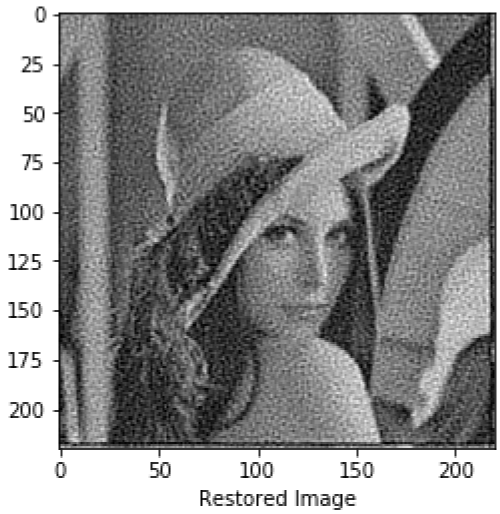


Blurred Image

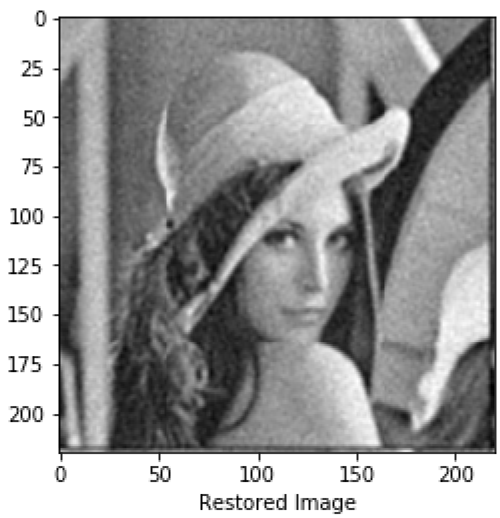


Blurred and Noisy Image

Restored Images :



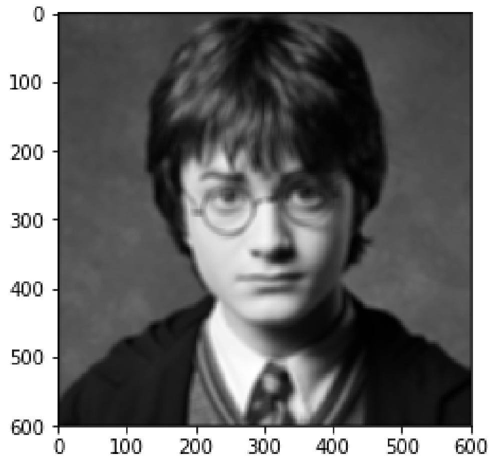
- $K = 0.02$
- MSE : 1406.29
- $PSNR$: 16.65



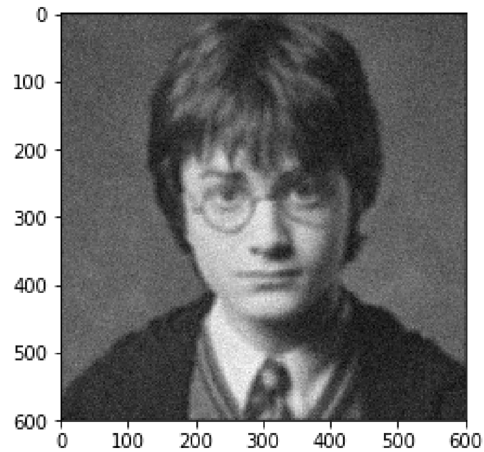
- $K = 0.508$
- MSE : 2606.28
- $PSNR$: 13.97

Testing :

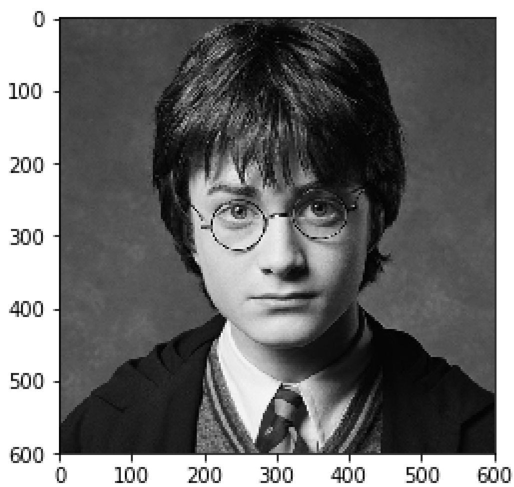
1. Harry Potter Image



Blurred Image

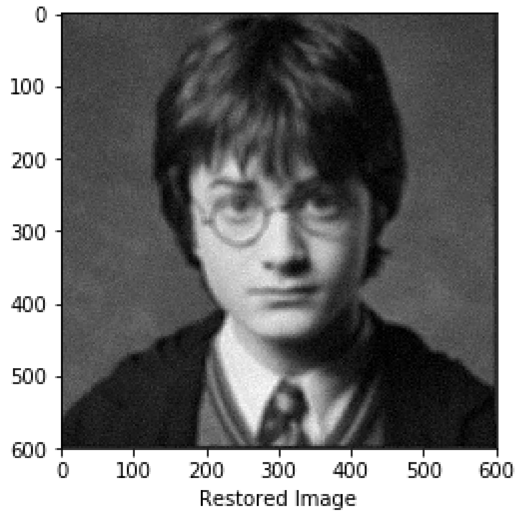


Blurred and Noisy Image



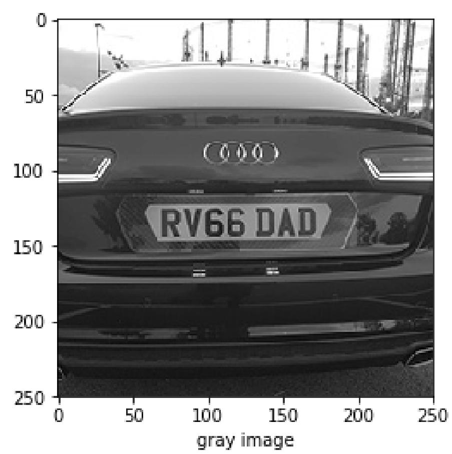
gray image

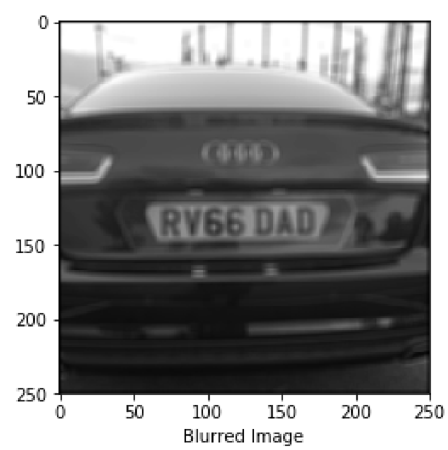
The wiener filter formed during training is used to restore this image.

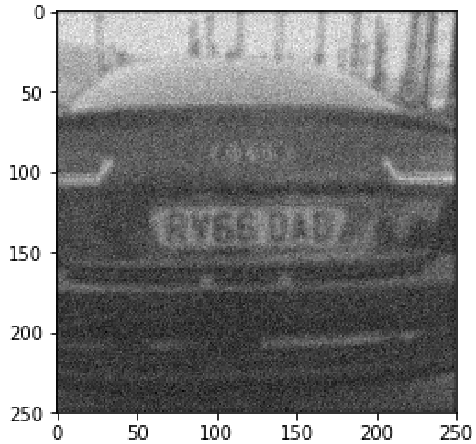


- $K = 0.508$
- MSE : 393.80
- $PSNR$: 22.17

2. Car Image

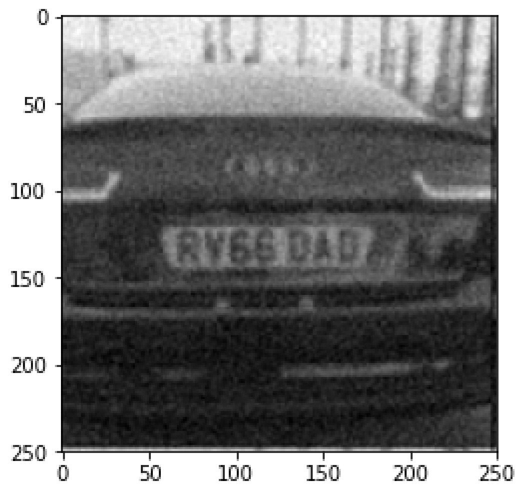






Blurred and Noisy Image

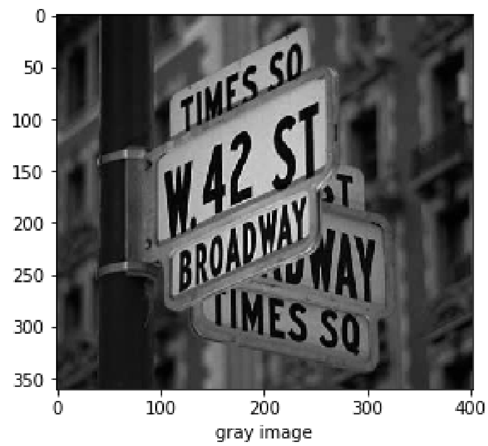
The wiener filter formed during training is used to restore this image.

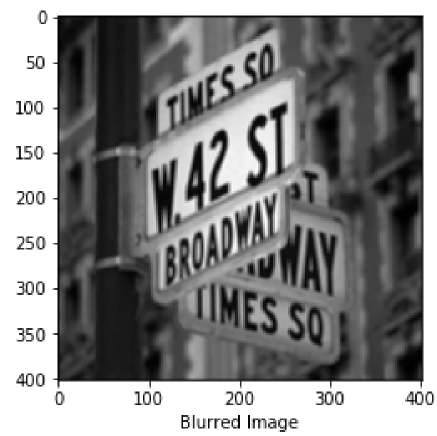


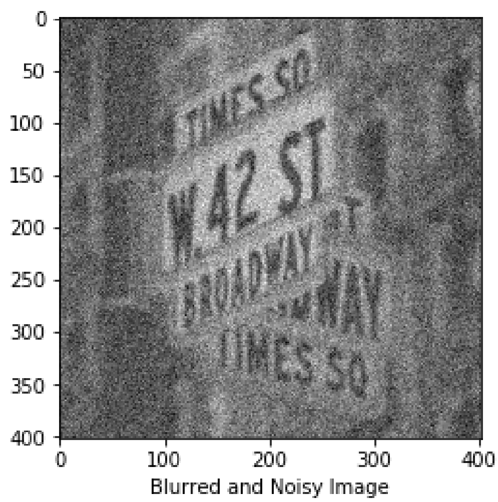
Restored Image

- $K = 0.508$
- MSE : 4074.34
- $PSNR$: 12.03

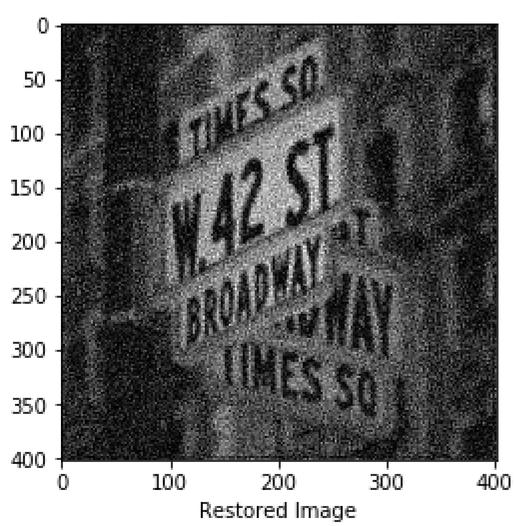
3. Street sign Image







The wiener filter formed during training is used to restore this image.



- $K = 0.508$
- MSE : 1454.77
- $PSNR$: 16.50

Conclusions :

The wiener filter is able to remove noise from blurry and noisy images. The choice of K is important in determining the performance of the filter on a particular image.

References :

1. https://lib.dr.iastate.edu/cgi/viewcontent.cgi?referer=https://www.google.com/&httpsredir=1&article=3004&context=qnde
2. https://researchweb.iiit.ac.in/~aabhas.majumdar/mywebsite/ImageRes.pdf
3. http://www.robots.ox.ac.uk/~az/lectures/ia/lect3.pdf