Name- Pranjal vijay Bramhankar 2301010083.

Q.1.

Find the rank of the matrix by reducing in Row reduced echelon From.

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

Apply elementary row operations to get the matrix into row echelon from then reduce it to RREF.

First column

1) $R_2 = R_2 - 2R_1$
2) $R_3 = R_3 - 3R_1$
3) $R_4 = R_4 - 6R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -5 & 5 \end{bmatrix}$$

second column

1) $R_3 = R_3 - 4/3 R_2$
2) $R_4 = R_4 - 4/3 R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & 0 & -1 \\ 0 & -4 & 0 & 7/3 \end{bmatrix}$$

Third column

1) $R_4 = R_4 - R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & 0 & -1 \\ 0 & 0 & 0 & \frac{13}{3} \end{bmatrix}$$

Name : Pranjal vijay Bramhankar 2301010083

This is now row echelon From.

To convert it to RREF

1) $R_3 = R_3 + 4/3 R_2$

2) $R_1 = R_1 - 2R_2$

$$\begin{bmatrix} 1 & 2 & 0 & -4 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & 0 & -1 \\ 0 & 0 & 0 & 13/3 \end{bmatrix}$$

divide third row by -4

$R_3 = -\frac{1}{4} R_3$

$$\begin{bmatrix} 1 & 2 & 0 & -4 \\ 0 & 0 & -3 & 2 \\ 0 & 1 & 0 & 1/4 \\ 0 & 0 & 0 & 13/3 \end{bmatrix}$$

we have the matrix in RREF.

so rank of matrix A is 3

Name = Pranjal vijay Bramhankar 2301010083.

Q.2.

let W be the vector space of all symmetric 2X2 matrices and let $T: W \rightarrow P_2$ be the linear transformation defined by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b)+(b-c)x+(c-a)x^2$. Find the rank and nullity of T

$$M = \begin{bmatrix} a & b \\ \cancel{b}c & d \end{bmatrix}$$

By rank-nullity theorem
rank (T) + nullity(T) = dim(W)

Find the images of the basic vectors of W under T

1) For the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$:

$$T\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = (1-0)+(0-0)x+(0-1)x^2 = 1-x^2$$

2) For the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$:

$$T\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = (0-1)+(1-1)x+(1-0)x^2 = -1+x$$

3) For the matrix $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$:

$$T\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = (0-0)+(0-0)x+(0-0)x^2 = 0$$

The images of the basis vectors $\{1-x^2, -1+x, 0\}$ are linearly independent.
They span the image of T. So, the rank T is 3

Name - Pranjal vijay Bramhankar 2301010083.

using the rank - nullity theorem

$$\text{rank}(T) + \text{nullity}(T) = \dim(W)$$

$$3 + \text{nullity}(T) = 3$$

$$\text{nullity}(T) = 0$$

rank of T is 3 and nullity of T is 0

Name - Pranjal vijay Bramhankar 2301010083

Q.3.

let $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ Find the eigenvalues and eigenvectors of A^{-1} and $A+4I$

Eigenvalues and Eigenvectors of matrices A^{-1} and $A+4I$

(1) Eigenvalues and Eigenvectors of A

$\det(A-\lambda I) = 0$

$$\det(A-\lambda I) = \begin{vmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix}$$

$$= (2-\lambda)^2 - (-1)(-1)$$

$$= \lambda^2 - 4\lambda + 4 - 1$$

$$= \lambda^2 - 4\lambda + 3$$

$$= (\lambda-1)(\lambda-3) = 0$$

so $\lambda_1 = 1$ and $\lambda_2 = 3$

$(A-\lambda I)v = 0$

solve v we get

For $\lambda_1 = 1$

$$(A-\lambda_1 I)v_1 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} v_1 = 0$$

we get $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

For $\lambda_2 : 3$

$$(A-\lambda_2 I)v_2 = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} v_2 = 0$$

we get $v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(2) Eigenvalues and Eigenvectors of A^{-1}

IF λ is eigenvalue of A, then λ^{-1} is For A^{-1}

same for the Eigenvectors.

Name- Pranjal vijay Bramhankar 2301010083

eigenvalues of A^{-1} and λ_1^{-1} and λ_2^{-1}

Eigenvalues of A^{-1} : $\frac{1}{\lambda_1}, \frac{1}{\lambda_2} = \frac{1}{1}, \frac{1}{3} = 1, \frac{1}{3}$

The Eigenvectors remains the same.

(3) Eigenvalues and Eigenvectors of $A + 4I$:

If μ is eigenvalues of A, then $\mu + 4$ is an eigenvalue of $A + 4I$, with same eigenvectors

so, eigenvalues of $A+4I$ are $\mu_1 + 4$ and $\mu_2 + 4$

Eigenvalues of $A+4I$: $\lambda_1 + 4, \lambda_2 + 4 = 1 + 4, 3 + 4 = 5, 7$

The eigenvectors remains the same.

For A^{-1}

Eigenvalue $\lambda_1 = 1$, Eigenvector $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Eigenvalue $\lambda_2 = 3$, Eigenvector $v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

For $A + 4I$

Eigenvalue $\lambda_1 + 4 = 5$, Eigenvector $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Eigenvalue $\lambda_2 + 4 = 7$, Eigenvector $v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Name- Pranjal vijay Bramhankar 2301010083

Q.4.

Slove by Gauss - Seidel Method (Take 3 iteration)

$$3x - 0.1y - 0.2z = 7.85$$
$$0.1x + 7y - 0.3z = -19.3$$
$$0.3x - 0.2y + 10z = 71.4$$

with initial values $x(0) = 0$, $y(0) = 0$, $z(0) = 0$

Equation is

$$3x - 0.1y - 0.2z = 7.85$$
$$0.1x + 7y - 0.3z = -19.3$$
$$0.3x - 0.2y + 10z = 17.4$$

equation to isolate x, y. and z

1) $x = (7.85 + 0.1y + 0.2z) / 3$

2) $y = (-19.3 - 0.1x + 0.3z) / 7$

3) $z = (71.4 - 0.3x - 0.2y) / 10$

Iteration 1.

Using initial values $x(0) = 0$ $y(0) = 0$ $z(0) = 0$

1) $x(1) = \frac{((7.85 + 0.1(0) + 0.2(0))}{3} = 2.61667$

2) $y(1) = \frac{(-19.3 - 0.1(2.61667) + 0.3(0))}{7} = -2.77295$

3) $z(1) = \frac{(71.4 - 0.3(2.61667) - 0.2(-2.77295)}{10} = 7.18943$

Iteration 2

using $x(1) = 2.61667$, $y(1) = -2.77295$, $z(1) = 7.18943$

1) $x(2) = \frac{(7.85 + 0.1(-2.77295) + 0.2(7.18943))}{3} = 3.00056$

2) $y(2) = \frac{(-19.3 - 0.1(3.00056) + 0.3(7.18943))}{7} = -2.99984$

3) $z(2) = \frac{(71.4 - 0.3(3.00056) - 0.2(-2.99984))}{10} = 7.00004$

Name - Pranjal vijay Bramhankar 2301010083

Iteration 3

using $x(2) = 3.00056$ $y(2) = -2.99984$ $z(2) = 7.00004$

1) $x(3) = \dfrac{(7.85 + 0.1(-2.99984) + 0.2(7.00004))}{3} = 3.00002$

2) $y(3) = \dfrac{(-19.3 - 0.1(3.00002) + 0.3(7.00004))}{7} = -3$

3) $z(3) = \dfrac{(71.4 - 0.3(3.00002) - 0.2(-3))}{10} = 7$

After 3 iteration

$x = 3$

$y = -3$

$z = 7.$

Pranjal vijay Bramhankar 2301010083

Q.5.

Define consistent and inconsistent system of equations. Hence slove the following system of equations if consistent

$x+3y+2z=0$
$2x-y+3z=0$
$3x-5y+4z=0$
$x+17y+4z=0$

rewrite in matrix from

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} : \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Use row reduction

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{array}\right]$$

subtracting twice ~~row~~ First row from second.
subtracting thrice first row from third

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 1 & 17 & 4 & 0 \end{array}\right]$$

subtract 1st row from 4th row (1R - 4R)

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{array}\right]$$

Name - Pranjal vijay Bramhankar 2301010083

add twice the second row to the third row

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 14 & 2 & 0 \end{array}\right]$$

Now, multiply second row by $-1/7$ to get leading 1

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & 1 & 1/7 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 14 & 2 & 0 \end{array}\right]$$

subtract 14 times the second row from forth row

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & 1 & 1/7 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

We see that last row is $0=0$. This means it is dependent equation. the system of equation infinitely many solutions.

let's denote $z=t$, t is a real number. from second row, we have $y = -\frac{1}{7}t$. Finally, from first row have

$$x = -3y - 2z = -3\left(-\frac{1}{7}t\right) - 2t = \frac{3}{7}t - 2t = \frac{3}{7}t - \frac{14}{7}t = -\frac{11}{7}t$$

solution of system of equation is $x = -\frac{11}{7}t$, $y = -\frac{1}{7}t$, $z = t$

A system of equations is said to be consistent if it has at least one solution, meaning the equations can be satisfied simultaneously. on the other hand, if system has no. solution, it is said to be inconsistent.

Name - Pranjal vijay Bromhankar 2301010083

Q.6.

Determine whether the function $T: P_2 \to P_2$ given is linear transformation or not. where $T(a+bx+cx^2) = (a+1)+(b+1)x+(c+1)x^2$

1) Additivity: $T(u+v) = T(u)+T(v)$ for all u, v in the domain.
2) Homogeneity: $T(k.u) = k.T(u)$ For all k in the field of scalars and all u in the domain

Additivity
$u = a_1+b_1x+c_1x^2$ and $v = a_2+b_2x+c_2x^2$, $a_1, b_1, c_1, a_2, b_2, c_2$ are scalar.

$T(u+v) = T((a_1+a_2)+(b_1+b_2)x+(c_1+c_2)x^2)$
$= ((a_1+a_2)+1)+((b_1+b_2)+1)x+((c_1+c_2)+1)x^2$
$= (a_1+1)+(b_1+1)x+(c_1+1)x^2+(a_2+1)+(b_2+1)x+(c_2+1)x^2$
$= T(u)+T(v)$

Thus $T(u+v) = T(u)+T(v)$, so function is additive.

Homogeneity
let $u = a+bx+cx^2$ be a polynomial and k is scalar.
$T(k.u) = T(k.(a+bx+cx^2))$
$= T(ka+kbx+kcx^2)$
$= (ka+1)+(kb+1)x+(kc+1)x^2$
$= k(a+1)+k(b+1)x+k(c+1)x^2$
$= k.T(u)$

Thus $T(k.u) = k.T(u)$, so function is homogeneous. since T satisfies both additivity and Homogeneity, it is a linear transformation.

Name = Pranjal vijay Bramhankar 2301010083

Q.7.

Determine the whether the set $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$ is a basic of $V_3(R)$ in case s is S is not a basis determine the dimensions and the basic of sub space spanned by s.

We need to chect it satisfied 2 ~~to~~ conditions.

1) linear independence
2) Spanning

checking for linear independence by constructing the augmented matrix and performing row operations

$$\left(\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 2 & 1 & 1 & 0 \\ 3 & 0 & 3 & 0 \end{array}\right)$$

performing row operation

$$\left(\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 0 & -5 & 5 & 0 \\ 0 & -9 & 9 & 0 \end{array}\right)$$

row operation lead to

$$\left(\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

system has infinitely many solutions. vectors are linearly dependent. S is not basic for $V_3(R)$
To determine dimension and basis subspace ~~around~~ spanned by s. we consider span vectors is S.
from row-reduced echelon from, two linearly independent vectors, namely (1,2,3) and (3,1,0)
Hence, these two vectors from basis for the subspace spanned by s.
The dimension of subspace is 2.

Name- Pranjal vijay Bramhankar 2301010083

Q.8.

Using Jacobi's method (perfrom 3 iterations) slove

$3x - 6y + 2z = 23$

$-4x + y - z = -15$

$x - 3y + 7z = 16$

with initial values $x_0 = 1$, $y_0 = 1$, $z_0 = 1$

Iteration 1

1) From eqn 1

$$x^{(1)} = \frac{23 + 6y^{(0)} - 2z^{(0)}}{3}$$

2) From eqn 2

$$y^{(1)} = \frac{-15 + 4x^{(0)} + z^{(0)}}{-3}$$

3) From eqn 3

$$z^{(1)} = \frac{16 - x^{(0)} + 3y^{(0)}}{7}$$

subsituting Initial values

$$x^{(1)} = \frac{23 + 6(1) - 2(1)}{3} = \frac{27}{3} = 9$$

$$y^{(1)} = \frac{-15 + 4(1) + 1}{-3} = \frac{-10}{-3} = 3.333$$

$$z^{(1)} = \frac{16 - 1 + 3(1)}{7} = \frac{18}{7} = 2.571$$

Name- Pranjal vijay Bramhankar 2301010083.

Iteration 2:

1) From eq^n 1

$$x^{(2)} = \frac{23 + 6y^{(1)} - 2z^{(1)}}{3}$$

2) From eq^n 2

$$y^{(2)} = \frac{-15 + 4x^{(1)} + z^{(1)}}{3}$$

3) From eq^n 3

$$z^{(2)} = \frac{16 - x^{(1)} + 3y^{(1)}}{7}$$

Substituting values from iteration 1

$$x^{(2)} = \frac{23+6(3.3333)-2(2.5714)}{3} = \underline{\underline{8.9524}}$$

$$y^{(2)} = \frac{-15+4(9)+2.5714}{-3} = \underline{\underline{-5.8571}}$$

$$z^{(2)} = \frac{16-9+3(3.3333)}{7} = \underline{\underline{3.9524}}$$

Iteration 3:

1) From eq^n 1

$$x^{(3)} = \frac{23+6y^{(2)}-2z^{(2)}}{3} = \underline{\underline{20.5079}}$$

2) From eq^n 2

$$z^{(3)} = \frac{16 - x^{(2)} + 3y^{(2)}}{7} = \underline{\underline{9.2108}}$$

3) From eq^n 3

$$\cancel{z^{(3)}} = \cancel{\frac{16-x^{(2)}+3y^{(2)}}{7}} =$$

$$y^{(3)} = \frac{-15+4x^{(2)}+z^{(2)}}{-3} = \underline{\underline{37.762}}$$

Pranjal vijay Bramhankar 2301010083

Q. 9.

Explain one application of matrix operations in image processing with example.

One application of matix operations in image processing is in the manupalition of images using transformation such as scaling, rotation, translation, and shearing. These transformations can be represented by matrices and applied to the coordinates of each pixel in image.

Application = Scaling

Scaling involves resizing an image either by enlarging or shrinking it. This operation changes the size of image while maintaining its aspect ratio.

Matrix Representation:

lets consider a 2D image represented by matrix A. where each element of matrix corresponds to the intensity of pixel To scale of image by a factor of as S in both horizontal and vertical directions. we can use following scaling matrices.

For scaling up

$$\text{Scale\_Up} = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix}$$

scaling down

$$\text{Scale\_Down} = \begin{pmatrix} 1/s & 0 \\ 0 & 1/s \end{pmatrix}$$

Example -

consider a simple 3x3 image represented by matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

If we want to scale this image up by a factor 2, we multiply image matrix A by scaling matrix Scale_Up:

$$\text{Scale\_Up} \times A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \times \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = \begin{pmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & 18 \end{pmatrix}$$

so. After scaling up the image. the new matrix A' represents the scaled image.

Name - Pranjal vijay Bramhankar 2301010083.

Q.10.

Give a brief description of linear transformation for computer Vision for rotating 2D image.

linear transformations plays a fundamental role in computer vision, particularly in manipulating and analyzing images. Rotating a 2D image is a linear transformation used in computer vision tasks, such as image recognition, object detection, image registration.

Rotating a 2D image.
Rotating a 2D image invovoles transforming each pixel's coordinates in the image by a specified angle around a chosen pivot point. This transformation can be achieved using a rotation matrix.

$$R(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$$

where,

- θ is the angle of rotation
- $\cos(\theta)$ and $\sin(\theta)$ are trignometric functions representating cosine and sine of the ~~tri~~angle respectively.

Example -
let consider a simple 2D image represented by a matrix A. To rotate this image by an angle of θ we multiply coordinates of each pixel in original image by rotation matrix $R(\theta)$

$A' = R(\theta) \times A$

where A' is the rotated image.