

Quadratic Programming: Manufacturing Problem

Reference:

Book Title: Engineering Optimization theory and practice

Author(s): Singiresu S.Rao

Publisher : John Wiley and sons Inc

Edition : Fourth

Problem:

A manufacturing firm produces two products, A and B, using two limited resources. The maximum amounts of resources 1 and 2 available per day are 1000 and 250 units, respectively. The production of 1 unit of Product A requires 1 unit of resource 1 and 0.2 unit of resource 2, and the production of 1 unit of product B requires 0.5 unit of resource 1 and 0.5 unit of resource 2. The unit costs of resources 1 and 2 are given by the relations $(0.375 - 0.00005u_1)$ and $(0.75 - 0.0001u_2)$, respectively, u_i denotes the number of units of resource i used ($i=1, 2$). The selling prices per unit of products A and B, p_a and p_b are given by

$$p_a = 2.00 - 0.0005x_a - 0.00015x_b$$

$$p_b = 3.50 - 0.0002x_a - 0.0015x_b$$

where x_a and x_b indicate, respectively, the number of units of product A and B sold. Formulate the problem of maximizing the profit assuming that the firm can sell the units it manufactures.

How to use the code:

This code is made to test quadprog function which solves the linear quadratic problem. The code is tested using a manufacturing problem specified above.

To use the function, we need to pass some parameters which are needed to be specified by the user.

First, the number of variables and then, number of constraints are to be provided.

Now, the user is asked to put the coefficients of the non-linear variables in the objective function which can be taken from the objective function mentioned in the mathematical Formulation below. This will create a matrix having dimension of $[nbVar][nbCon]$.

The user is asked to enter the coefficients of linear variables used in the objective function and a vector is created.

The user is then asked to specify the coefficients of the variables present in the constraints in the formulation which are stored in the vector of dimension $nbVar$.

Then user needs to enter the values present in the right hand side of the constraints. As the parameters are entered, the program will calculate the values of the units and the cost using the quadprog function which are displayed on the screen.

Mathematical Formulation:

Objective Function : $F = -x_b(3.5 - 0.0002x_a - 0.0015x_b)$ subject to constraints

$-0.5x_b \leq 1000$

$0.2x_a + 0.5x_b \leq 250$

$x_a \geq 0$

$x_b \geq 0$

Function used:

`xopt = quadprog(nbVar,nbCon,H,f,lb,ub,A,conLB,conUB,x0,options)`

where

- `nbVar` : A double, denoting the number of variables
- `nbCon` : A double, denoting the number of constraints
- `H` : A symmetric matrix of doubles, representing the Hessian of the quadratic problem.
- `f` : A vector of doubles, representing coefficients of the linear terms in the quadratic problem.
- `lb` : A vector of doubles, containing the lower bounds of the variables.
- `ub` : A vector of doubles, containing the upper bounds of the variables.
- `A` : A matrix of doubles, representing the constraint matrix in $\text{conLB} \leq A \cdot x \leq \text{conUB}$.
- `conLB` : A vector of doubles, containing the lower bounds of the constraints $\text{conLB} \leq A \cdot x \leq \text{conUB}$.
- `conUB` : A vector of doubles, containing the upper bounds of the constraints $\text{conLB} \leq A \cdot x \leq \text{conUB}$.
- `x0` : A vector of doubles, containing the starting values of variables of size $(1 \times n)$ or $(n \times 1)$ where 'n' is the number of variables.