ASSIGNMENT 3

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1. x $\equiv$ 3 mod 5 and x $\equiv$ 4 mod 11, will have solution of type x $\equiv$ a mod N, where N = 5 $\times$ 11 = 55.
   $N_1 = 55/5 = 11$ and $N_2 = 55/11 = 5$.
   $11 \times x_1 \equiv 1$ mod 5 and $5 \times x_2 \equiv 1$ mod 11.
   Therefore, $x_1 = 1$ and $x_2 = 9$ will satisfy the equation.

   Now, a = $\sum x_i \times N_i \times b_i = 1 \times 11 \times 3 + 9 \times 5 \times 4 = 213$.
   So, the solution is $x \equiv 213$ mod 55 which is simplified as $x \equiv 48$ mod 55.
2. 7-safe numbers are $\{3,4\}$; 11-safe numbers are $\{3,4,5,6,7,8\}$ and 13-safe numbers are $\{3,4,5,6,7,8,9,10\}$.
   By the Chinese Remainder Theorem they will have a unique solution of the form x $\equiv x_i$ mod N_i for each combination as gcd(7,11,13) = 1.
   Hence, there are 2 $\times 6 \times$ 8 solutions mod 7 $\times 11 \times$ 13, which means 96 solutions mod 1001.
   So, there are 960 solution uptil 10010.
   But we need values upto 10000. So the numbers from 10001 to 10010 must be checked individually.
   10001 mod 7 = 5, not a solution.
   10002 mod 7 = 6, not a solution.
   10003 mod 7 = 0, not a solution.
   10004 mod 7 = 1, not a solution.
   10005 mod 7 = 2, not a solution.
   10006 mod 7 = 3, 10006 mod 11 = 7 and 10006 mod 13 = 9, a solution.
   10007 mod 7 = 4, 10007 mod 11 = 8 and 10007 mod 13 = 10, a solution.
   10008 mod 7 = 5, not a solution.
   10009 mod 7 = 6, not a solution.
   10010 mod 7 = 0, not a solution.
   Hence, total solutions = 960 - 2 = 958.
3. $\binom{n}{7} = \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)}{2^4.3^2.5.7}$
   If $\binom{n}{7}$ is to be divisible by 12, then numerator must be divisible by $2^6.3^3$.

   If the numerator is to be divisible by 27, any of the number should be divisible by 9, and hence as these are 7 consecutive numbers, there are 7 possibilities for mod 9 of n.
   Now, we have to check when the numerator is divisible by 64.
   If n is even then n,n-2,n-4 and n-6 are each divisible by 2 and two of them are divisible by 4 (either n and n-4 or n-2 and n-6), and hence it is divisble by 64.
   If n is odd, n-1,n-3 and n-5 are each divisible by 2.
   Now, if n-3 is divisible by 4 then it must also be divisible by 16 for the numerator to be divisible by 64, and hence there is 1 possibility for mod

16 of n.
If n-1 is divisible by 4, then so is n-5 and hence the number is divisible by 32, and so any of n-1 or n-5 must be divisible by 8 for the numerator to be divisible by 64.Hence there are 2 possibilities for mod 8 of n, and 4 for mod 16 of n.

Combining all these, there are 2 requirements, one for divisibility by 27 and other for divisibility by 64. They are :
n $\equiv$ 0,1,2,3,4,5,6 mod 9
n $\equiv$ 0,1,2,3,4,5,6,8,9,10,12,13,14 mod 16
Therefore for $\binom{n}{7}$ to be divisible by 12, there are 7 $\times$13 possibilities mod (9 $\times$16).
Hence there are 91 solutions for every 144 numbers and hence the fraction of checked numbers that are punched reaches a limiting value $\frac{91}{144}$ and hence m + n = 91 + 144 = 235.