Numbers Made Dumber

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Assignment 3

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1. Find the solution to the linear congruence

$$x \equiv 3 \mod 5$$
$$x \equiv 4 \mod 11$$

2. For a positive integer p, define the positive integer n to be p-safe if n differs in absolute value by more than 2 from all multiples of p. For example, the set of 10-safe numbers is $3, 4, 5, 6, 7, 13, 14, 15, 16, 17, 23...$ Find the number of positive integers less than or equal to 10,000 which are simultaneously 7-safe, 11-safe, and 13-safe.

3. Consider a number line consisting of all positive integers greater than 7. A hole punch traverses the number line, starting frrom 7 and working its way up. It checks each positive integer n and punches it if and only if $\binom{n}{7}$ is divisible by 12. As the hole punch checks more and more numbers, the fraction of checked numbers that are punched approaches a limiting number ρ. If ρ can be written in the form m/n, where m and n are positive integers, find $m + n$.

4. Call a lattice point "visible" if the greatest common divisor of its coordinates is 1. Prove that there exists a 100×100 square on the board none of whose points are visible.