Arithmetic Modulo 2 (binary arithmetic)

0+0=0 0+1=1 1+0=1 1+1=0	0*0=0 0*1=0 1*0=0 1*1=1
Add = exclusive or	Mult = and

Carry-Look Ahead Addition (Babbage 1800's)

Example						
1	0	1	1	1		Carry
	1	0	1	1	1	First Int
	1	0	1	0	1	Second Int
1	0	1	1	0	0	Sum

Goal: Add Two n-bit Integers

Carry-Look Ahead Addition (Babbage 1800's)

Goal: Add Two n-bit Integers

Example						Notation			
1	0	1	1	1	Carry	c_2	c_1	c_0	
	1	0	1	1	1 First Int	a_3	a_2	a_1	a_0
	1	0	1	0	1 Second Int	b_3	b_2	b_1	b_0

Carry-Look Ahead Addition (Babbage 1800's)

Goal: Add Two n-bit Integers

Example						Notation			
1	0	1	1	1	Carry	c_2	c_1	c_0	
	1	0	1	1	1 First Int	a_3	a_2	a_1	a_0
	1	0	1	0	1 Second Int	a_3	b_2	b_1	b_0

(addition mod 2)

$c_{-1} = 0$

for $i = 0 : n-1$

$s_i = a_i + b_i + c_{i-1}$

$c_i = a_i b_i + c_{i-1}(a_i + b_i)$

end

$s_n = c_{n-1}$

Goal: Add Two n-bit Integers

Example						Notation			
1	0	1	1	1	Carry	c_2	c_1	c_0	
	1	0	1	1	1 First Int	a_3	a_2	a_1	a_0
	1	0	1	0	1 Second Int	a_3	b_2	b_1	b_0

$c_{-1} = 0$

(addition mod 2)

for i = 0 : n-1

$s_i = a_i + b_i + c_{i-1}$

$c_i = a_i b_i + c_{i-1}(a_i + b_i)$

$$\begin{bmatrix} c_i \\ 1 \end{bmatrix} = \begin{bmatrix} a_i + b_i & a_i b_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_{i-1} \\ 1 \end{bmatrix}$$

end

$s_n = c_{n-1}$

Goal: Add Two n-bit Integers

Example						Notation			
1	0	1	1	1	Carry	c_2	c_1	c_0	
	1	0	1	1	1 First Int	a_3	a_2	a_1	a_0
	1	0	1	0	1 Second Int	a_3	b_2	b_1	b_0

$c_{-1} = 0$ (addition mod 2)

for i = 0 : n-1

$s_i = a_i + b_i + c_{i-1}$

$c_i = a_i b_i + c_{i-1}(a_i + b_i)$

end

$s_n = c_{n-1}$

$$\begin{bmatrix} c_i \\ 1 \end{bmatrix} = \begin{bmatrix} a_i + b_i & a_i b_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_{i-1} \\ 1 \end{bmatrix}$$

Matmul prefix with binary arithmetic is equivalent to carry-look ahead!

Compute c_i by prefix, then
$s_i = a_i + b_i + c_{i-1}$ in parallel