Arithmetic Modulo 2 (binary arithmetic)

0+0=0	0*0=0
0+1=1	0*1=0
1+0=1	1*0=0
1+1=0	1*1=1
Add = exclusive or	Mult = and



Carry-Look Ahead Addition (Babbage 1800's)

		Exa	mple)		
1	0	1	1	1		Carry
	1	0	1	1	1	First Int
	1	0	1	0	1	Second Int
1	0	1	1	0	0	Sum

Goal: Add Two n-bit Integers



Carry-Look Ahead Addition (Babbage 1800's)

Goal: Add Two n-bit Integers

		Exai	mple)				No	tation	
1	0	1	1	1		Carry	$\mathbf{c_2}$	\mathbf{c}_1	\mathbf{c}_0	
	1	0	1	1	1	First Int	$\mathbf{a_3}$	$\mathbf{a_2}$	\mathbf{a}_1	$\mathbf{a_0}$
	1	0	1	0	1	Second Int	$\mathbf{b_3}$	$\mathbf{b_2}$	$\mathbf{b_1}$	$\mathbf{b_0}$

Carry-Look Ahead Addition (Babbage 1800's) Goal: Add Two n-bit Integers

Example							Notation			
1	0	1	1	1		Carry	$\mathbf{c_2}$	$\mathbf{c_1}$	$\mathbf{c_0}$	
	1	0	1	1	1	First Int	\mathbf{a}_3	\mathbf{a}_{2}	\mathbf{a}_1	\mathbf{a}_0
	1	0	1	0	1	Second Int	$\mathbf{a_3}$	$\mathbf{b_2}$	$\mathbf{b_1}$	$\mathbf{b_0}$

(addition mod 2)

for
$$i = 0 : n-1$$

 $s_i = a_i + b_i + c_{i-1}$
 $c_i = a_i b_i + c_{i-1} (a_i + b_i)$
end

$$s_n = c_{n-1}$$

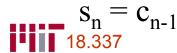
Goal: Add Two n-bit Integers

$$c_{-1} = 0$$
for $i = 0$ (addition mod 2)
$$s_{i} = a_{i} + b_{i} + c_{i-1}$$

$$c_{i} = a_{i}b_{i} + c_{i-1}(a_{i} + b_{i})$$

$$\begin{bmatrix} c_{i} \\ 1 \end{bmatrix} = \begin{bmatrix} a_{i} + b_{i} & a_{i}b_{i} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_{i-1} \\ 1 \end{bmatrix}$$

end



Goal: Add Two n-bit Integers

$$c_{-1} = 0$$
 (addition mod 2)
for $i = 0$: n-1
 $s_i = a_i + b_i + c_{i-1}$
 $c_i = a_i b_i + c_{i-1} (a_i + b_i)$
end

18.337 C_{n-1}

$$\begin{bmatrix} c_i \\ 1 \end{bmatrix} = \begin{bmatrix} a_i + b_i & a_i b_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_{i-1} \\ 1 \end{bmatrix}$$

Matmul prefix with binary arithmetic is equivalent to carry-look ahead!

Compute c_i by prefix, then $s_i = a_i + b_i + c_{i-1}$ in parallel