Unsupervised Learning of Manifolds

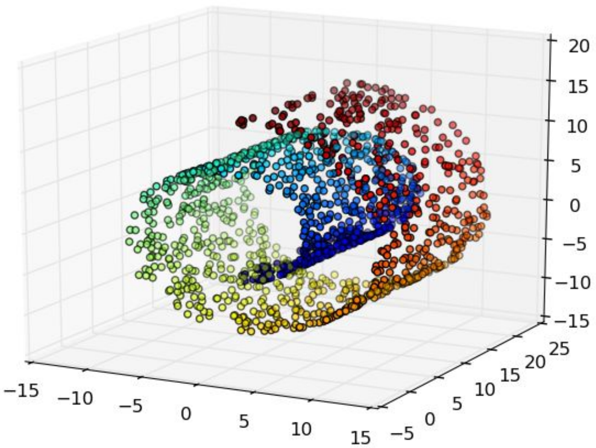
CS365A

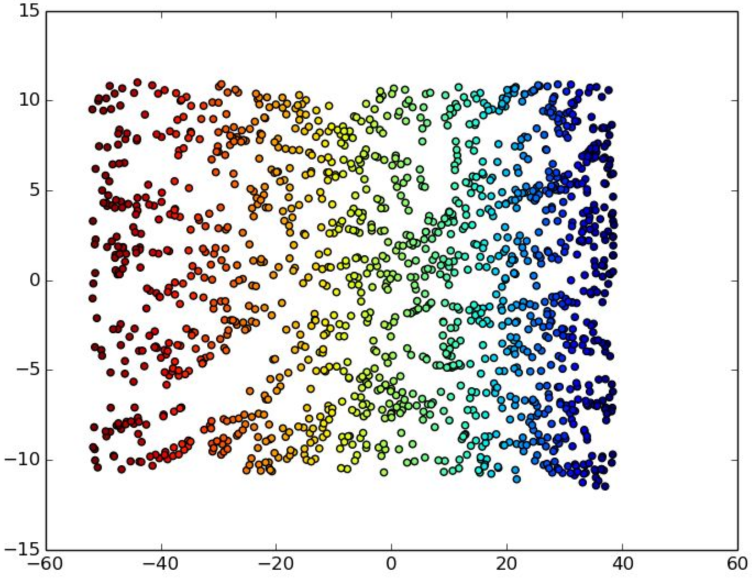
What are manifolds?

- A manifold is a topological space that is locally Euclidean (i.e., around every point, there is a neighborhood that is topologically the same as the open unit ball in R^n)
- Consider the ancient belief that the earth is flat - it is locally flat at any point on the small scale that we see.
- We will be trying to find these manifolds from images obtained from a robot’s camera

Visualizing manifolds

The Swiss roll

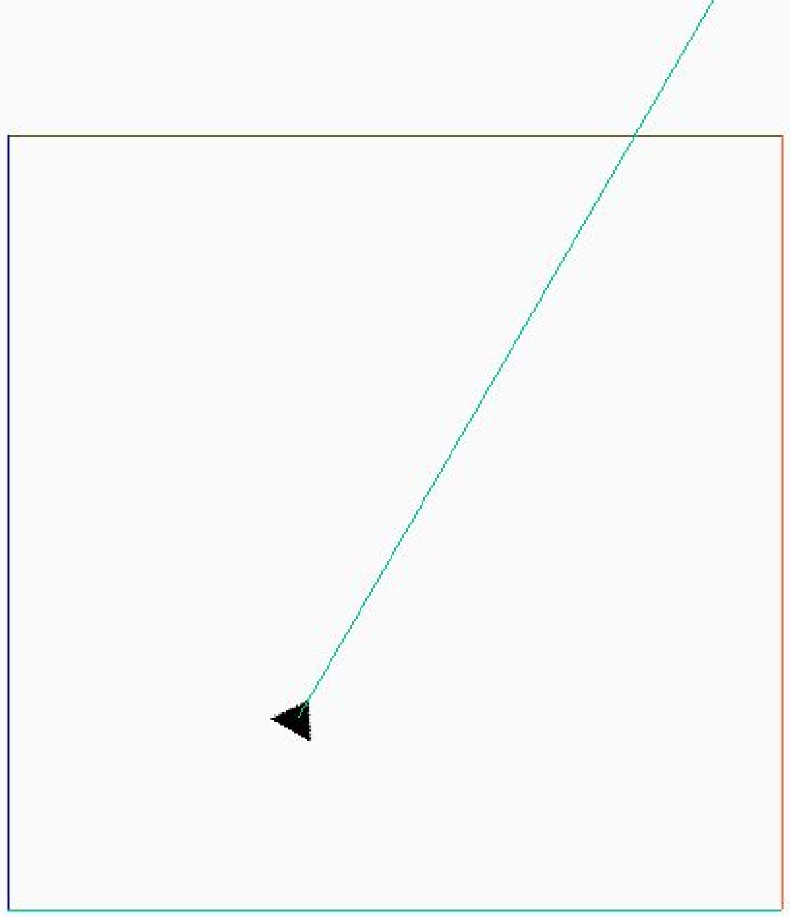




Why learn manifolds?

- Easy to analyse and compute in the manifold space
- Removes the curse of dimensionality
- Gives easy way of inferring certain information (the spatial location and orientation of the robot in this HW)

$(x,y,\theta) = (-50,-100,60^{o})$
FOV = 120



The Isomap algorithm

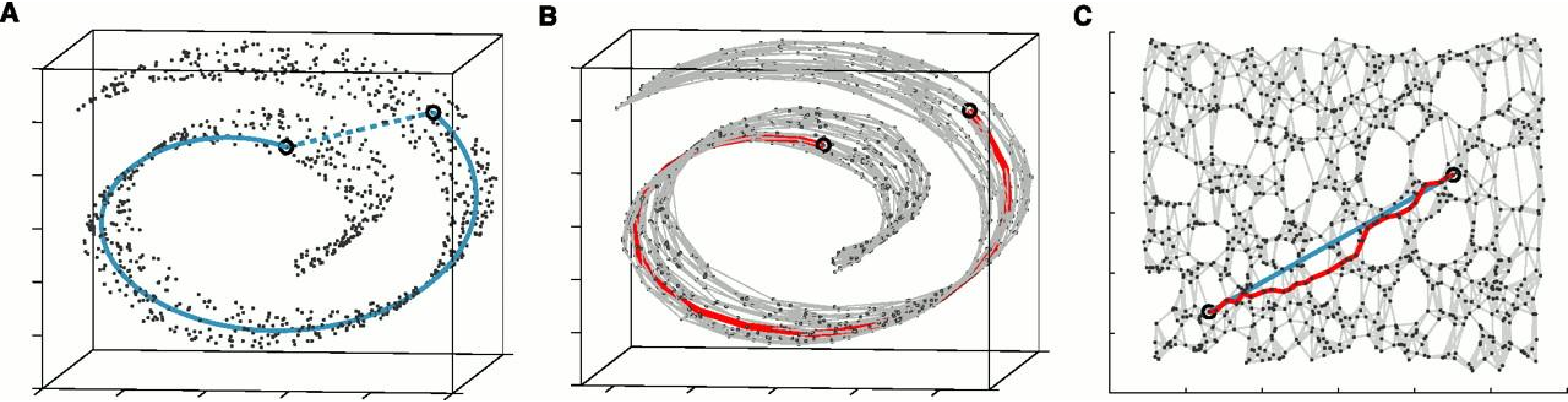
- Find neighbours of a point and construct neighbourhood graph
  - All points within sphere of some radius
  - K-nearest neighbours
  - The neighbourhood graph is denoted by G

The Isomap algorithm

- Compute shortest path
  - Initialize $d_G(i,j)$ - $d_X(i,j)$ if i,j are connected by an edge
    - ∞ otherwise
  - Then for each value of $k = [1, 2, . . ., N]$ in turn, replace all entries $d_G(i, j)$ by $\min\{d_G(i, j), d_G(i,k)+d_G(k, j)\}$.
    The matrix of final values $D_G = \{d_G(i, j)\}$ will contain the shortest path distances between all pairs of points in G

The Isomap algorithm

- ➔ Construct d-dimensional embedding
  {Using MDS - multidimensional scaling}
  - Find coordinate vectors y_i that best preserves the manifold's estimated intrinsic geometry
    - Minimize $E = ||\tau(D_G)-\tau(D_Y)||_{L2}$
      τ - converts distances to inner products
    - Reaches global minima on setting D_Y to top d eigenvectors of $\tau(D_G)$



The Isomap algorithm

Fig. taken from http://science.sciencemag.org/content/sci/290/5500/2319.full.pdf

The homework

- Find this lower dimensional manifolds in a variety of settings
- Visualize the obtained manifolds - rings? cylinder? torus?
- Given an input image infer the coordinate vector that would have generated this image using your learned manifold.