Group testing with incomplete block designs and imperfect tests

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Joint work with
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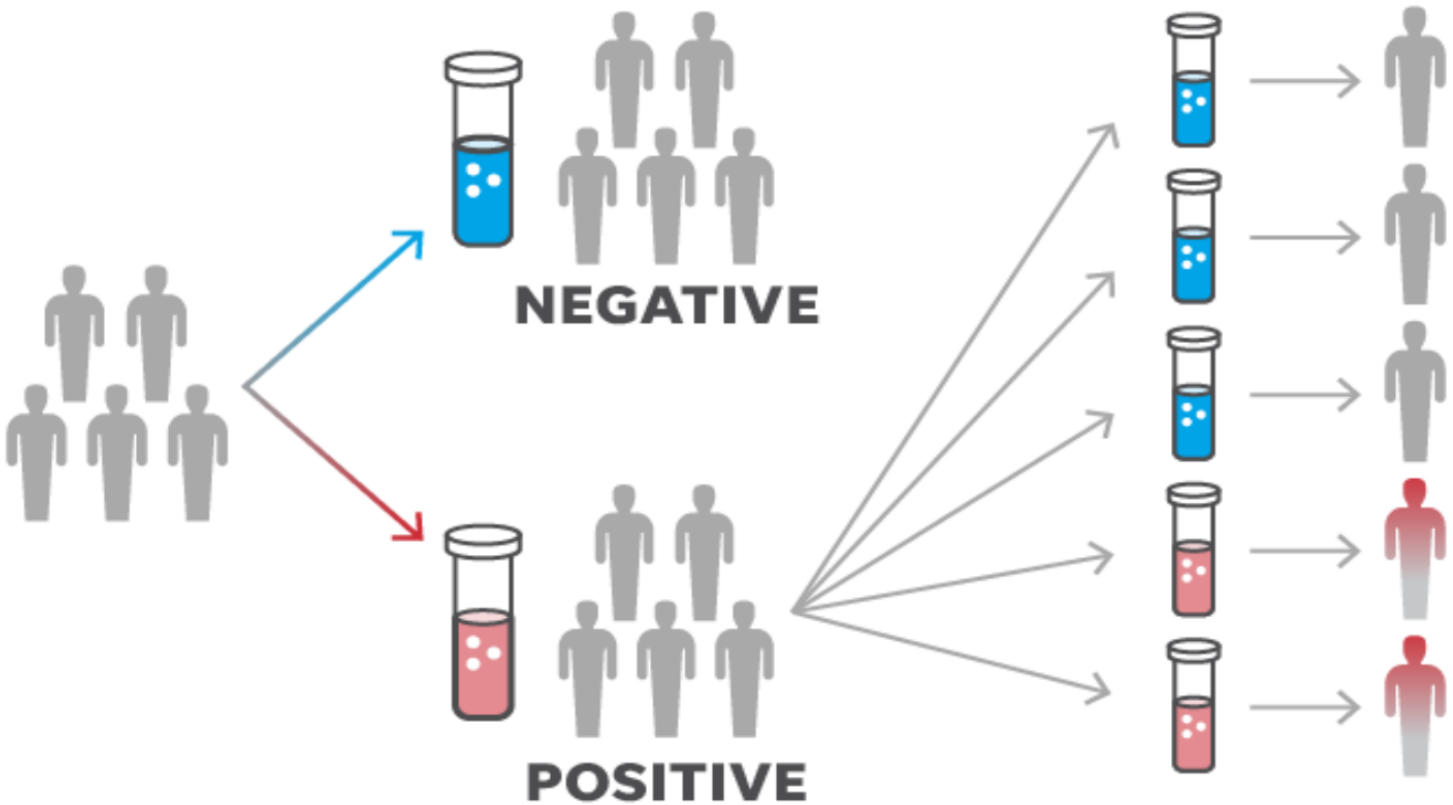
What is Group testing?

- Also known as "pooled testing"
- Screening for infectious diseases



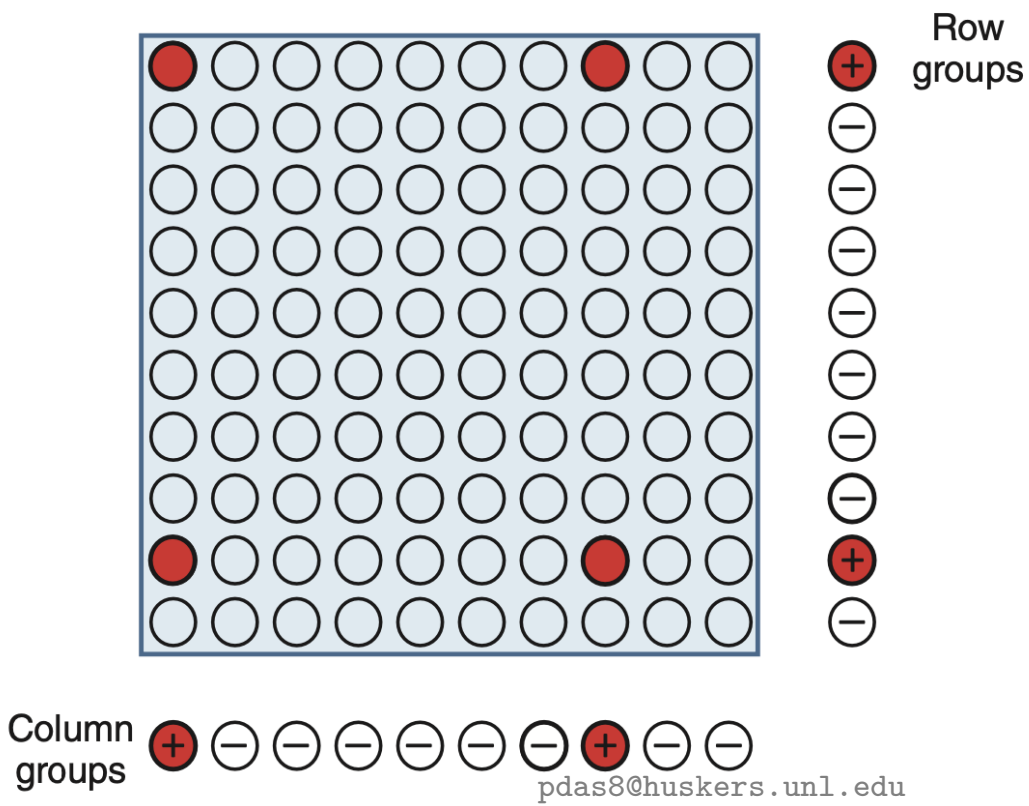
SOURCE USA TODAY research
Karl Gelles/USA TODAY

Importance

- Decrease number of tests
  - Increase in testing capacity
  - Works when disease prevalence is low
- A recent instance: COVID-19 pandemic
  - News media outlets: *The New York Times*, *The Washington Post*, ABC News, PBS NewsHour, USA Today, ...
  - Over 1 GB of papers published during the first 2 years of the pandemic!
    - Abdalhamid et al. (*American Journal of Clinical Pathology*, 2020)
    - Hogan et al. (*Journal of the American Medical Association*, 2020)
    - Lohse et al. (*Lancet Infectious Diseases*, 2020)
- Widely used elsewhere: Blood donation screening, computer network assessments, ...

Algorithms

- Hierarchical algorithm
  - Dorfman testing (two stages)
  - Three stages and higher are possible
- Non-hierarchical algorithm
  - Array testing



Algorithms

- Non-hierarchical algorithm (continued)
  - Incomplete block designs (IBDs)

	Treatments					
Blocks	1	2	3	4	5	6
1	1	0	1	0	1	0
2	0	1	0	1	1	0
3	0	1	1	0	0	1
4	1	0	0	1	0	1

  - Group testing setting

	Specimens					
Groups	1	2	3	4	5	6
1	1	0	1	0	1	0
2	0	1	0	1	1	0
3	0	1	1	0	0	1
4	1	0	0	1	0	1

Purpose

- Previous work using IBDs
  - Sudbury (*Biometrics*, 2010) and Eskridge et al. (*Biotechnology Progress*, 2018)
  - Did not incorporate **false positive** or **false negative**
- Our purpose
  - Examine IBDs when false positive or false negative possible
  - Compare with Dorfman testing and array testing
  - Use the expected number of tests as the performance measure

M matrix

- Matrix for specimen arrangements

M:

	Specimens					
Groups	1	2	3	4	5	6
1	1	0	1	0	1	0
2	0	1	0	1	1	0
3	0	1	1	0	0	1
4	1	0	0	1	0	1

- Referred as "**M** without immediate ambiguity"

M matrix

- With immediate ambiguity

M:

	Specimens						
Groups	1	2	3	4	5	6	7
1	1	0	1	0	1	0	1
2	0	1	1	0	0	1	1
3	0	1	0	1	1	0	0
4	1	0	0	1	0	1	0

  - Specimens 3 and 7 in same groups

Comparisons

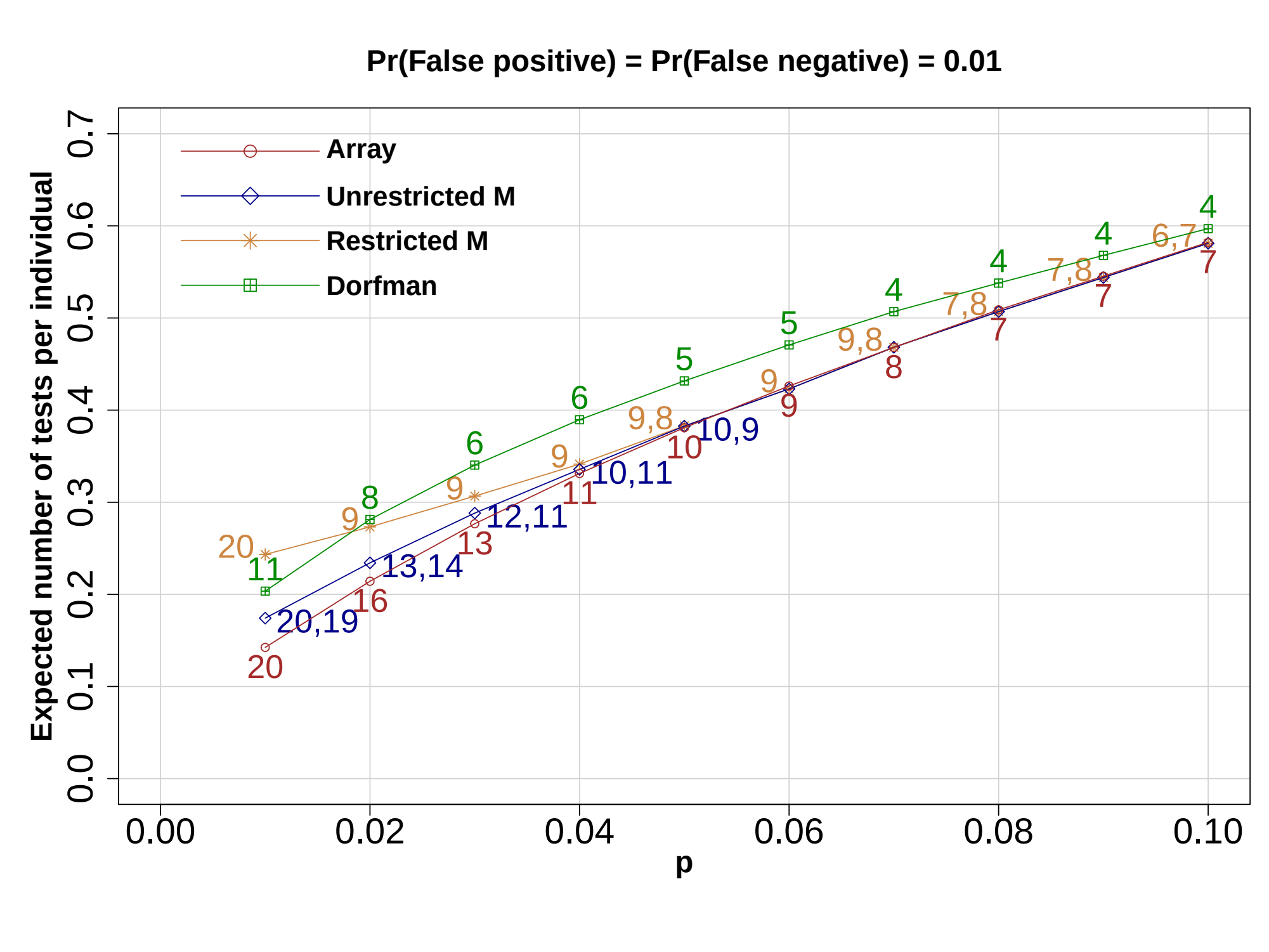
- Cases of **M** and other algorithms
  - IBD using restricted **M** (without ambiguity)
  - IBD using unrestricted **M** (with or without ambiguity)
  - Dorfman testing
  - Array testing
- Expected number of tests per individual
  - Let T be the number of tests for an initial set of I individuals
    - For Dorfman, stage 1 group size
    - For array, number of individuals in an array
    - For IBD, number of individuals represented by **M**
  - $\mathrm{E}(T)/I$ is the expected number of tests per individual
    - $\mathrm{E}(T)/I = 1$ when testing each individual separately
    - Want $\mathrm{E}(T)/I < 1$ for group testing
    - Example: $\mathrm{E}(T)/I = 0.25$ ...
  - Closed form expressions exist for Dorfman and array testing (Kim et al., *Biometrics*, 2007)

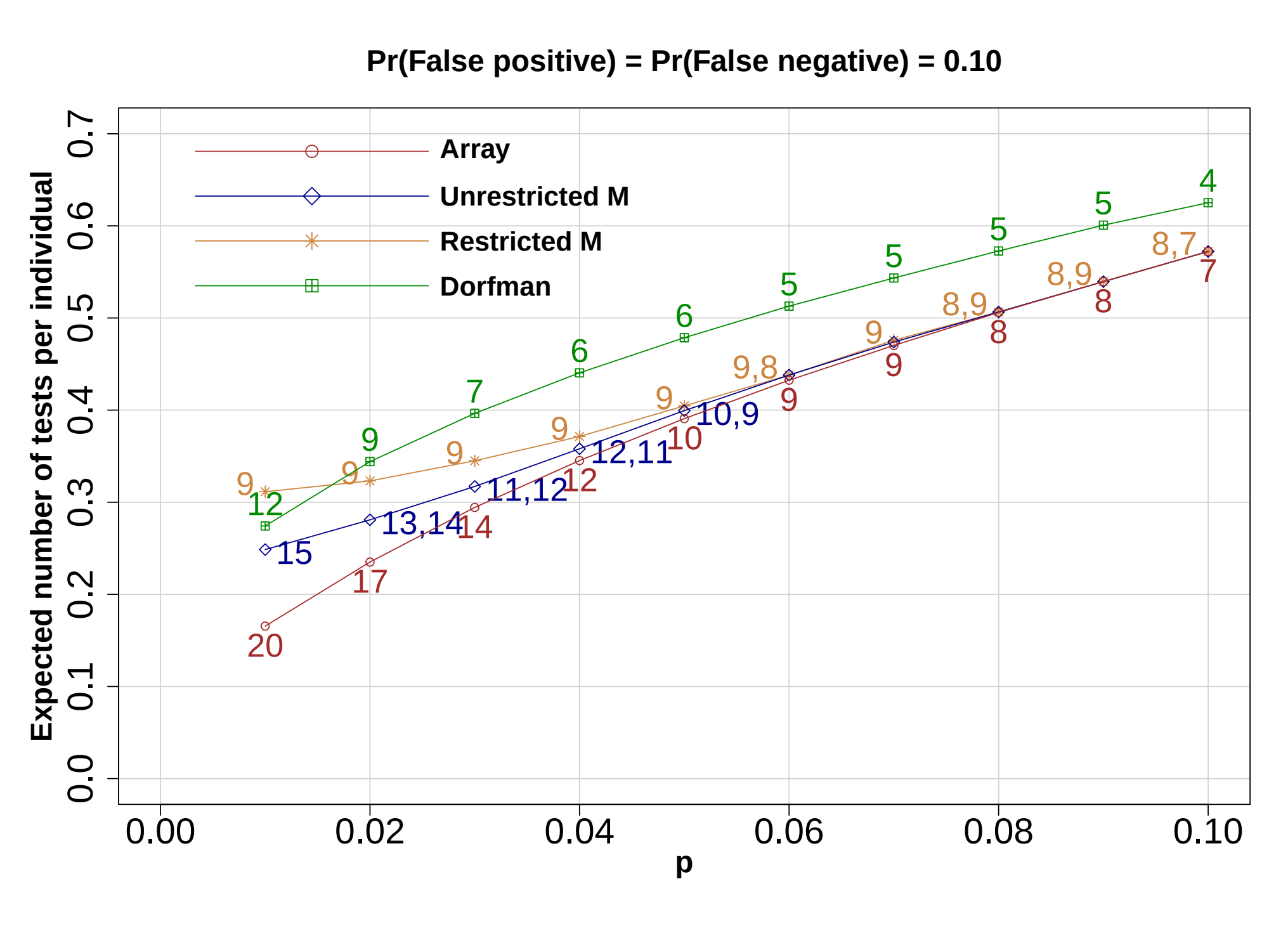
Comparisons

- A closed form expression for E(T)/I does not exist currently for IBDs
  - Simulate the testing process for an **M**
  - Use Monte Carlo simulation
  - Repeat this process 10,000 times and observe number of tests each time
  - Estimate E(T)/I
- **Best algorithm** has the smallest E(T)/I

Steps

1. Find optimal testing configuration (OTC) for each algorithm at a prevalence (p)
   - $p = 0.01, 0.02, ..., 0.10$
   - $Pr(\text{False positive}) = Pr(\text{False negative}) = 0.01, 0.10$
   - IBD
     - **Number of specimens:** 5 - 60
     - **Number of groups:** 4 - 10
     - **Number of groups a specimen can be in:** 2 - 4
   - Dorfman testing
     - **Group size:** 2 - 100
   - Array testing:
     - **Group size:** 2 - 20
2. Compare OTCs by expected number of tests per individual


Pr(False positive) = Pr(False negative) = 0.01
Array
Unrestricted M
Restricted M
Dorfman
Expected number of tests per individual
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.00
0.02
0.04
0.06
0.08
0.10
p
20
16
13
11
9
8
6
5
4
20,19
13,14
12,11
10,11
10,9
9,8
7,8
6,7
9
8
7


Pr(False positive) = Pr(False negative) = 0.10
Array
Unrestricted M
Restricted M
Dorfman
Expected number of tests per individual
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.00
0.02
0.04
0.06
0.08
0.10
p
20
17
14
12
9
8
7
15
13,14
11,12
12,11
10,9
9,8
8,9
8,7
12
9
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5
4

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