Tutorial-4 (DAA)

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1). $T(n) = 3T(n/2) + n^2$

Answer) $a = 3,\ b = 2,\ f(n) = n^2$

$n^{\log_b a} = n^{\log_2 3}$

comparing $n^{\log_2 3}$ and n^2

$n^{\log_2 3} < n^2$ (case 3)

∴ according to master Theorem

$T(n) = \theta(n^2)$

2). $T(n) = 4T(n/2) + n^2$

$a = 4,\ b = 2$

$n^{\log_b a} = n^{\log_2 4} = n^2 = f(n)$ (Case 2)

∴ according to master Theorem

$T(n) = O(n^2 \log n)$

3.) $T(n) = T(n/2) + 2^n$

$a = 1,\ b = 2$

$n^{\log_2 1} = n^0 = 1$

$1 < 2^n$ (case 3)

∴ according to master Theorem

$T(n) = \theta(2^n)$.

4.) $T(n) = 2^n T(n/2) + n^n$

∴ Master's Theorem is Not applicable as a is function.

5.) $T(n) = 16T(n/4) + n$

$a = 16,\ b = 4,\ f(n) = n$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 > f(n)$ (Case 1)

$\therefore T(n) = \theta(n^2)$

6.) $T(n) = 2T(n/2) + n \log n$

$a = 2,\ b = 2,\ F(n) = n \log n$

$n^{\log_b^a} = n^{\log_2^2} = n$

Now, $f(n) > n$

$\therefore$ According to masters Theorem $T(n) = \theta(n \log n)$

7.) $T(n) = aT\left(\frac{n}{2}\right) + \frac{n}{\log n}$

$a = 2,\ b = 2,\ f(n) = \frac{n}{\log n}$

$n^{\log_b a} = n^{\log_2^2} = n$

$n > f(n)$

$\therefore$ According to masters Theorem $T(n) = \theta(n)$

8.) $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

$a = 2,\ b = 4,\ f(n) = n^{0.51}$

$n^{\log_b a} = n^{\log_4 2} = n^{0.5}$

$n^{0.5} < f(n)$

$\therefore$ According to masters Theorem $T(n) = \theta(n^{0.5})$

9.) $T(n) = 0.5\,T(n/2) + \frac{1}{n}$.

$\therefore$ Masters Not applicable as $a < 1$.

10.) $T(n) = 16T(n/4) + n!$

$a = 16,\ b = 4,\ f(n) = n!$

$n^{\log_b^a} = n^{\log^{16}} = n^2$

$n^2 < n!$

$\therefore$ According to masters Theorem $T(n) = \theta(n!)$

11.) $T(n) = 4T\left(\frac{n}{4}\right) + \log n$

$a = 4,\ b = 2,\ f(n) = \log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > f(n)$

$\therefore$ According to Master's theorem, $T(n) = \Theta(n^2)$

12.) $T(n) = \text{sqrt}(n) + (n/2) + \log n$

$\therefore$ Master's Not applicable as a is Not constant.

13.) $T(n) = 3T(n/2) + n$

$a = 3,\ b = 2 \qquad f(n) = n$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$n^{1.58} > f(n)$

$\therefore$ According to Master's theorem, $T(n) = \Theta(n^{\log_2 3})$

14.) $T(n) = 3T(n/3) + \sqrt{n}$.

$a = 3,\ b = 3,\ f(n) = \sqrt{n}$

$n^{\log_b a} = n^{\log_3 3} = n$

$n > \sqrt{n}$

$\therefore$ According to Master's Theorem, $T(n) = \Theta(n)$

15.) $T(n) = 4T(n/2) + cn$

$a = 4,\ b = 2,\ f(n) = c*n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > c*n$

$\therefore$ According to Master's Theorem, $T(n) = \Theta(n^2)$

16.) $T(n) = 3T(n/4) + n\log n$

$\Rightarrow$ $a = 3,\ b = 4,\ f(n) = n\log n$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$n^{0.79} < n \log n$

$\therefore$ According to Master's theorem, $T(n) = \Theta(n \log n)$

17) $T(n) = 3T(n/3) + n/2$

$a = 3, \quad b = 3, \quad f(n) = \frac{n}{2}$

$n^{\log_b^a} = n^{\log_3^3} = n$

$\Theta(n) = \Theta\left(\frac{n}{2}\right)$

$\therefore$ According to Master's theorem $T(n) = \Theta(n \log n)$.

18) $T(n) = 6T(n/3) + n^2 \log n$

$\rightarrow$ $a = 6, \quad b = 3, \quad f(n) = n^2 \log n$

$n^{\log_b^a} = n^{\log_3^6} = n^{1.63}$

$n^{1.63} < n^2 \log n$

$\therefore$ According to Master's theorem $T(n) = \Theta(n^2 \log n)$

19.) $T(n) = 4T(n/2) + n/\log n$.

$a = 4, \quad b = 2, \quad f(n) = n/\log n$.

$n^{\log_b^a} = n^{\log_2^4} = n^2$

$n^2 > n/\log n$.

$\therefore$ According to Master's theorem $T(n) = \Theta(n^2)$.

20). $T(n) = 64T(n/8) - n^2 \log n$.

Master's theorem is Not applicable as $f(n)$ is Not Increasing function.

21.) $T(n) = 7T(n/3) + n^2$

$\Rightarrow$ $a = 7, \quad b = 3, \quad f(n) = n^2$

$n^{\log_b^a} = n^{\log_3^7} = n^7$

$n^7 < n^2$

$\therefore$ According to Master's theorem, $T(n) = \Theta(n^2)$

22) $T(n) = T(n/2) + n(2 - \cos n)$

Master's Theorem isn't applicable since regularity condition is violated in Case 3.