Table 1: Table of parameters used to model message propogation

Parameter	**Description**
λ	Rate at which susceptible nodes acquire the infection
β	Direct transmission rate (contact rate)
ε	Transmission reducing factor
σ^{-1}	Average time period of a exposed node
γ^{-1}	Average time period of infection
ω^{-1}	Average time period of recovery
τ^{-1}	Average time period of a carrier node
μ	Death rate of nodes
ρ	Probability that an infected node becomes carrier

The CISER equations are as follows

$$\frac{dS}{dt} = -(\beta I + \varepsilon\beta C)S - \mu S + \omega R \tag{1}$$

$$\frac{dE}{dt} = (\beta I + \varepsilon\beta C)S - (\sigma + \mu)E \tag{2}$$

$$\frac{dI}{dt} = \sigma E - (\gamma + \mu)I \tag{3}$$

$$\frac{dC}{dt} = \rho\gamma I - (\tau + \mu)C \tag{4}$$

$$\frac{dR}{dt} = (1-\rho)\gamma I + \tau C - (\omega + \mu)R \tag{5}$$

The real parts of the roots of the cubic polynomial $P(\lambda) = \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3$ are negative if the coefficients of this polynomial satisfy the following four conditions:

1. $a_1 = c_1 + c_2 + c_3 > 0$
2. $a_2 = c_1c_2 + c_1c_3 + c_2c_3 - \sigma\beta > 0$
3. $a_3 = c_1c_2c_3(1 - R_0) > 0$
4. $a_1a_2 - a_3 > 0$

By the definition of the coefficients of c, the first condition holds true. Let us consider the following lemma.

Lemma:

If the basic reproduction number is less than one and the condition (3) is true, then the conditions (2) and (4) are also true.

Proof:

$$\frac{dS}{dt}=\frac{dy_1}{dt},\frac{dE}{dt}=\frac{dy_2}{dt},\frac{dI}{dt}=\frac{dy_3}{dt},and\frac{dC}{dt}=\frac{dy_4}{dt},$$

$$\begin{aligned}\frac{dy_1}{dt}&=-(\beta y_3+\varepsilon\beta y_4)\overline{S}-(\beta y_3+\varepsilon\beta y_4)y_1-(\beta\overline{I}\\&+\varepsilon\beta\overline{C}-(\omega+\mu))y_1-w(y_2+y_3+y_4)\end{aligned}\tag{6}$$

$$\begin{aligned}\frac{dy_2}{dt}&=(\beta y_3+\varepsilon\beta y_4)\overline{S}+(\beta\overline{I}+\varepsilon\beta\overline{C})y_1\\&+(\beta y_3+\varepsilon\beta y_4)y_1-(\sigma+\mu)y_2\end{aligned}\tag{7}$$

$$\frac{dy_3}{dt}=\sigma y_2-(\gamma+\mu)y_3\tag{8}$$

$$\frac{dy_4}{dt}=\rho\gamma y_3-(\tau+\mu)y_4\tag{9}$$

Let us define the Lyapunov function as:

V : $IR^4\to IR^+$ by V(y) = $y_1^2+\ y_2^2+y_3^2+y_4^2$

Evaluating the condition on the parameters that make $\frac{dV(y)}{dt}\le\ 0,\ \forall y\in IR^4$ and if the directional derivative of the solution y = y(t_0, y_0, t) of the IVP along the closed curve V(y) is negative at any time t, then the endemic equilibrium point $\overline{x}$ of the original IVP is asymptotically stable.

$$\frac{dV(y)}{dt}=\frac{\partial V(y)}{\partial y}.\frac{dy}{dt}\tag{10}$$

$$\begin{aligned}\frac{dV(y)}{dt}&=2[-\beta Sy_1y_3-\varepsilon\beta Sy_1y_4-(\beta\overline{I}+\varepsilon\beta\overline{C}-(\omega+\mu))y_1^2\\&-w(y_1y_2+y_1y_3+y_1y_4)]+2[\beta Sy_2y_3+\varepsilon\beta Sy_2y_4\\&+(\beta\overline{I}+\varepsilon\beta\overline{C})y_1y_2-(\sigma+\mu)y_2^2]+2[\sigma y_3y_2-(\gamma+\mu)y_3^2]\\&+2[\rho\gamma y_3-(\tau+\mu)y_4^2]\end{aligned}\tag{11}$$

This above identity is including the variable S $\in\Omega$ and its limit value in the neighborhood of $\overline{x}$ is $\overline{S}$. Hence,

$$\begin{aligned}\frac{dV(y)}{dt}&=2[-\beta\overline{S}y_1y_3-\varepsilon\beta\overline{S}y_1y_4-(\beta\overline{I}+\varepsilon\beta\overline{C}-(\omega+\mu))y_1^2\\&-w(y_1y_2+y_1y_3+y_1y_4)]+2[\beta\overline{S}y_2y_3+\varepsilon\beta\overline{S}y_2y_4\\&+(\beta\overline{I}+\varepsilon\beta\overline{C})y_1y_2-(\sigma+\mu)y_2^2]+2[\sigma y_3y_2-(\gamma+\mu)y_3^2]\\&+2[\rho\gamma y_3-(\tau+\mu)y_4^2]\end{aligned}\tag{12}$$

Using the property $\pm\ ab\ \le\ a^2\ +b^2$, $\forall\ a, b\ \in\ IR$, yield the following inequality:

$$
\begin{aligned}
\frac{dV(y)}{dt} \le & -[\beta\overline{S}(y_1^2+y_3^2)+\varepsilon\beta\overline{S}(y_1^2+y_4^2)+2(\beta\overline{I}+\varepsilon\beta\overline{C}-(\omega+\mu))y_1^2 \\
& + w(3y_1^2+y_2^2+y_3^2+y_4^2)]+[-\beta\overline{S}(y_2^2+y_3^2)-\varepsilon\beta\overline{S}(y_2^2+y_4^2) \\
& -(\beta\overline{I}+\varepsilon\beta\overline{C})(y_1^2+y_2^2)+2(\sigma+\mu)(y_2^2] \\
& +[-\sigma(y_2^2+y_3^2)+2(\gamma+\mu)y_3^2] \\
& +[-\rho\gamma(y_3^2+y_4^2)+2(\tau+\mu)y_4^2]
\end{aligned} \tag{13}
$$

The endemic equilibrium point $\overline{x}$ is asymptotically stable only if the RHS of the above inequality is positive definite. That is, every coefficient C_i of the variable y_i^2, i = 1, 2, , 4 must be positive.

$$C_1 = \beta\overline{S}+\varepsilon\beta\overline{S}+2(\beta\overline{I}+\varepsilon\beta\overline{C}-(\omega+\mu))+3w-(\beta\overline{I}+\varepsilon\beta\overline{C})\ge 0 \tag{14}$$

$$C_2 = w-\beta\overline{S}-\beta\overline{S}-(\beta\overline{I}+\varepsilon\beta\overline{C})+2(\sigma+\mu)-\sigma\ge 0 \tag{15}$$

$$C_3 = \beta\overline{S}+w-\beta\overline{S}-\sigma+2(\gamma+\mu)-\rho\gamma\ge 0 \tag{16}$$

$$C_4 = \varepsilon\beta\overline{S}+w-\varepsilon\beta\overline{S}-\rho\gamma+2(\tau+\mu)\ge 0 \tag{17}$$

It follows that the message propagation endemic equilibrium point is asymptotically stable if the following conditions hold:

1. $\beta(\overline{S}+\overline{I})+\varepsilon\beta(\overline{S}+2\overline{C})+2w-\mu\ge 0$
2. $w-\beta(\overline{S}+\overline{I})-\varepsilon\beta(\overline{S}+\overline{C})+\sigma+2\mu\ge 0$
3. $w-\sigma+2(\gamma+\mu)-\rho\gamma\ge 0$
4. $w-\rho\gamma+2(\tau+\mu)\ge 0$