Solving 2D geometry using MATRICES

Matrix Project

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February 14, 2019

Geometric Question

A tangent at apoint on the ellipse

$$X^T VX = 51 \longrightarrow (1)$$

Where

$$V = \begin{bmatrix} 3 & 0 \\ 0 & 27 \end{bmatrix}$$

meets the coordinates axes at A and B.If O be the origin,find the minimum area of ΔOAB.

Parametric Matrix

Let,

$$X = \begin{bmatrix} a\cos\theta \\ b\sin\theta \end{bmatrix} \longrightarrow (2)$$

substituting (2) in (1)
$\Rightarrow 3a^2\cos^2\theta + 27b^2\sin^2\theta = 51$
$\Rightarrow a^2\cos^2\theta + b^2\sin^2\theta = 17$
$\rightarrow \theta = 0 \Rightarrow a = \sqrt{17} \rightarrow \theta = \pi/2 \Rightarrow b = \sqrt{17}/3$
$\Longrightarrow$

$$X = \begin{bmatrix} \sqrt{17}\cos\theta \\ \sqrt{17}\sin\theta/3 \end{bmatrix} \longrightarrow (3)$$

Tangent Matrix at a parametric point

Direction Matrix=$d(X)/d(\theta)$,

$$d(X)/d(\theta) = \begin{bmatrix} -\sqrt{17}\sin\theta \\ \sqrt{17}\cos\theta/3 \end{bmatrix} \longrightarrow (4)$$

$$Norm - Vector = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} (4)$$

$$\Rightarrow Norm - Vector = \begin{bmatrix} \sqrt{17}\cos\theta/3 \\ \sqrt{17}\sin\theta \end{bmatrix}$$

$$\Rightarrow Equationoftangentatpoint\theta =$$

$$\left[\sqrt{17}\cos\theta/3 \quad \sqrt{17}\sin\theta\right] X_T = \left[\sqrt{17}\cos\theta/3 \quad \sqrt{17}\sin\theta\right] \begin{bmatrix} \sqrt{17}\cos\theta \\ \sqrt{17}\sin\theta/3 \end{bmatrix} =$$

$$\left[17/3\right] \longrightarrow (5)[here - X_T istangentspace]$$

Finding Points A and B

Let the respective equations be

$$n_1^T = p_1 and n_2^T = p_2$$

This can be written as the matrix equation

$$\begin{bmatrix} n_1^T \\ n_2^T \end{bmatrix} x = P$$

$\Rightarrow N^T x = P$
Where,

$$N = \begin{bmatrix} n_1 & n_2 \end{bmatrix}$$

The point of intersection is then obtained as

$$x = (N^T)^-1P$$

$$= N^- TP$$

Here,

$$n_1 = \begin{bmatrix} \sqrt{17}\cos\theta/3 \\ \sqrt{17}\sin\theta \end{bmatrix}$$

$$n_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \longrightarrow forY - axis$$

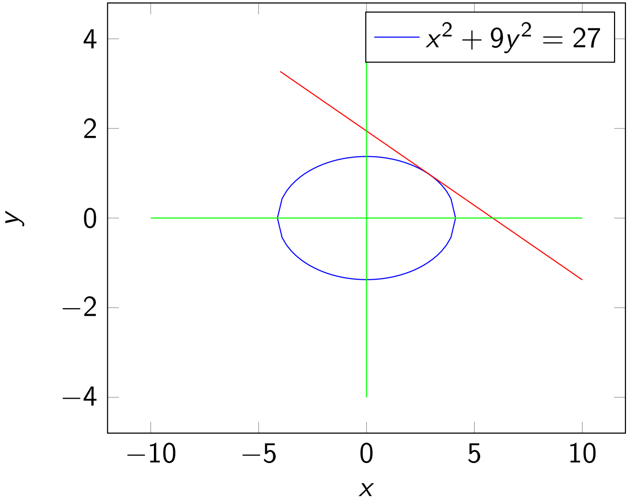
OR

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \longrightarrow forX - axis$$

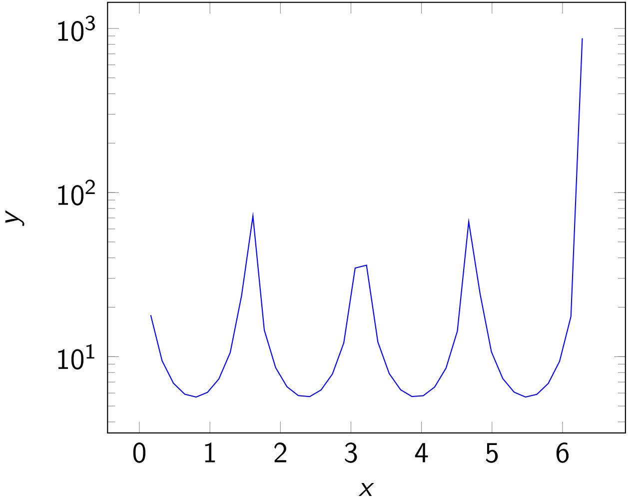
By soving

$$A = \begin{bmatrix} \sqrt{17}/\cos\theta \\ 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \sqrt{17}/(3\sin\theta) \end{bmatrix}$$


$x^2 + 9y^2 = 27$
4
2
0
−2
−4
y
−10
−5
0
5
10
x

Area of ΔOAB$=(0.5)(\sqrt{17}/\cos\theta)(\sqrt{17}/(3\sin\theta))$
$=17/3\sin 2\theta$



Area has min value when $\sin 2\theta hasmaxvalue \longrightarrow |\sin 2\theta| = 1$