EE386 Digital Signal Processing Lab **Jul-Dec 2021**

Experiment 3

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1 Introduction

The exercise focuses on FFT and its applications. We begin by plotting the magnitude spectrum of the given instrument tracks using FFT and find their fundamental frequency by peak picking. A similar technique is then used to match the fundamental frequencies of two instruments and also to design and implement a keylock. In the last problem we analyse the temporal variations of the frequency peaks in the spectrum of an opera track using Short Term Fourier Transform. In our example we have $\alpha = 1 + mod(260, 4) = 1$.

All the code for this exercise and the relevant files are included in the `.zip` file submitted along with this report.

2 Problems

1. Plotting signal spectra

(Part 1: Plotting magnitude spectrum using FFT)

(Solution)

The Fast Fourier Transform

A fast Fourier transform (FFT) is an algorithm that calculates the discrete Fourier transform (DFT) of some sequence effciently. FFTs commonly change the time domain into the frequency domain.

In Python FFT is calculated using `numpy.fft.fft(signal)`

The FFT operates by decomposing an N point time domain signal into N time domain signals each composed of a single point. The second step is to calculate the N frequency spectra corresponding to these N time domain signals. Lastly, the N spectra are synthesized into a single frequency spectrum. Figure 1 displays the Flow diagram of FFT.

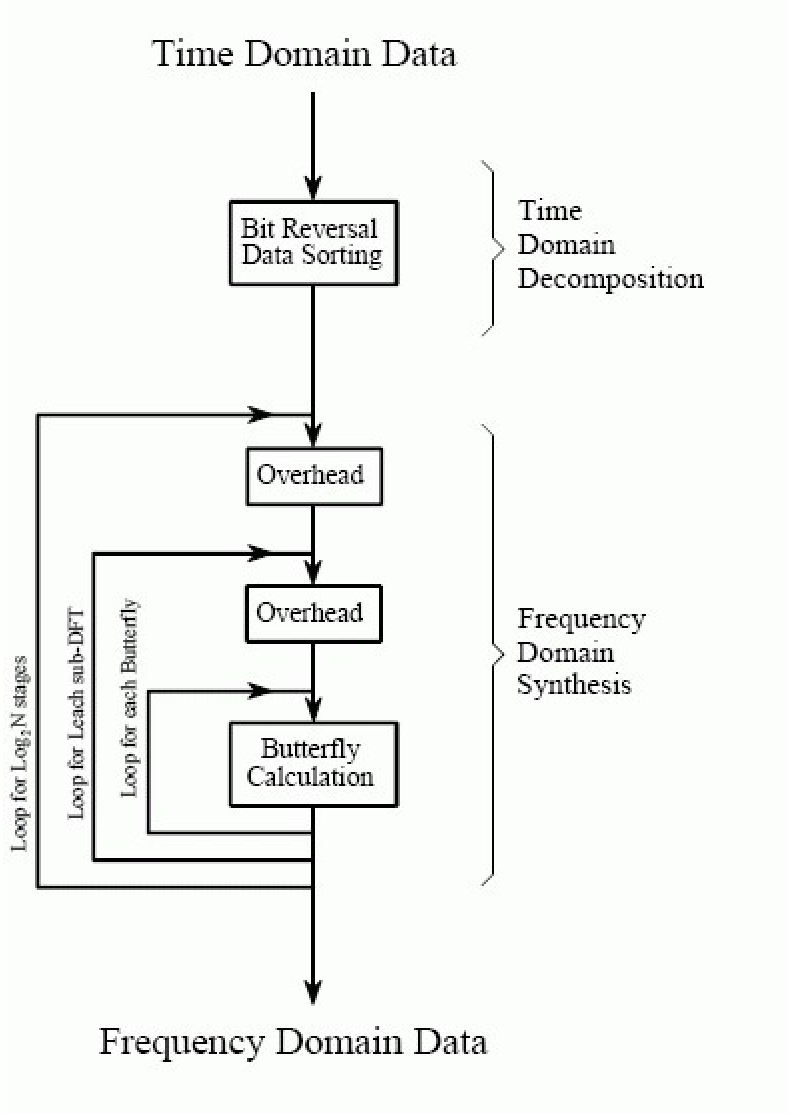


Figure 1: Flow diagram of FFT

We use FFT to compute the single sided magnitude spectrum of the Piano, Trumpet, Violin and Flute tracks. Figure 2 shows the corresponding spectra

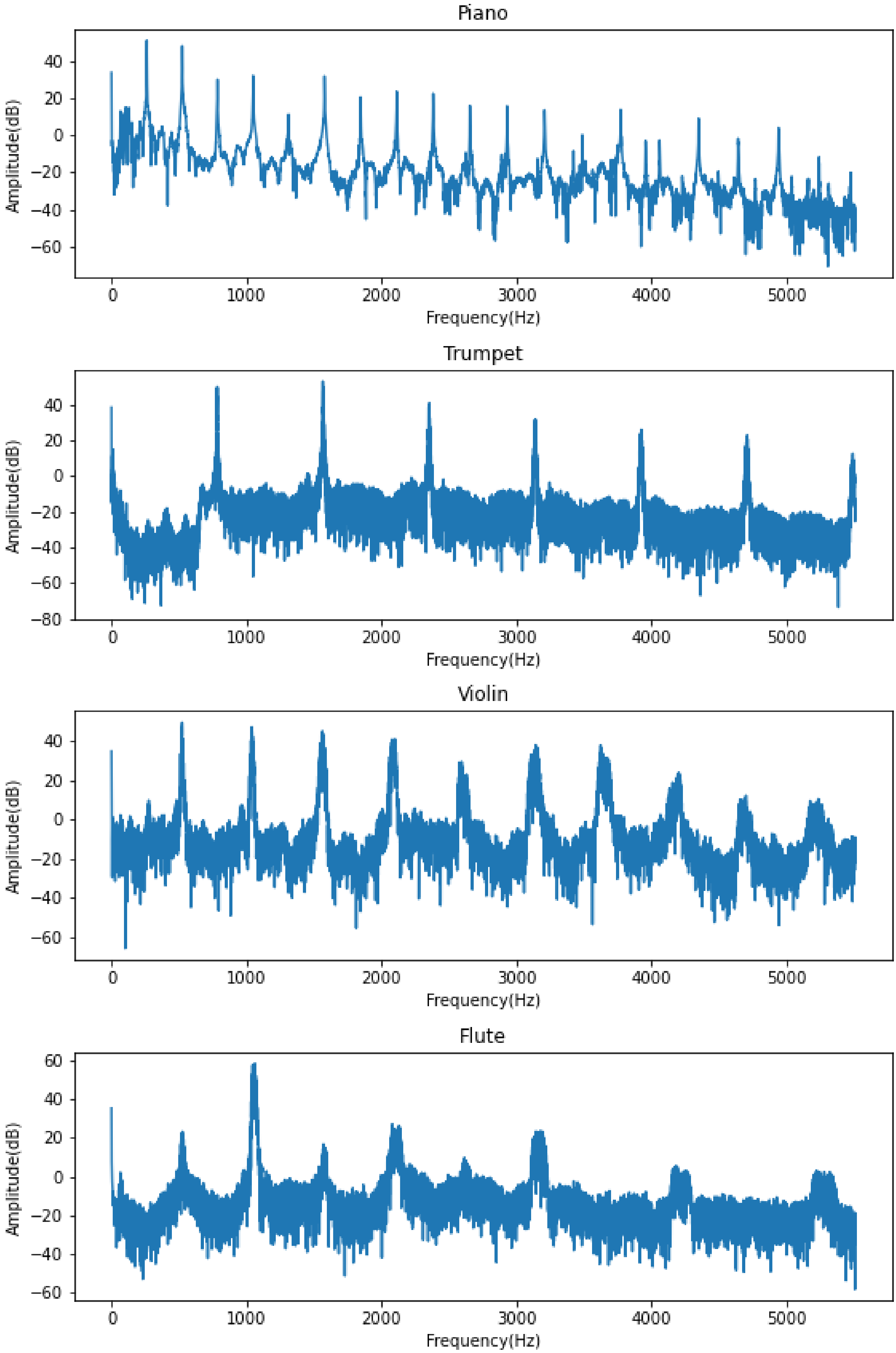


Figure 2: Magnitude Spectrum of the four different tracks with $\alpha = 1$

From the spectra above we can eyeball the frequency range which contains the Fundamental Frequency. For example in the spectrum of trumpet in Figure 2 it can be said that the fundamental frequency lies in the range 136 Hz to 938 Hz. Hence we only investigate this section of the spectrum to find the the frequency corresponding to the local maxima which for the trumpet turns out to be 784.5289 Hz which we can verify using a tuner or pitch pipe to be G_5. Table 1 shows the fundamental frequencies for the other instruments obtained using the same method.

Instrument	**Fundamental Frequency(Hz)**
Piano	261.6353
Trumpet	784.5289
Violin	522.6397
Flute	1063.2000

Table 1: Fundamental frequencies corresponding to each of the instrument tracks

(Part 2: Determining the value of β)
(Solution)
The single sided spectrum of all the four flute tracks is obtained using FFT and we compare the spectrum with that of `piano1.wav`. Figure 3 shows the two spectra plotted on the same figure. It is clearly observed that both the tracks have same fundamental frequency of 261.6353 Hz and almost similar magnitude of the fundamental. Hence in terms of the fundamental frequency we can say that `flute2.wav` is the closest to `piano1.wav`. Thus $\beta = 2$

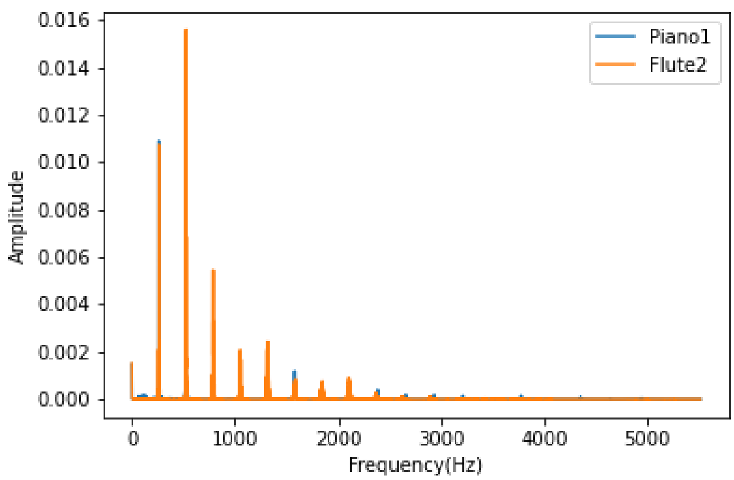


Figure 3: Magnitude Spectrum of flute2.wav and piano1.wav on top of each other showing similar fundamental frequency

2. Whistling keylock

(Part 1: Plot of Fourier Spectrum of reference signal)

(Solution)

To simulate the whistle reference signal we generate the tone A_4 at 440 Hz. Figure 4 shows the Magnitude spectrum of our reference signal. The fundamental frequency can be visually verified to be 440 Hz. This simplifies our analysis, however, in the presence of harmonics we can use peak picking explained in Problem 1 to obtain the fundamental frequency.

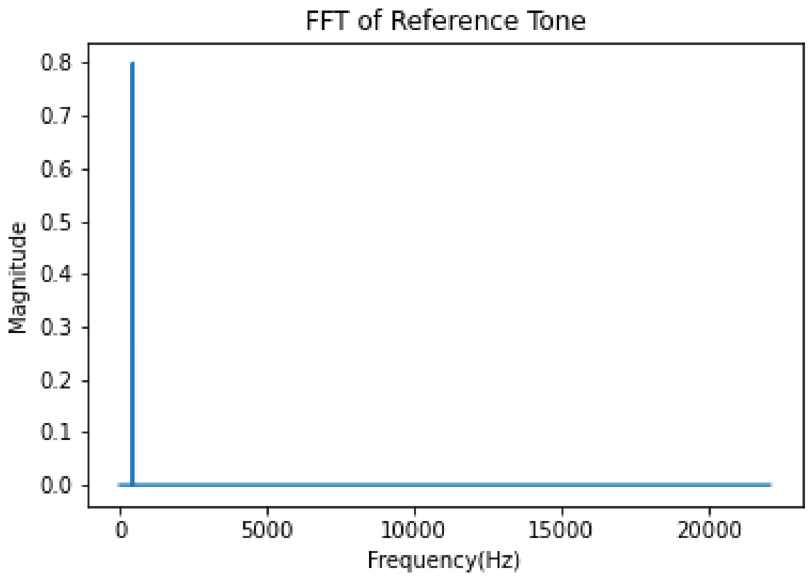
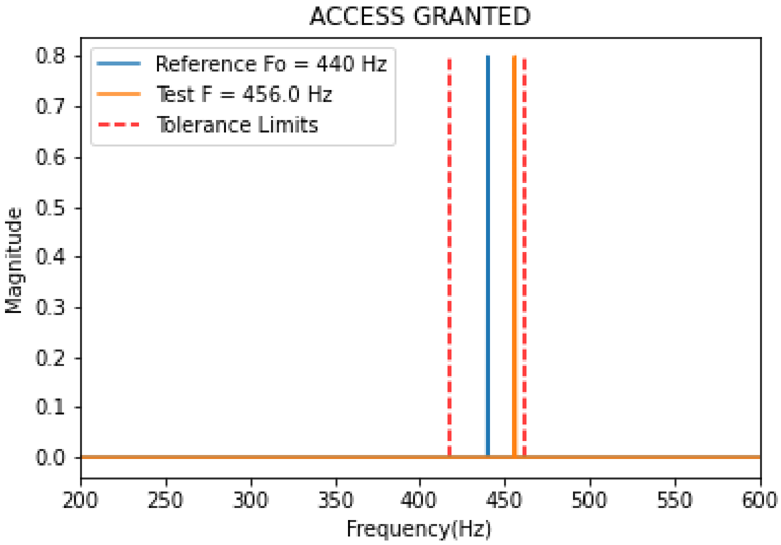


Figure 4: Magnitude Specturm of the tone generated at 440 Hz serving as reference frequency for the lock

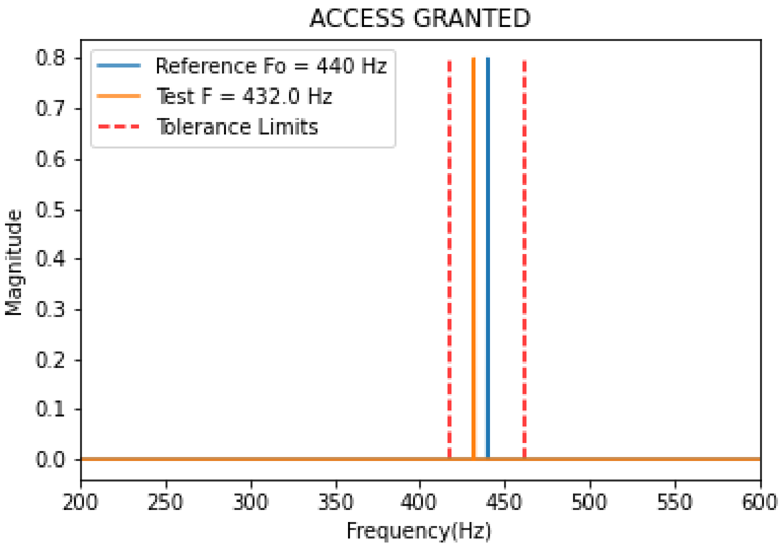
(Part 2: Design and testing of the keylock)

(Solution)

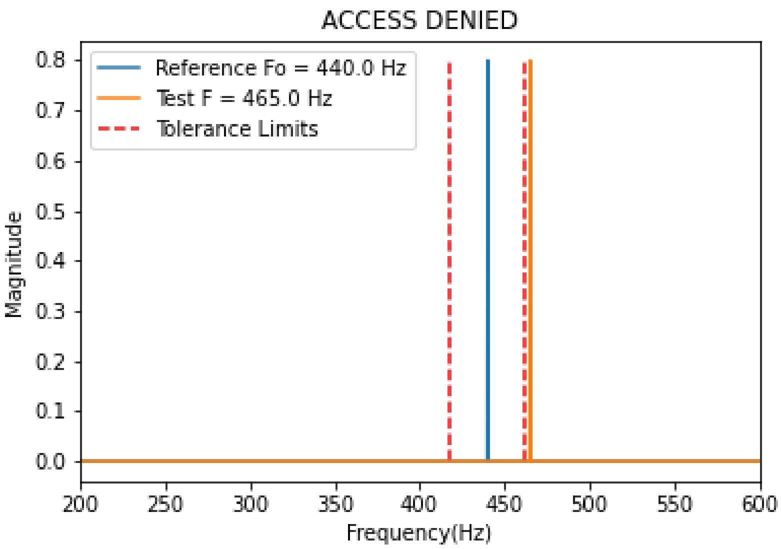
To successfully implement the lock we wish to grant access to any signal with a fundamental frequency in $440 \pm 5\%$ Hz. This is achieved by a function that determines the fundamental frequency of the input using peak picking and compares it with our required range [418, 462]. Inorder to test our function we use three cases with fundamental frequencies at 456 Hz, 432 Hz and 465 Hz. Figure 5 illustrates the lock in action.



(a) $F = 456$ Hz



(b) $F = 432$ Hz



(c) $F = 465$ Hz

Figure 5: Results when the lock is tested with tones of different input fundamental frequencies

3. Simultaneous time and frequency representations

(Part 1: Plot of Fourier Spectrum of Opera.wav)

(Solution)

The Fourier Transform is very good at identifying sinusoidal components of a time-domain signal $f(t)$. However, the basic building blocks of the Fourier Transform, the complex exponentials oscillate over all of time (between $-\infty$ to $+\infty$). For this reason, it is difficult for Fourier Transform to represent signals that are localized in time. Figure 6 shows the magnitude spectrum of the given Opera track. This informal observation in fact reflects a fundamental property (uncertainty principle) of the Fourier Transform. In this problem, we will describe a Fourier based approach to overcome this limitation of the Fourier Transform.

The Short Term Fourier Transform

In the STFT, we perform a series of windowing and FFT operations. We divide the signal into intervals and then calculate FFT. The choice of window presents important trade-offs. Intuitively, we expect the Opera signal to have rapidly fluctuating properties, hence, in order to enable fine temporal resolution we should use a narrow window. In contrast, if we expect our signal to have slowly fluctuating properties, we should use a wide window. There is no optimal way to pick the STFT window. Indeed, this trade-off between temporal and frequency resolution is a fundamental feature of the STFT. For our signal we choose a window of 22050 samples which divides the signal of length 5 seconds into 10 intervals. Figures 7 and 8 illustrate the STFT of all the intervals.

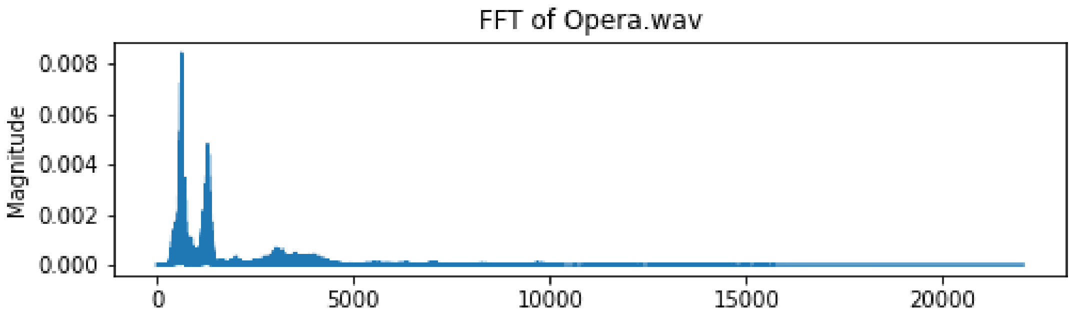


Figure 6: Fourier Spectrum of complete Opera track

(Part 2: Temporal variation of the spectrum)

(Solution)

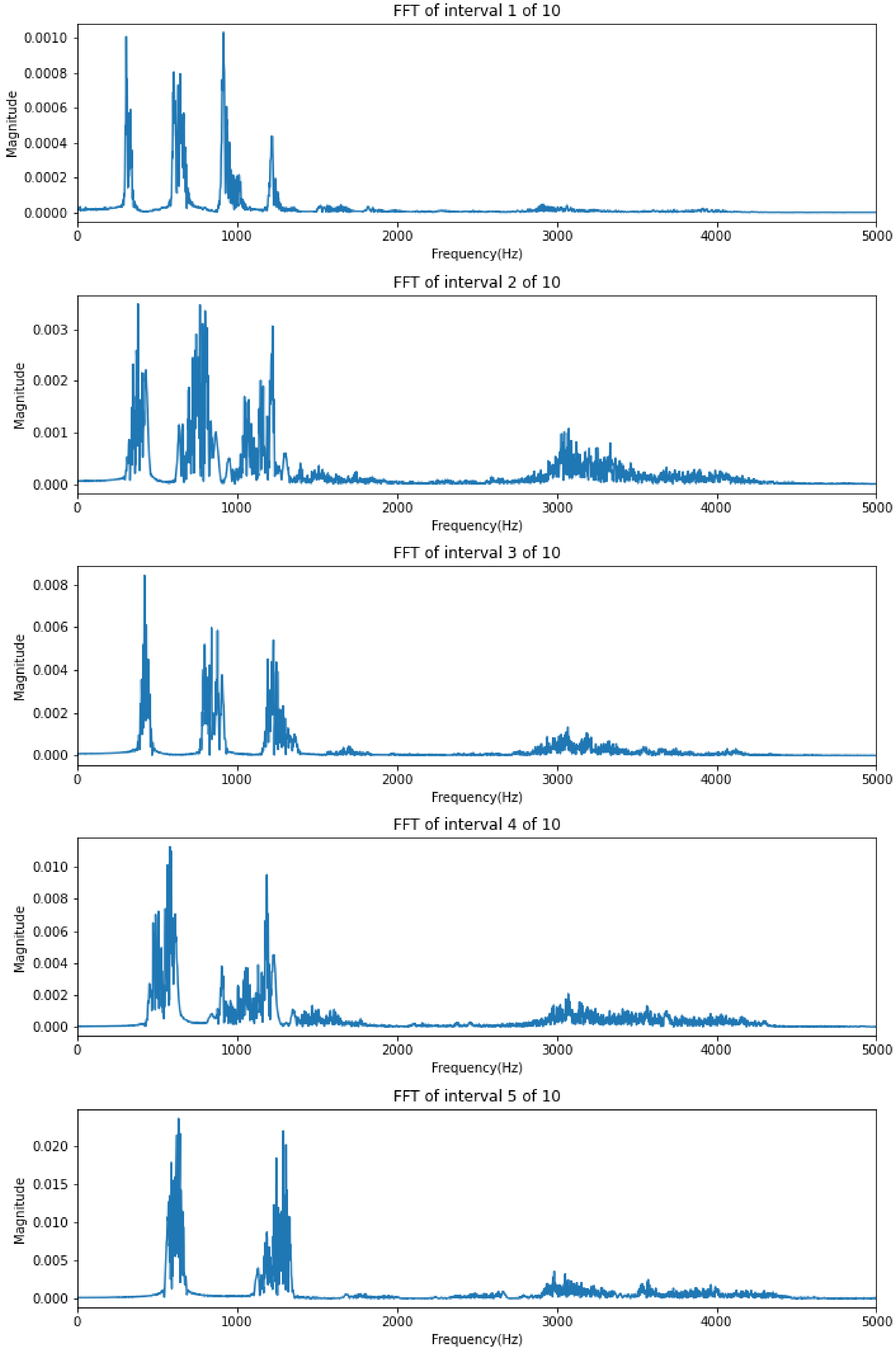


Figure 7: Fourier Spectrum of the first 5 intervals of Opera track

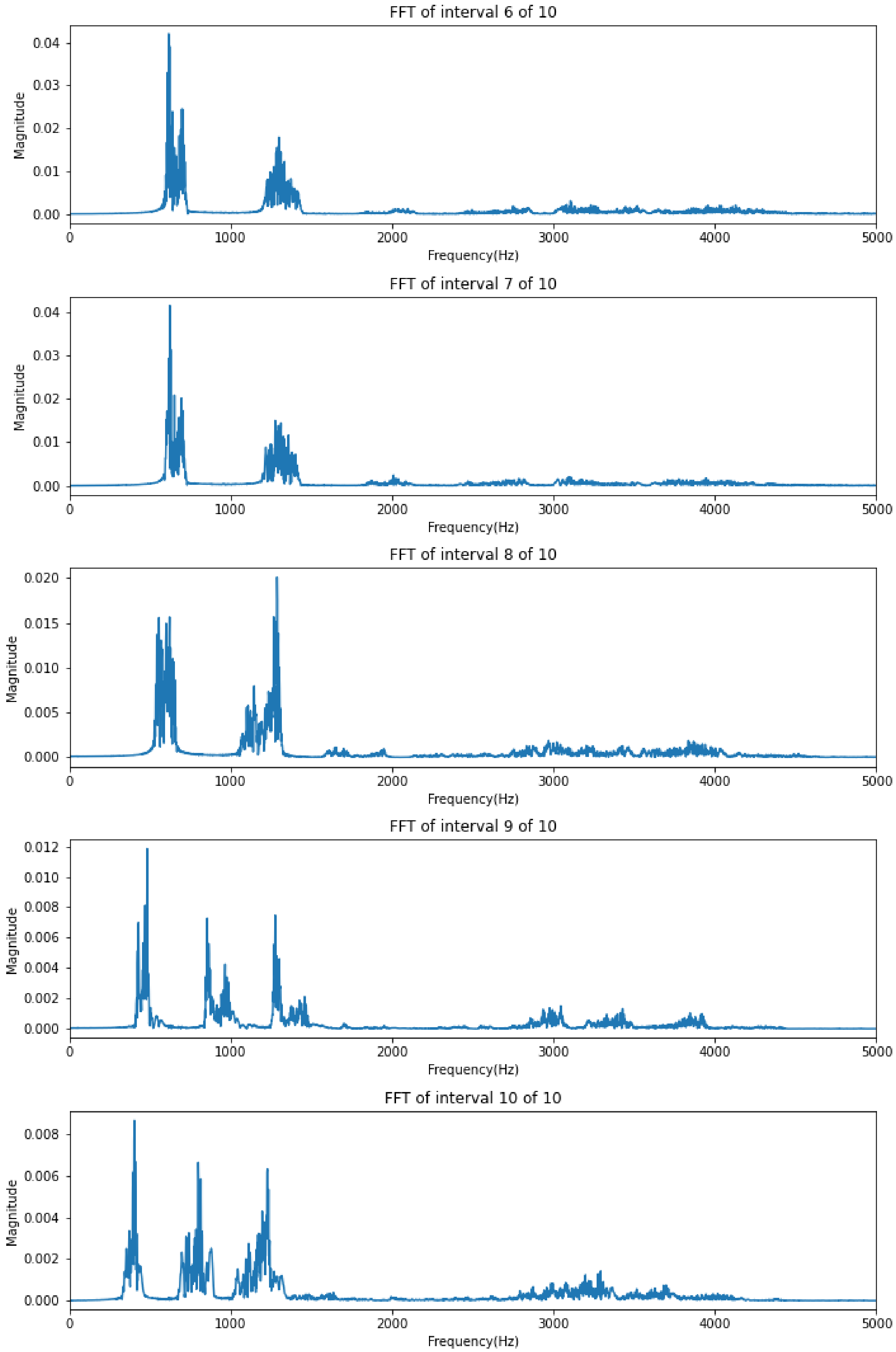


Figure 8: Fourier Spectrum of the last 5 intervals of Opera track

At a sampling rate $F_s = 44100$ Hz, 22050 samples correspond to $t = \frac{1}{44100} \times 22050 = 0.5$ s. Thus each subfigure in 7 and 8 represents the FFT of t seconds interval in the sequence of their interval numbers. This enables us to capture the temporal variations of the spectrum for each t seconds interval.

From the FFTs we can compare the locations of the dominant peaks every 0.5 seconds. As The peaks move to the right we hear the Opera singer sing a higher note. As the peaks shift to the left the note gets lower and bassy. The presence of multiple peaks indicates voice modulation and changing of notes. We can also notice the difference in the magnitude scales in the above plots which indicates the dynamics of the singing. Some notes are sang softer and some are loud. All in all we get an idea of the frequencies and their intensities in each time interval.

- From interval 1 to 2 the there is a slight rightward shift in frequency and gradual increase in magnitude.
- In intervals 2 and 3, the magnitude in 3 is greater and frequency is slightly higher.
- From interval 3 to 4 the frequency increases and gets even louder.
- From interval 4 to 5 frequency shifts to right and magnitude increases.
- From interval 5 to 6 the frequencies are somewhat similar but the magnitude is higher in interval 6.
- From interval 6 to 7 there is slight increase in frequency but the magnitude in 7 is the highest in the whole track.
- From interval 7 to 8 the frequency and magnitude begin to decrease.
- From interval 8 to 9 the frequency peaks shift leftwards indicating singing lower notes and also magnitude decreases.
- From interval 9 to 10 there is further decrease in frequency and lowering of magnitude and the track ends.