Wherever relevant, use $\alpha = 1 + \text{mod}(x, 4)$, where x is the last three digits of your registration number. Wherever relevant, plot signals with normalised axes, with an appropriate resolution for time and with appropriate labels and legends.

Problem 1. (Generating signals)
Create a user-defined function $x(t)$ to generate a decaying exponential with α as the time constant. Plot the following in different subplots:

1. Plot the signal $x(t)$ for $t \in [0, 10]$.
2. Plot the signal $x(t - 1.5\alpha)$ for $t \in [0, 10]$.
3. Plot the signal $x(2t)$ for $t \in [0, 10]$.

Compute the Fourier transform $X(\Omega)$ of the signal $x(t)$ analytically. Plot the magnitude and phase spectra of the signal. For delayed and dilated signals, write its Fourier transform as a function of $X(\Omega)$.

Problem 2. (Generating tones)
Create 5000 samples of two sinusoids of 200αHz and 220αHz with a time resolution of 0.001s. Append them (to make a 2×5000 array), and write it into a `.wav` file. In two subplots, (i) use the `plot` function to plot the first 100 samples of the signal and (ii) use the `stem` function to plot the first 100 samples of the signal.

Problem 3. (Convolution)
Load the data `Track00`α`.wav`. Load the data from the text file `ConvFile`α`.txt` and then convolve the two data streams. Use the convolution function from SciPy and try different modes. Store the result into a `.wav` file. What do you observe? Can you guess the type of filter?

Problem 4. (Amplitude Modulation)
Import the file `speech.wav` that contains the speech signal $s(n)$ with F_s as the sampling frequency. Write a user-defined function to obtain

$$y(n) = s(n) \cos\left(2\pi \frac{F}{F_s} n\right).$$

Generate $y(n)$ for a particular choice of $F = 500$Hz. Plot $s(n)$ and $y(n)$. Can you notice the differences between the signals? Compute, analytically, the Fourier transform of $y(t) = s(t)\cos(2\pi F_0 t)$ in terms of the Fourier transform of $s(t)$ and explain the observation. *Try:* plotting the spectrum of the signals using the `fft` function and verify the claims.