EE386 Digital Signal Processing Lab | Jul-Dec 2021

Experiment 6

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1 Introduction

The exercise is based on IIR filters. We begin by designing a Butterworth low pass filter for the given specifications by finding its transfer function. We then investigate the stability from the pole-zero plot and compare the unit step and impulse responses of Butterworth and Chebyshev type1 filters. Next we filter an ECG signal using the designed filter and compare the filtered and original signals. We then design a band pass filter to extract the fundamental frequency from an instrument signal. Finally we design a Chebyshev filter for the same specifications and compare the orders. In our example we have $\alpha = 1 + mod(260, 3) = 3$.
All the code for this exercise and the relevant files are included in the `.zip` file submitted along with this report.

2 Problems

1. Butterworth filter design

(Part 1: Transfer function of the filter)

(Solution)

We begin the filter design process from the given specifications $\delta_p = \frac{1}{\sqrt{2}}$, $\delta_s = 0.01$, $\Omega_p^{'} = 20\pi$ rad/sec, $\Omega_s^{'} = 40\pi$ rad/sec, $F_s = 720$ samples/second. We then compute $\omega_p^{'}$ and $\omega_s^{'}$ using formula $\omega^{'} = \Omega' T$ where $T = \frac{1}{F_s}$ and $\epsilon = \sqrt{\frac{1-\delta_p^2}{\delta_p^2}}$. In order to overcome the problems of warping, we conduct pre-warping to compute Ω_p" and Ω_s" using formula $\Omega^{"} = \frac{2}{T}\tan(\frac{\omega^{'}}{2})$. The order of Butterworth filter is given by

$$N = \frac{\log(\frac{1}{\epsilon}\sqrt{\frac{1-\delta_s^2}{\delta_s^2}})}{\log(\frac{\Omega_s}{\Omega_p})} \tag{2.1}$$

where $\frac{\Omega_s}{\Omega_p} = \frac{\Omega_s^{"}}{\Omega_p^{"}}$ for low pass filter. From equation 2.1 we get $N = 7$. Also $\Omega_p = \sqrt[N]{\epsilon} = 1$. For $N = 7$, we get the standard transfer function as

$$H(s) = \frac{1}{(1+s)(1+0.445s+s^2)(1+1.247s+s^2)(1+1.802s+s^2)} \tag{2.2}$$

In order to avoid complex calculations, the following steps are implemented in MATLAB in the file `filter_poly.m` in the `.zip` file included with this report.
Next we replace $s \rightarrow 1 \times \frac{s^{'}}{\Omega_p^{"}}$ to get required analog transfer function. Finally we do the bilinear transformation $s \rightarrow \frac{2}{T}(\frac{1-z^{-1}}{1+z^{-1}})$ to get the transfer function of the digital filter in the z-domain.

(Part 2: Pole Zero Plot)

(Solution)

A causal system can be defined as $h(n) = 0, n < 0$. For causal system, ROC will be outside the circle in Z-plane.

$$H(Z) = \sum_{n=0}^{\infty} h(n)Z^{-n} \tag{2.3}$$

Expanding the above equation,

$$H(Z) = h(0) + h(1)Z^{-1} + h(2)Z^{-2} + ... = N(Z)/D(Z) \tag{2.4}$$

For causal systems, expansion of Transfer Function does not include positive powers of Z and the order of numerator cannot exceed order of denominator. This can be written as-

$$\lim_{z\rightarrow\infty} H(Z) = h(0) = 0 \text{ or Finite} \tag{2.5}$$

For stability of causal system, poles of Transfer function should be inside the unit circle in Z-plane. Figure 1 shows the pole zero plot of our 7th order low pass filter. Our system is causal and we can observe that all poles lie within the unit circle. Hence the absolute values of the poles are less than one and the system is stable.

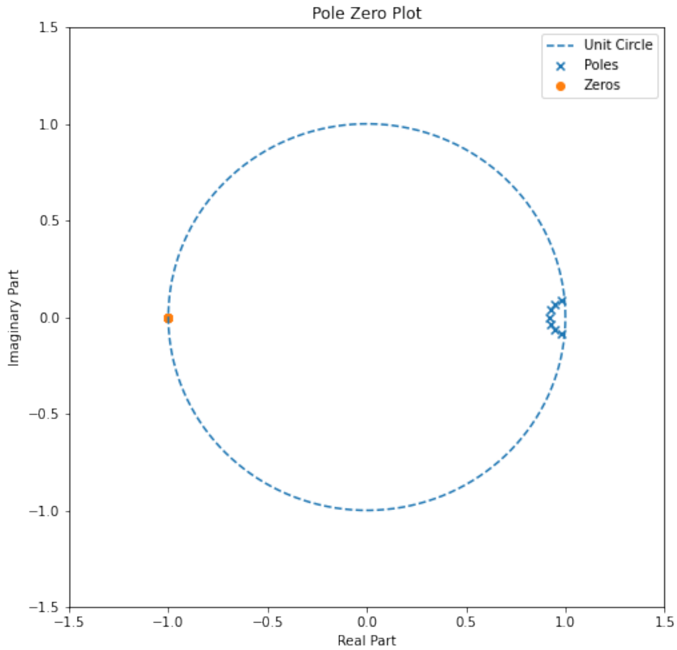

Figure 1: Pole Zero Plot of ButterWorth filter in Z-plane

(Part 3: Bode Plot)

(Solution)

Bode plots are a very useful way to represent the frequency response of a system as a function of frequency. The advantage of using Bode plots is that they provide a straightforward and common way of describing the frequency response of a linear time invariant system. Figure 2 shows the magnitude Bode plot of our 7th order low pass filter. The vertical axis is the magnitude or gain in dB and the horizontal axis is the frequency in hertz. From the Bode plot we observe and confirm that our filter meets the design requirements.

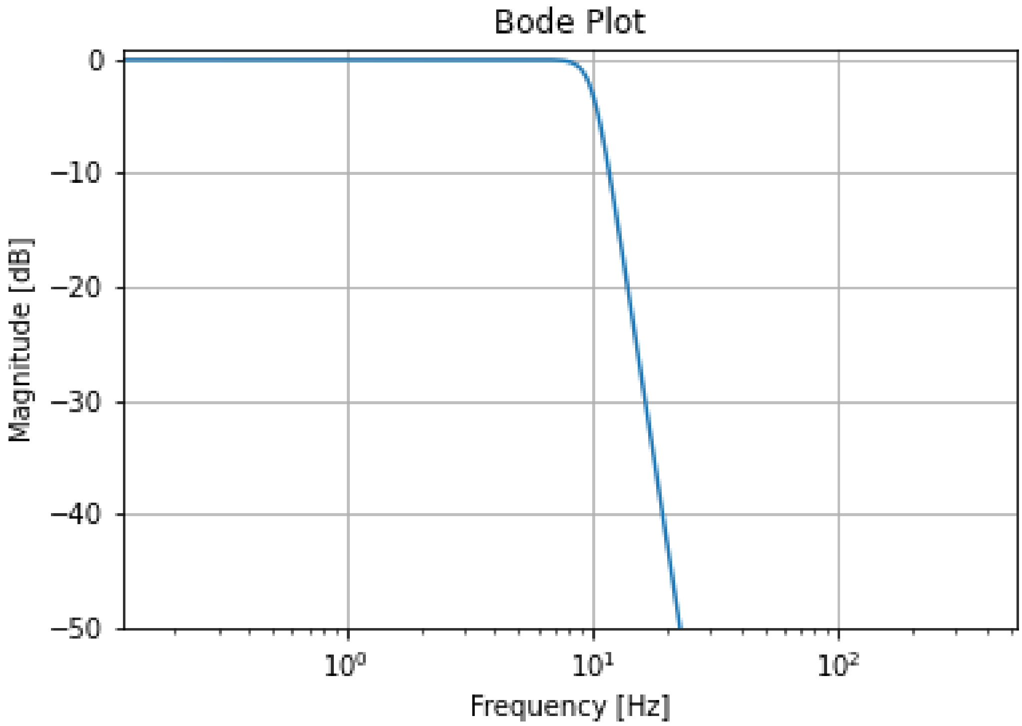

Figure 2: Bode plot of Butterworth Filter

(Part 4: Impulse and Step Response)

(Solution)

The impulse response, or impulse response function (IRF), of a dynamic system is its output when presented with a brief input signal, called an impulse. Figure 3 shows the impulse response of the Butterworth and Chebyshev Type1(Problem 4) filters for the given design requirements. We can compare the responses of the two filters based on peak overshoot. Peak overshoot M_p is defined as the deviation of the response, at the time required for the response to reach the peak value for the first time, from the final value of response. It is also called the maximum overshoot. From the figure we can observe that the peak overshoot of the Butterworth filter is more than that of Chebyshev type1. Another factor to compare the two filters is the settling time. It is the time required for the response to reach the steady state and stay within the specified tolerance bands around the final value. In general, the tolerance bands are 2% and 5%. The settling time is denoted by t_s. We can observe that the settling time of Butterworth filter is less than that of Chebyshev type1.

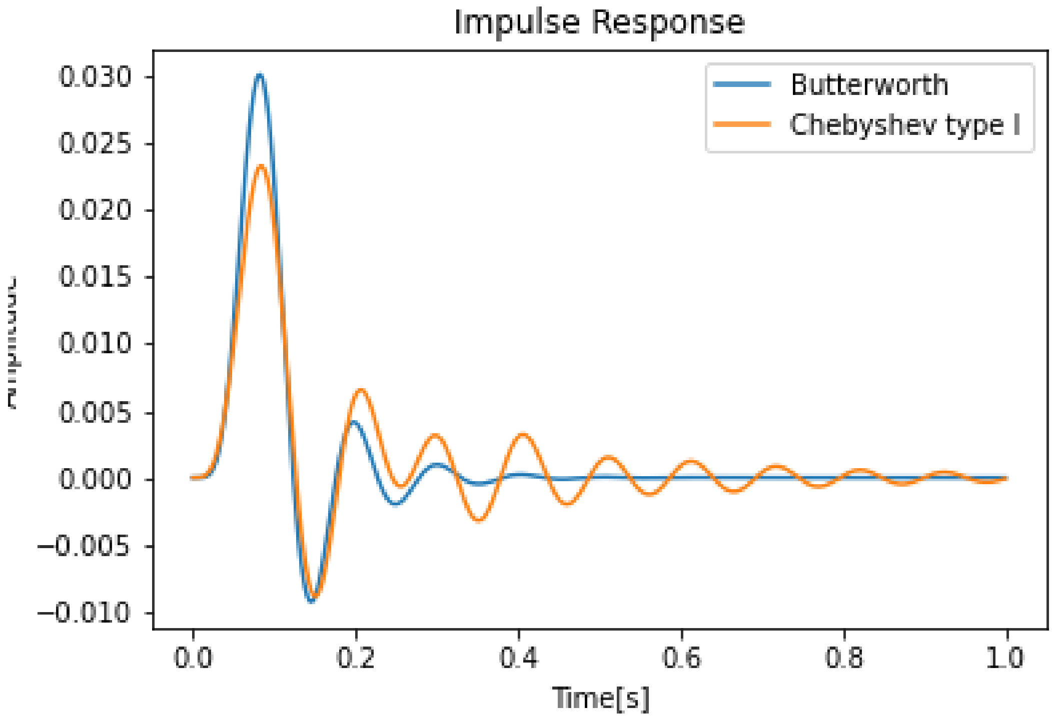

Figure 3: Impulse Response of the two filters

The response of a system (with all initial conditions equal to zero at $t = 0^-$, i.e., a zero state response) to the unit step input is called the unit step response. Figure 4 shows the unit step response of the Butterworth and Chebyshev Type1 filters for the given design requirements. Here again we can compare the responses based on peak overshoot and settling time. We can observe that the peak overshoot of Butterworth filter is more than that of Chebyshev Type1 and the settling time of Butterworth filter is less than that of Chebyshev type1 filter. Ideally we would require less peak overshoot and quicker settling time. Hence if peak overshoot is a major design consideration than Chebyshev type1 is a better choice among the two and if settling time is a major design consideration than Butterworth filter is a better choice among the two.

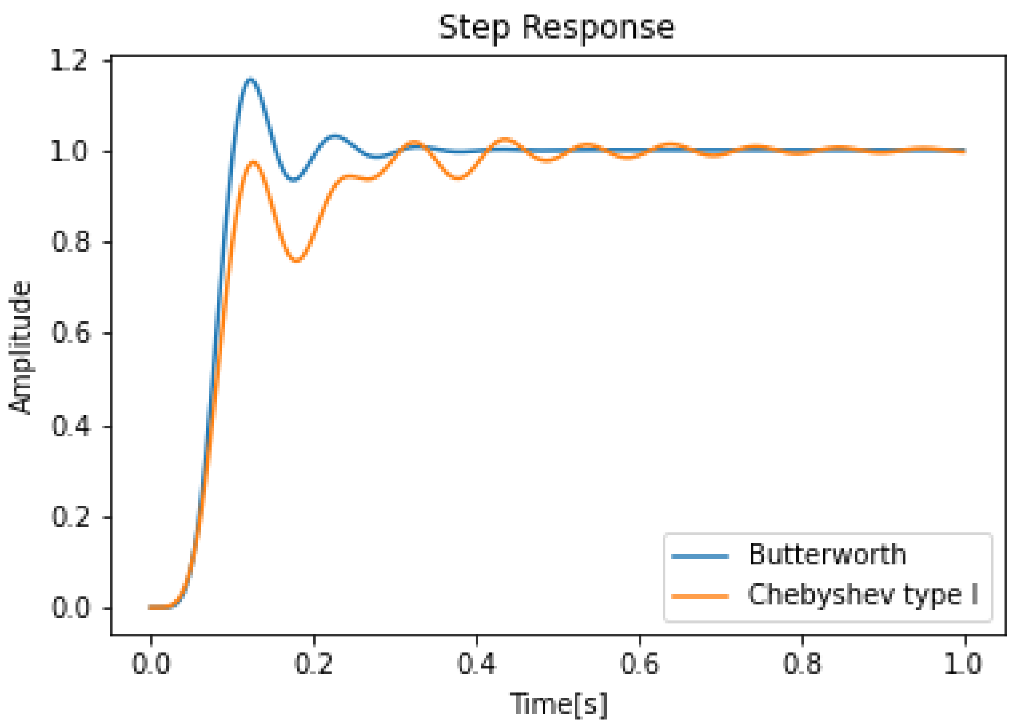

Figure 4: Impulse Response of the two filters

2. Filtering

(Filtering the ECG signal)

(Solution)

We filter the given ECG signal with the Butterworth filter designed in Problem 1. Figure 5 shows the magnitude spectrum of the original and filtered signal. We can observe that the frequency components over the cutoff frequency of 10 Hz have been attenuated.

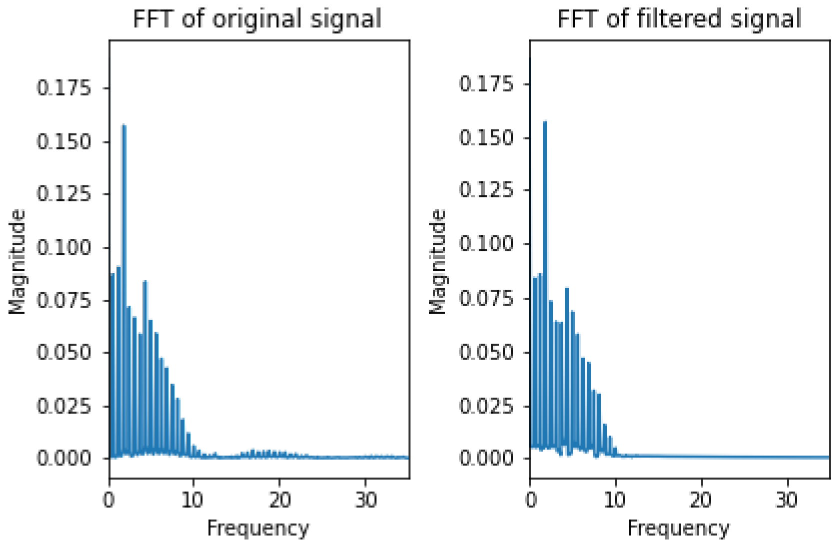

Figure 5: FFT of the original and filtered signals

The phase delay of the filter is the amount of time delay each frequency component of the signal suffers in going through the filter. The group delay is the average time delay the composite signal suffers at each frequency. Hence the group delay is the derivative of the phase with respect to angular frequency and is a measure of the distortion in the signal introduced by phase differences for different frequencies. Butterworth filters have a linear phase response, hence same slope in the pass-band. Figure 6 shows the original and filtered signals in the time domain. We can observe that the shape of the signal has not changed significantly after filtering due to the linear phase response of Butterworth filter. This makes it a good choice for analyzing signals like ECG where the shape of the signal is significant.

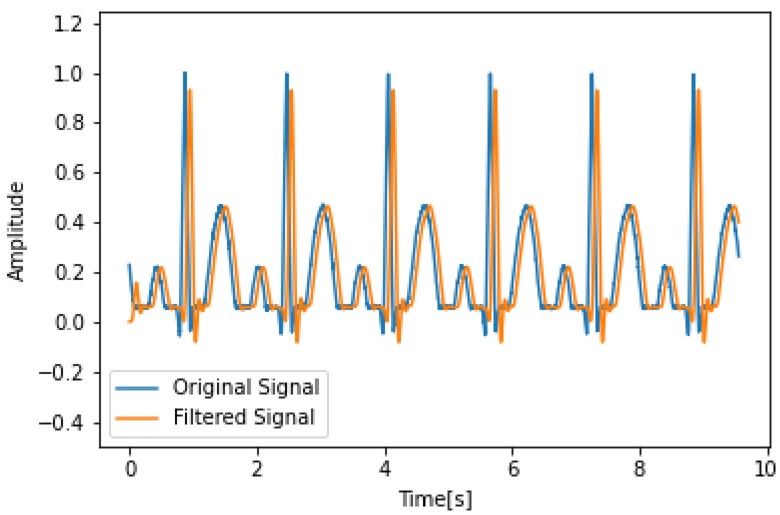

Figure 6: Original and filtered signals in time domain

3. Filtering — Time-Frequency Analysis
(Extracting the fundamental frequency)

(Solution)

Figure 7 shows the spectrogram of the instrument signal using Hanning window of length 512 samples and 25% overlap.

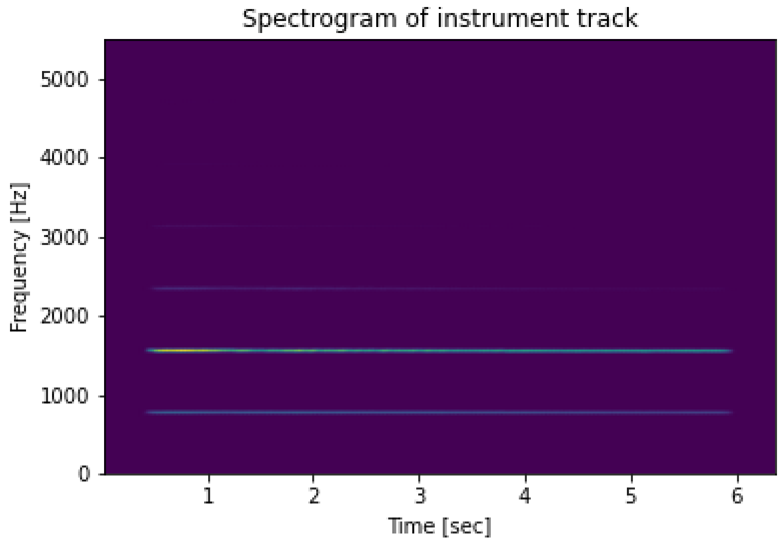

Figure 7: Spectrogram of instrument track

In order to extract the fundamental frequency, we design a band pass Butterworth filter with $F_l = 500$ Hz and $F_u = 900$ Hz. Figure 8 shows the Bode plot of the filter.

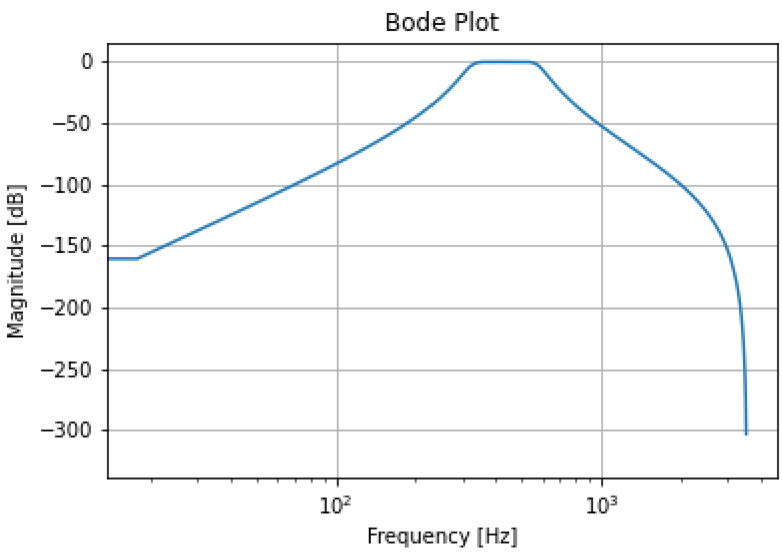

Figure 8: Bode plot of band pass filter

In music theory, pitch is the perceived fundamental frequency of a sound, however the actual fundamental frequency may differ from the perceived because of the overtones. It is these overtones together that give an instrument its timbre. When we listen to the audio after filtering, it sounds almost like computer generated audio. The timbre of the instrument is lost indicating the attenuation of overtones. Figure 9 shows the spectrogram of the filtered signal and it confirms that only the fundamental is present in the signal.

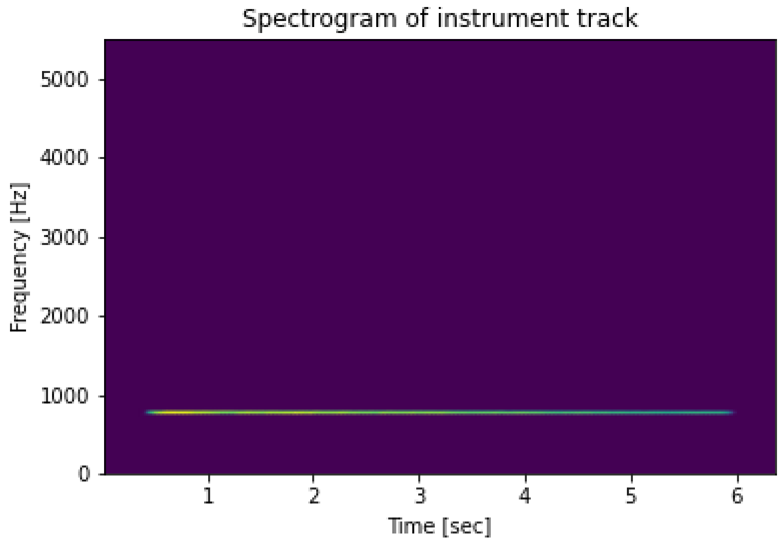

Figure 9: Spectrogram of filtered signal

4. Chebyshev filter design

(Designing low pass filter)

(Solution)

We begin the filter design process from the given specifications $\delta_p = \frac{1}{\sqrt{2}}$, $\delta_s = 0.01$, $\Omega_p^{'} = 20\pi$ rad/sec, $\Omega_s^{'} = 40\pi$ rad/sec, $F_s = 720$ samples/second. We then compute $\omega_p^{'}$ and $\omega_s^{'}$ using formula $\omega^{'} = \Omega' T$ where $T = \frac{1}{F_s}$ and $\epsilon = \sqrt{\frac{1-\delta_p^2}{\delta_p^2}}$. In order to overcome the problems of warping, we conduct pre-warping to compute Ω_p" and Ω_s" using formula $\Omega^{"} = \frac{2}{T}\tan(\frac{\omega^{'}}{2})$. The order of Chebyshev Type1 filter is given by

$$N = \frac{\cosh^{-1}(\frac{1}{\epsilon}\sqrt{\frac{1-\delta_s^2}{\delta_s^2}})}{\cosh^{-1}(\frac{\Omega_s}{\Omega_p})} \tag{2.6}$$

where $\frac{\Omega_s}{\Omega_p} = \frac{\Omega_s^{"}}{\Omega_p^{"}}$ for low pass filter. From equation 2.6 we get $N = 5$. Hence we can say that for a given filter requirement, the order of Chebyshev Type1 filter will be less than the Butterworth filter. This is because the Chebyshev filter has a steeper roll-off rate compared to Butterworth filter. We use the built in SciPy function to get a 5th order digital Chebyshev Type1 filter with a cutoff frequency of 10 Hz. Figure 10 shows the Bode plot of the filter which confirms that the design requirements are met.

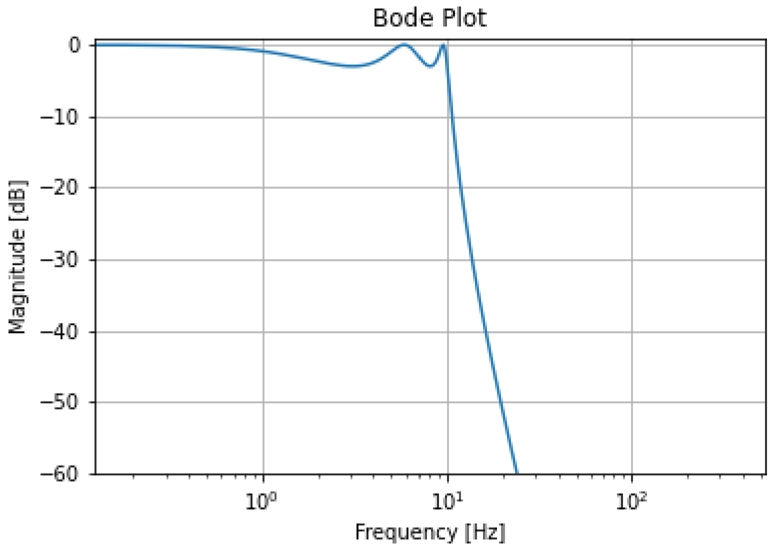

Figure 10: Bode plot of Chebyshev Type1 Filter