In[∘]:= $$\mathtt{powerLaw} = u^{+} == C\,(y^{+})^{m};$$

In[∘]:= $$\mathtt{bcs} = \left\{u^{+} \to \frac{U_{\infty}}{u_{\tau}},\ y^{+} \to \frac{u_{\tau}\,\delta[x]}{\nu}\right\};$$

In[∘]:= `Off[Solve::ifun]`

In[∘]:= $$\mathtt{Cf} = \mathtt{Solve}\left[\{\mathtt{powerLaw}\ /.\ \mathtt{bcs}\}\ /.\ \left\{u_{\tau} \to \left(C_f\,\frac{U_{\infty}^{2}}{2}\right)^{1/2}\right\},\ C_f\right][[1]][[1]][[2]]$$

Out[∘]=

$$\left(2^{-\frac{1}{2}-\frac{m}{2}}\,C\,\nu^{-m}\,U_{\infty}^{m}\,\delta[x]^{m}\right)^{-\frac{2}{1+m}}$$

In[∘]:= $$f = \left(\frac{y}{\delta[x]}\right)^{1/7};$$

In[∘]:= $$\mathtt{momInt} = \partial_x \int_0^{\delta[x]} f\,(1-f)\,dy == \frac{\mathtt{Cf}}{2}$$

Out[∘]=

$$\frac{7\,\delta'[x]}{72} == \frac{1}{2}\left(2^{-\frac{1}{2}-\frac{m}{2}}\,C\,\nu^{-m}\,U_{\infty}^{m}\,\delta[x]^{m}\right)^{-\frac{2}{1+m}}$$

In[∘]:= $$\mathtt{sol} = \mathtt{DSolve}[\mathtt{momInt},\ \delta,\ \{x,\ 0,\ \infty\}][[1]][[1]][[2]]$$

Out[∘]=

$$\mathtt{Function}\left[\{x\},\ e^{\frac{(1+m)\left(i\pi+\mathrm{Log}\left[-\frac{36x}{7}-c_1\right]-\mathrm{Log}\left[\frac{C^{\frac{2}{1+m}}\,(1+m)\,\nu^{-\frac{2m}{1+m}}\,U_{\infty}^{\frac{2m}{1+m}}}{2\,(1+3m)}\right]\right)}{1+3m}}\right]$$

In[∘]:= $$\delta\mathtt{Byx} = \mathtt{Simplify}\left[\left(\frac{\mathtt{sol}[x]}{x}\right)\ /.\ \left\{\nu \to U_{\infty}\,\frac{x}{Re_x},\ m \to 1/7\right\},\ \{x>0,\ C>0,\ U_{\infty}>0,\ c_1>0\}\right]$$

Out[∘]=

$$\frac{\left(-\frac{5}{14}\right)^{4/5}\,(-36\,x-7\,c_1)^{4/5}\,\left(\frac{1}{Re_x}\right)^{1/5}}{C^{7/5}\,x^{4/5}}$$