In[ ]:= `v = v0[y] (x / δ[y]) (1 - x / δ[y]) ^2`

Out[ ]=

$$\frac{x\,v0[y]\left(1-\frac{x}{\delta[y]}\right)^2}{\delta[y]}$$

In[ ]:= `dT = qw δ[y] / κ`

Out[ ]=

$$\frac{qw\,\delta[y]}{\kappa}$$

In[ ]:= `T = T0 + dT (1 - x / δ[y]) ^2`

Out[ ]=

$$T0+\frac{qw\left(1-\frac{x}{\delta[y]}\right)^2\delta[y]}{\kappa}$$

In[ ]:= `momInt` $= \partial_y \int_0^{\delta[y]} v^2\,dx == -\nu\,(\partial_x v \;/.\; x \to 0) + \int_0^{\delta[y]} g\,\beta\,(T - T0)\,dx$

Out[ ]=

$$\frac{2}{105}\,v0[y]\times\delta[y]\,v0'[y]+\frac{1}{105}\,v0[y]^2\,\delta'[y]==-\frac{\nu\,v0[y]}{\delta[y]}+\frac{g\,qw\,\beta\,\delta[y]^2}{3\,\kappa}$$

In[ ]:= `egyInt` $= \partial_y \int_0^{\delta[y]} v\,(T - T0)\,dx == -\alpha\,(\partial_x T \;/.\; x \to 0)$ `// Simplify[#, qw / κ > 0] &`

Out[ ]=

$$60\,\alpha==\delta[y]\,(\delta[y]\,v0'[y]+2\,v0[y]\,\delta'[y])$$

In[ ]:=
```
rules = {
   v0[y] → A y^m,
   v0'[y] → D[A y^m, y],
   δ[y] → B y^n,
   δ'[y] → D[B y^n, y]
  }
```

Out[ ]=

$$\{v0[y]\to A\,y^m,\ v0'[y]\to A\,m\,y^{-1+m},\ \delta[y]\to B\,y^n,\ \delta'[y]\to B\,n\,y^{-1+n}\}$$

In[ ]:= `eq1 = momInt /. rules`

Out[ ]=

$$\frac{2}{105}\,A^2\,B\,m\,y^{-1+2\,m+n}+\frac{1}{105}\,A^2\,B\,n\,y^{-1+2\,m+n}==\frac{B^2\,g\,qw\,y^{2\,n}\,\beta}{3\,\kappa}-\frac{A\,y^{m-n}\,\nu}{B}$$

In[ ]:= `eq2 = (egyInt /. rules)`

Out[ ]=

$$60\,\alpha==B\,y^n\left(A\,B\,m\,y^{-1+m+n}+2\,A\,B\,n\,y^{-1+m+n}\right)$$

Comparing the powers of y since above equation has to be true for all values of y

In[ ]:= `mnSol = Solve[{2 m + n - 1 == 2 n, 2 n == m - n, 0 == m + 2 n - 1}, {m, n}]`

Out[ ]=

$$\left\{\left\{m\to\frac{3}{5},\ n\to\frac{1}{5}\right\}\right\}$$

In[]:=
```
ABSol = Solve[{
    Simplify[eq1 /. mnSol, y > 0] // (#[[1]] &),
    Simplify[eq2 /. mnSol, {y > 0, Tw - T0 > 0}] // (#[[1]] &)},
    {A, B}]
```

Out[]=

$$\left\{\left\{A \to \frac{10\,(-6)^{1/5}\,g^{2/5}\,qw^{2/5}\,\beta^{2/5}\,(\alpha\,\kappa\,(4\,\alpha+5\,\nu))^{3/5}}{-4\,\alpha\,\kappa-5\,\kappa\,\nu},\ B \to \frac{(-6)^{2/5}\,\left(4\,\alpha^2\,\kappa+5\,\alpha\,\kappa\,\nu\right)^{1/5}}{g^{1/5}\,qw^{1/5}\,\beta^{1/5}}\right\},\right.$$

$$\left\{A \to \frac{10\times 6^{1/5}\,g^{2/5}\,qw^{2/5}\,\beta^{2/5}\,(\alpha\,\kappa\,(4\,\alpha+5\,\nu))^{3/5}}{4\,\alpha\,\kappa+5\,\kappa\,\nu},\ B \to \frac{6^{2/5}\,\left(4\,\alpha^2\,\kappa+5\,\alpha\,\kappa\,\nu\right)^{1/5}}{g^{1/5}\,qw^{1/5}\,\beta^{1/5}}\right\},$$

$$\left\{A \to \frac{10\,(-1)^{3/5}\,6^{1/5}\,g^{2/5}\,qw^{2/5}\,\beta^{2/5}\,(\alpha\,\kappa\,(4\,\alpha+5\,\nu))^{3/5}}{-4\,\alpha\,\kappa-5\,\kappa\,\nu},\right.$$
$$\left.B \to -\frac{(-1)^{1/5}\,6^{2/5}\,\left(4\,\alpha^2\,\kappa+5\,\alpha\,\kappa\,\nu\right)^{1/5}}{g^{1/5}\,qw^{1/5}\,\beta^{1/5}}\right\},$$

$$\left\{A \to \frac{10\,(-1)^{4/5}\,6^{1/5}\,g^{2/5}\,qw^{2/5}\,\beta^{2/5}\,(\alpha\,\kappa\,(4\,\alpha+5\,\nu))^{3/5}}{4\,\alpha\,\kappa+5\,\kappa\,\nu},\right.$$
$$\left.B \to -\frac{(-1)^{3/5}\,6^{2/5}\,\left(4\,\alpha^2\,\kappa+5\,\alpha\,\kappa\,\nu\right)^{1/5}}{g^{1/5}\,qw^{1/5}\,\beta^{1/5}}\right\},$$

$$\left\{A \to \frac{10\,(-1)^{2/5}\,6^{1/5}\,g^{2/5}\,qw^{2/5}\,\beta^{2/5}\,(\alpha\,\kappa\,(4\,\alpha+5\,\nu))^{3/5}}{4\,\alpha\,\kappa+5\,\kappa\,\nu},\right.$$
$$\left.\left.B \to \frac{(-1)^{4/5}\,6^{2/5}\,\left(4\,\alpha^2\,\kappa+5\,\alpha\,\kappa\,\nu\right)^{1/5}}{g^{1/5}\,qw^{1/5}\,\beta^{1/5}}\right\}\right\}$$

In[]:=
```
ASol = Simplify[ABSol[[2]][[1]][[2]]]
```

Out[]=

$$\frac{10\times 6^{1/5}\,g^{2/5}\,qw^{2/5}\,\beta^{2/5}\,(\alpha\,\kappa\,(4\,\alpha+5\,\nu))^{3/5}}{4\,\alpha\,\kappa+5\,\kappa\,\nu}$$

In[]:=
```
BSol = Simplify[ABSol[[2]][[2]][[2]]]
```

Out[]=

$$\frac{6^{2/5}\,(\alpha\,\kappa\,(4\,\alpha+5\,\nu))^{1/5}}{g^{1/5}\,qw^{1/5}\,\beta^{1/5}}$$

In[]:=
```
Tw = T /. {x → 0, δ[y] → BSol y^(1/5)} // # /. g → Ra ν α κ / (β qw y^4) & //
    # /. ν → Pr α & // Simplify[#, {qw > 0, κ > 0, α > 0, y > 0, β > 0}] &
```

Out[]=

$$T0+\frac{6^{2/5}\,(4+5\,Pr)^{1/5}\,qw\,y}{(Pr\,Ra)^{1/5}\,\kappa}$$

In[]:=
```
Tw - T0 // N[#] &
```

Out[]=

$$\frac{2.04767\,(4.+5.\,Pr)^{1/5}\,qw\,y}{(Pr\,Ra)^{1/5}\,\kappa}$$