In[•]:=
```
v = v0[y] (x / δ[y]) (1 - x / δ[y]) ^ 2
```
Out[•]=

$$\frac{x\, v0[y] \left(1 - \frac{x}{\delta[y]}\right)^2}{\delta[y]}$$

In[•]:=
```
T0 = T00 + γ y
```
Out[•]=

$$T00 + y\,\gamma$$

In[•]:=
```
T = T0 + (Tw - T0) (1 - x / δ[y]) ^ 2
```
Out[•]=

$$T00 + y\,\gamma + (-T00 + Tw - y\,\gamma)\left(1 - \frac{x}{\delta[y]}\right)^2$$

In[•]:=
```
momInt = ∂y ∫0^δ[y] v^2 dx == -ν (∂x v /. x → 0) + ∫0^δ[y] g β (T - T0) dx
```
Out[•]=

$$\frac{2}{105}\, v0[y] \times \delta[y]\, v0'[y] + \frac{1}{105}\, v0[y]^2\, \delta'[y] == -\frac{\nu\, v0[y]}{\delta[y]} - \frac{1}{3}\, g\,\beta\, (T00 - Tw + y\,\gamma)\, \delta[y]$$

In[•]:=
```
egyInt = ∂y ∫0^δ[y] v (T - T0) dx == -α (∂x T /. x → 0)
```
Out[•]=

$$-\frac{1}{30}\,\gamma\, v0[y] \times \delta[y] - \frac{1}{30}\,(T00 - Tw + y\,\gamma)\,\delta[y]\, v0'[y] - \frac{1}{30}\,(T00 - Tw + y\,\gamma)\, v0[y]\, \delta'[y] == \frac{2\,\alpha\,(-T00 + Tw - y\,\gamma)}{\delta[y]}$$

In[•]:=
```
nonDimRules = {
    y → yt H,
    δ[y] → δt[yt] H RaH^(-1 / 4),
    δ'[y] → δt'[yt] RaH^(-1 / 4),
    v0[y] → (α / H) RaH^(1 / 2) v0t[yt],
    v0'[y] → (α / H^2) RaH^(1 / 2) v0t'[yt],
    γ → b (Tw - T00) / H,
    g → RaH ν α / (β (Tw - T00) H^3)};
simplifyRules = {
    Tw - T00 > 0,
    α > 0, H > 0, RaH > 0, Pr > 0};
```

In[•]:=
```
momNonDim = momInt /. nonDimRules // # /. ν → α Pr & // Simplify[#, simplifyRules] &
```
Out[•]=

$$35\,Pr\,(-1 + b\,yt)\,\delta t[yt] + v0t[yt]\left(\frac{105\,Pr}{\delta t[yt]} + 2\,\delta t[yt]\, v0t'[yt]\right) + v0t[yt]^2\,\delta t'[yt] == 0$$

In[•]:=
```
momPrInf = momNonDim // #[[1]] / Pr & // Limit[#, Pr → ∞] & // # /. b → 1 & // # == 0 &
```
Out[•]=

$$\frac{105\, v0t[yt]}{\delta t[yt]} + 35\,(-1 + yt)\,\delta t[yt] == 0$$

In[•]:=
```
v0tSol = Solve[momPrInf, v0t[yt]][[1]][[1]][[2]]
```
Out[•]=
$$-\frac{1}{3}\ (-1+\text{yt})\ \delta\text{t}[\text{yt}]^2$$

In[•]:=
```
egyNonDim = egyInt /. nonDimRules // # /. ν → α Pr & // Simplify[#, simplifyRules] &
```
Out[•]=
$$\frac{(-1+\text{b yt})\ \left(-60+\delta\text{t}[\text{yt}]^2\ \text{v0t}'[\text{yt}]\right)}{\delta\text{t}[\text{yt}]}+\text{v0t}[\text{yt}]\ (\text{b}\ \delta\text{t}[\text{yt}]+(-1+\text{b yt})\ \delta\text{t}'[\text{yt}])==0$$

In[•]:=
```
problemDetails = {b → 1, v0t[yt] → v0tSol, v0t'[yt] → D[v0tSol, yt]}
```
Out[•]=
$$\left\{\text{b}\to 1,\ \text{v0t}[\text{yt}]\to -\frac{1}{3}\ (-1+\text{yt})\ \delta\text{t}[\text{yt}]^2,\right.$$
$$\left.\text{v0t}'[\text{yt}]\to -\frac{1}{3}\ \delta\text{t}[\text{yt}]^2-\frac{2}{3}\ (-1+\text{yt})\ \delta\text{t}[\text{yt}]\ \delta\text{t}'[\text{yt}]\right\}$$

In[•]:=
```
egyPrInf = egyNonDim // # /. problemDetails & // Simplify[#, {δt[yt] > 0}] &
```
Out[•]=
$$(-1+\text{yt})\ \left(180+2\ \delta\text{t}[\text{yt}]^4+3\ (-1+\text{yt})\ \delta\text{t}[\text{yt}]^3\ \delta\text{t}'[\text{yt}]\right)==0$$

In[•]:=
```
transformationRules = {δt[yt] → Δ[yt]^(1/4), δt'[yt] → Δ'[yt] × Δ[yt]^(-3/4)/4}
```
Out[•]=
$$\left\{\delta\text{t}[\text{yt}]\to\Delta[\text{yt}]^{1/4},\ \delta\text{t}'[\text{yt}]\to\frac{\Delta'[\text{yt}]}{4\ \Delta[\text{yt}]^{3/4}}\right\}$$

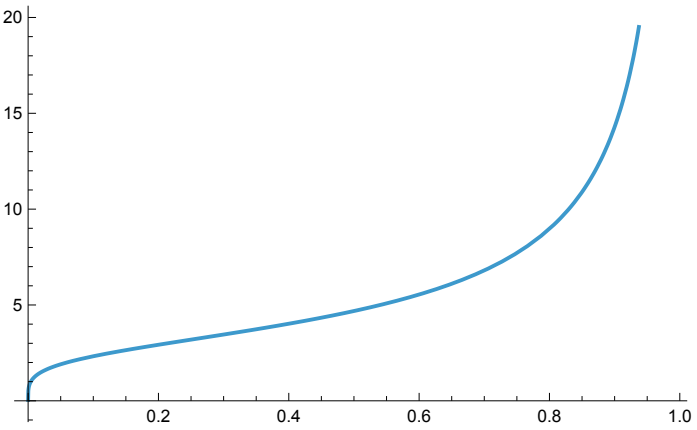
In[•]:=
```
ΔP = egyPrInf /. transformationRules // FullSimplify[#, {Δ[yt] > 0, yt < 1}] &
```
Out[•]=
$$720+8\ \Delta[\text{yt}]+3\ (-1+\text{yt})\ \Delta'[\text{yt}]==0$$

In[•]:=
```
Solve[ΔP, Δ'[yt]]
```
Out[•]=
$$\left\{\left\{\Delta'[\text{yt}]\to -\frac{8\ (90+\Delta[\text{yt}])}{3\ (-1+\text{yt})}\right\}\right\}$$

In[•]:=
```
sol = NDSolve[{ΔP, Δ[0] == 0}, Δ[yt], {yt, 0, 0.99}]
```
Out[•]=
$$\left\{\left\{\Delta[\text{yt}]\to \text{InterpolatingFunction}\left[\ \text{Domain: } \{\{0.,\ 0.99\}\}\ \text{Output: scalar}\ \right][\text{yt}]\right\}\right\}$$

In[•]:=
```
Plot[Evaluate[Δ[yt]^(1/4) /. sol], {yt, 0, 0.99}]
```

Out[•]=



In[•]:=
```
hy = ((-κ D[T, x] /. x → 0) / (Tw - T00)) // # /. nonDimRules & //
    # /. transformationRules & // Simplify[#, Tw > T00] & // # /. problemDetails &
```

Out[•]=

$$-\frac{2\,\mathrm{RaH}^{1/4}\,(-1+\mathrm{yt})\,\kappa}{\mathrm{H}\,\Delta[\mathrm{yt}]^{1/4}}$$

In[•]:=
```
hyNonDim = hy H / (κ RaH^(1/4))
```

Out[•]=

$$-\frac{2\,(-1+\mathrm{yt})}{\Delta[\mathrm{yt}]^{1/4}}$$

In[•]:=
```
hMeanNonDim = NIntegrate[hyNonDim /. sol, {yt, 0, 1}]
```

Out[•]=

```
{0.324668}
```

In[•]:=
```
NuMean = hMeanNonDim[[1]] RaH^(1/4)
```

Out[•]=

$$0.324668\,\mathrm{RaH}^{1/4}$$

In[•]:=
```
NuLiterature = 0.337 RaH^(1/4)
```

Out[•]=

$$0.337\,\mathrm{RaH}^{1/4}$$

In[•]:=
```
deviation = (1 - NuMean / NuLiterature)
```

Out[•]=

```
0.0365936
```