In[∘]:=
```
v = v0[y] (x / δ[y]) (1 - x / δ[y]) ^2
```
Out[∘]=

$$\frac{x\, v0[y] \left(1 - \frac{x}{\delta[y]}\right)^2}{\delta[y]}$$

In[∘]:=
```
T = T0 + (Tw - T0) (1 - x / δ[y]) ^2
```
Out[∘]=

$$T0 + (-T0 + Tw) \left(1 - \frac{x}{\delta[y]}\right)^2$$

In[∘]:=
```
lims = {x, 0, δ[y]};
momInt = ∂y ∫_0^δ[y] v^2 dx == -ν (∂x v /. x → 0) + ∫_0^δ[y] g β (T - T0) dx
```
Out[∘]=

$$\frac{2}{105}\, v0[y] \times \delta[y]\, v0'[y] + \frac{1}{105}\, v0[y]^2\, \delta'[y] == -\frac{\nu\, v0[y]}{\delta[y]} - \frac{1}{3}\, g\, T0\, \beta\, \delta[y] + \frac{1}{3}\, g\, Tw\, \beta\, \delta[y]$$

In[∘]:=
```
egyInt = ∂y ∫_0^δ[y] v (T - T0) dx == -α (∂x T /. x → 0) // Simplify[#, Tw - T0 > 0] &
```
Out[∘]=

$$\frac{60\, \alpha}{\delta[y]} == \delta[y]\, v0'[y] + v0[y]\, \delta'[y]$$

In[∘]:=
```
rules = {
   v0[y] → A y^m,
   v0'[y] → D[A y^m, y],
   δ[y] → B y^n,
   δ'[y] → D[B y^n, y]
  }
```
Out[∘]=

$$\left\{v0[y] \to A\, y^m,\ v0'[y] \to A\, m\, y^{-1+m},\ \delta[y] \to B\, y^n,\ \delta'[y] \to B\, n\, y^{-1+n}\right\}$$

In[∘]:=
```
eq1 = momInt /. rules
```
Out[∘]=

$$\frac{2}{105}\, A^2\, B\, m\, y^{-1+2m+n} + \frac{1}{105}\, A^2\, B\, n\, y^{-1+2m+n} == -\frac{1}{3}\, B\, g\, T0\, y^n\, \beta + \frac{1}{3}\, B\, g\, Tw\, y^n\, \beta - \frac{A\, y^{m-n}\, \nu}{B}$$

In[∘]:=
```
eq2 = (egyInt /. rules)
```
Out[∘]=

$$\frac{60\, y^{-n}\, \alpha}{B} == A\, B\, m\, y^{-1+m+n} + A\, B\, n\, y^{-1+m+n}$$

Comparing the powers of y since above equation has to be true for all values of y

In[∘]:=
```
mnSol = Solve[{2 m + n - 1 == n, n == m - n, m + n - 1 == -n}, {m, n}]
```
Out[∘]=

$$\left\{\left\{m \to \frac{1}{2},\ n \to \frac{1}{4}\right\}\right\}$$

In[ ]:=
```
ABSol = Solve[{
    Simplify[eq1 /. mnSol, y > 0] // (#[[1]] &),
    Simplify[eq2 /. mnSol, {y > 0, Tw - T0 > 0}] // (#[[1]] &)},
    {A, B}]
```

Out[ ]=

$$\left\{\left\{A \to \frac{4\left(-\frac{\sqrt{35}\, g\, T0\, \beta \sqrt{-\alpha\,(20\,\alpha+21\,\nu)}}{\sqrt{g\,(T0-Tw)\,\beta}}+\frac{\sqrt{35}\, g\, Tw\, \beta \sqrt{-\alpha\,(20\,\alpha+21\,\nu)}}{\sqrt{g\,(T0-Tw)\,\beta}}\right)}{20\,\alpha+21\,\nu},\right.\right.$$
$$\left.B \to -\frac{2\left(\frac{5}{7}\right)^{1/4}\left(-20\,\alpha^2-21\,\alpha\,\nu\right)^{1/4}}{(g\, T0\, \beta - g\, Tw\, \beta)^{1/4}}\right\},$$
$$\left\{A \to \frac{4\left(\frac{\sqrt{35}\, g\, T0\, \beta \sqrt{-\alpha\,(20\,\alpha+21\,\nu)}}{\sqrt{g\,(T0-Tw)\,\beta}}-\frac{\sqrt{35}\, g\, Tw\, \beta \sqrt{-\alpha\,(20\,\alpha+21\,\nu)}}{\sqrt{g\,(T0-Tw)\,\beta}}\right)}{20\,\alpha+21\,\nu},\right.$$
$$\left.B \to -\frac{2\,i\left(\frac{5}{7}\right)^{1/4}\left(-20\,\alpha^2-21\,\alpha\,\nu\right)^{1/4}}{(g\, T0\, \beta - g\, Tw\, \beta)^{1/4}}\right\},$$
$$\left\{A \to \frac{4\left(\frac{\sqrt{35}\, g\, T0\, \beta \sqrt{-\alpha\,(20\,\alpha+21\,\nu)}}{\sqrt{g\,(T0-Tw)\,\beta}}-\frac{\sqrt{35}\, g\, Tw\, \beta \sqrt{-\alpha\,(20\,\alpha+21\,\nu)}}{\sqrt{g\,(T0-Tw)\,\beta}}\right)}{20\,\alpha+21\,\nu},\right.$$
$$\left.B \to \frac{2\,i\left(\frac{5}{7}\right)^{1/4}\left(-20\,\alpha^2-21\,\alpha\,\nu\right)^{1/4}}{(g\, T0\, \beta - g\, Tw\, \beta)^{1/4}}\right\},$$
$$\left.\left\{A \to \frac{4\left(-\frac{\sqrt{35}\, g\, T0\, \beta \sqrt{-\alpha\,(20\,\alpha+21\,\nu)}}{\sqrt{g\,(T0-Tw)\,\beta}}+\frac{\sqrt{35}\, g\, Tw\, \beta \sqrt{-\alpha\,(20\,\alpha+21\,\nu)}}{\sqrt{g\,(T0-Tw)\,\beta}}\right)}{20\,\alpha+21\,\nu},\ B \to \frac{2\left(\frac{5}{7}\right)^{1/4}\left(-20\,\alpha^2-21\,\alpha\,\nu\right)^{1/4}}{(g\, T0\, \beta - g\, Tw\, \beta)^{1/4}}\right\}\right\}$$

In[ ]:=
```
ASol = Simplify[ABSol[[4]][[1]][[2]]]
```

Out[ ]=

$$\frac{4\sqrt{35}\,\alpha\sqrt{g\,(T0-Tw)\,\beta}}{\sqrt{-\alpha\,(20\,\alpha+21\,\nu)}}$$

In[ ]:=
```
BSol = Simplify[ABSol[[4]][[2]][[2]]]
```

Out[ ]=

$$\frac{2\left(\frac{5}{7}\right)^{1/4}(-\alpha\,(20\,\alpha+21\,\nu))^{1/4}}{(g\,(T0-Tw)\,\beta)^{1/4}}$$

In[ ]:=
```
h = -κ (D[T, x] /. x → 0) / (Tw - T0) // (# /. δ[y] → BSol y^(1/4) &)
```

Out[ ]=

$$\frac{\left(\frac{7}{5}\right)^{1/4}(g\,(T0-Tw)\,\beta)^{1/4}\,\kappa}{y^{1/4}\,(-\alpha\,(20\,\alpha+21\,\nu))^{1/4}}$$

In[ ]:=
```
Nu = h y / κ
```

Out[ ]=

$$\frac{\left(\frac{7}{5}\right)^{1/4}y^{3/4}\,(g\,(T0-Tw)\,\beta)^{1/4}}{(-\alpha\,(20\,\alpha+21\,\nu))^{1/4}}$$

In[ ]:=
```
NuMean = (4 / 3) Nu /. g → Ra ν α / (β (Tw - T0) y^3) // # /. ν → Pr α & //
    Simplify[#, {α > 0, Tw - T0 > 0, y > 0, Pr > 0, Ra > 0}] & // N[#] &
```

Out[ ]=

$$1.45034 \left(\frac{\text{Pr Ra}}{20. + 21.\,\text{Pr}}\right)^{1/4}$$

In[ ]:=
```
NuMean /. Pr → 0.71
```

Out[ ]=

$$0.547706\,\text{Ra}^{1/4}$$