A long, rectangular metallic blade has width L= 4 cm and temperature T_w = 40^{0}C. It is surrounded on both sides by atmospheric air at T_∞= 20^{0}C. The long side of the blade is always horizontal. Calculate the total heat transfer rate per unit of blade length when the short side of its rectangular shape (L) is (a) vertical, (b) inclined at 45^0 relative to the vertical, and (c) horizontal. Comment on the effect that blade orientation has on the total heat transfer rate.

In[•]:=
$$Nu = \left(0.825 + \frac{0.387\, Ra_L^{1/6}}{\left(1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right)^{8/27}}\right)^2;$$

In[•]:=
$$Ra_L = \frac{g\, Cos[\theta]\, \beta\, (T_w - T_\infty)\, L^3}{\nu\, \alpha};$$

In[•]:=
```
SetDirectory[NotebookDirectory[]];
airProps = Import["../air_props.csv"];
airProps[[2 ;;, 5]] = airProps[[2 ;;, 5]] 10^-3;
airProps[[2 ;;, 7]] = airProps[[2 ;;, 7]] 10^-6;
airProps[[2 ;;, 8]] = airProps[[2 ;;, 8]] 10^-6;
νI = Interpolation[airProps[[2 ;;, {1, 7}]]];
βI = Interpolation[airProps[[2 ;;, {1, 5}]] ];
PrI = Interpolation[airProps[[2 ;;, {1, 9}]]];
αI = Interpolation[airProps[[2 ;;, {1, 8}]]];
κI = Interpolation[airProps[[2 ;;, {1, 4}]]];
```

In[•]:=
$$TFilm = \frac{T_w + T_\infty}{2};$$

```
propertyVals = {ν → νI[TFilm], α → αI[TFilm],
   β → βI[TFilm], Pr → PrI[TFilm], κ → κI[TFilm], g → 9.81};
```

$$h = Nu\,\frac{\kappa}{L};\quad q = h\, L\, (T_w - T_\infty);$$

In[•]:=
```
partA = {Tw → 40, T∞ → 20, L → 0.04, θ → 0 Degree};
```

In[•]:=
```
h /. propertyVals // # /. partA &
```

Out[•]=
```
6.28886
```

In[•]:=
```
qPartA = 2 q /. propertyVals // # /. partA &
```

Out[•]=
```
10.0622
```

In[•]:=
```
partB = {Tw → 40, T∞ → 20, L → 0.04, θ → 45 Degree};
```

In[•]:=
```
RaL /. propertyVals // # /. partB &
```

Out[•]=
```
81015.9
```

In[•]:=
```
qPartB = 2 q /. propertyVals // # /. partB &
```

Out[•]=
```
9.2516
```

In[•]:=
$$Nu_{top} = 0.59\, Ra_L^{1/4};\quad Nu_{bottom} = 0.27\, Ra_L^{1/4};$$

In[ ]:=
$$h_{top} = Nu_{top} \frac{\kappa}{L_c}; \quad h_{bottom} = Nu_{bottom} \frac{\kappa}{L_c};$$
$$L_c = Limit\left[\frac{L\,W}{2\,(L+W)}, W \to \infty\right];$$
$$q_{top} = h_{top}\,L\,(T_w - T_\infty); \quad q_{bottom} = h_{bottom}\,L\,(T_w - T_\infty);$$
$$q_{total} = q_{top} + q_{bottom};$$

In[ ]:=
$$partC = \{T_w \to 40,\ T_\infty \to 20,\ L \to 0.04,\ \theta \to 0\ Degree\};$$

In[ ]:=
$$qPartC = q_{total} /. propertyVals\ //\ \# /. partC\ \&$$

Out[ ]=
16.7123

In[ ]:=
```
Grid[{{"Vertical", "45^0 inclined", "Horizontal"},
  {qPartA, qPartB, qPartC}}, Frame → All]
```

Out[ ]=

Vertical	45^0 inclined	Horizontal
10.0622	9.2516	16.7123