Fundamentals of Convective Heat Transfer

Week-07 (Live Session)

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Hydrodynamically developed and thermally developing flow through circular pipe with uniform wall heat flux

Problem Description

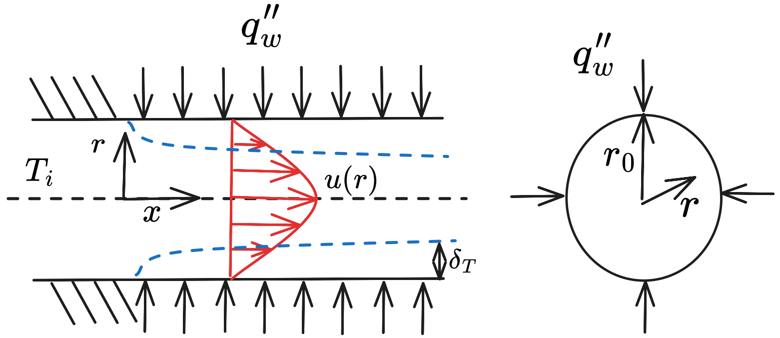


Figure 1: Illustration of the problem

- Graetz number: $Gz = \frac{Re_D Pr}{x/D} = \frac{D^2/\alpha}{x/u_m}$ where D is the inner diameter of the pipe
- Thermal boundary layer development has to be taken into account for larger values of Gz
- Temperature profile is given by

$$\frac{T(x,r) - T_i}{\frac{q''_w r_0}{k}} = \frac{4x/r_0}{Re_D Pr} + \frac{r^2}{r_0^2} - \frac{1r^4}{4r_0^4} - \frac{7}{24} + \sum_{n=1}^{\infty} C_n R_n e^{-\frac{\lambda_n^2 x/r_0}{Re_D Pr}}$$

Hydrodynamically developed and thermally developing flow through circular pipe with uniform wall heat flux

Nusselt number

$$Nu = \frac{2}{\frac{11}{24} + \sum_{n=1}^{\infty} C_n R_n(1) e^{-\frac{\lambda_n^2 x/r_0}{Re_D Pr}}}$$

Table 1: Eigen values and constants for uniform wall heat flux case

n	λ_n^2	$R_n(1)$	C_n
1	25.6796	-0.492597	0.403483
2	83.8618	0.395508	-0.175111
3	174.167	-0.345872	0.105594
4	296.536	0.314047	-0.732804
5	450.947	-0.291252	0.0550357
6	637.387	0.273808	-0.043483
7	855.850	-0.259852	0.035597

Problem 1

Consider water at 20^0C flowing through a pipe of diameter 25mm and length 2m with a mass flow rate of 10g/s. If the pipe is heated electrically resulting in a uniform wall heat flux of 0.1W/cm^2. (Take kinematic viscosity of water as 10^{-6} m^2/s, thermal conductivity as 0.6 W/mK, Prandlt number as 7).

1. Determine the overall heat transferred to water
2. Determine the mean outlet temperature of water
3. Plot the wall temperature as a function of length of the pipe

Hydrodynamically developed and thermally developing flow through circular pipe with uniform wall temperature

Problem Description

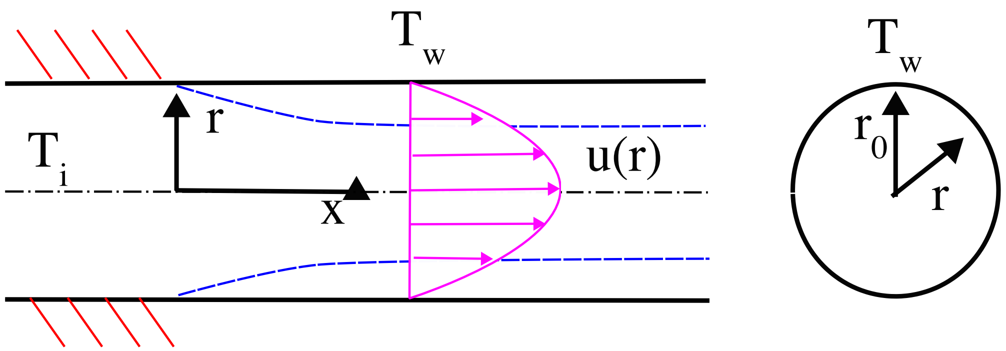


Figure 3: Illustration of the problem

$$\frac{T(x,r) - T_w}{T_i - T_w} = \sum_{n=0}^{\infty} C_n R_n(\eta) e^{-\frac{\lambda_n^2 x / r_0}{Re_D Pr}}, \quad C_n = -\frac{2}{\lambda_n \left(\frac{\partial R_n}{\partial \lambda_n} \right)_{\eta = 1}}$$

Hydrodynamically developed and thermally developing flow through circular pipe with uniform wall temperature

Table 2: Eigenvalues and constants for uniform wall temperature case

n	λ_n	C_n	G_n
0	2.7043644	+1.46622	0.74879
1	6.679032	-0.802476	0.54383
2	10.67338	+0.587094	0.46288
3	14.67108	-0.474897	0.41518
4	18.66987	+0.404402	0.38237

Table 3: Local and average Nusselt numbers

ξ	Nu	$\bar{Nu}$
0	∞	∞
0.001	12.8	19.29
0.01	6	8.92
0.1	3.71	4.64
0.2	3.66	4.15
∞	3.66	3.66

$$\bar{Nu}_D(x) = -\frac{1}{\frac{2x/r_0}{Re_D Pr}} ln(\theta_m), \quad \theta_m = \frac{T_m(x) - T_w}{T_i - T_w} = 8\sum_{n=0}^{\infty} G_n \frac{e^{-\frac{\lambda_n^2 x/r_0}{Re_D Pr}}}{\lambda_n^2}$$

Problem 2

As an application of the results obtained in this chapter for laminar flows in circular ducts under constant wall temperature boundary condition, consider an oil heat exchanger, in which the length of the tubes is 200 diameters long. The Prandtl number of the oil is given as 100, and the Reynolds number of the flow in the tubes is 1000:

a. Is it possible that the flow in the tubes in this heat exchanger can be assumed to be fully developed?

b. Find the value of the local Nusselt number at the end of the tubes by assuming hydrodynamically fully developed and thermally developing flow in the tubes.

c. Assuming that the average Nusselt number is approximately twice the local value at the end of the tubes, estimate the error that could have been introduced if fully developed conditions were used.

Heat Transfer in Plane Couette Flow

Isothermal top and bottom walls

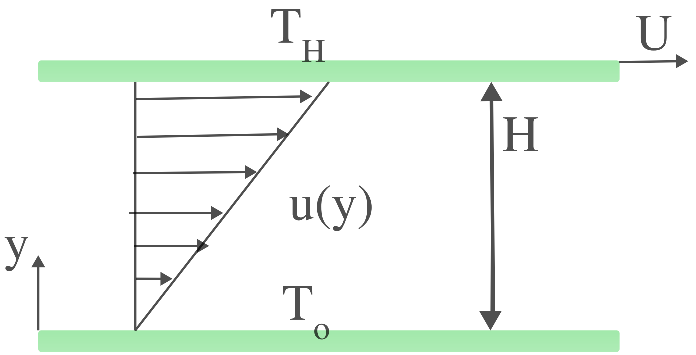


Figure 4: Illustration of the problem

The problem is characterised by Pr, Ec, Br numbers defined as

$$Pr = \frac{\nu}{\alpha}, \quad Ec = \frac{U^2}{c_p\left(T_H - T_o\right)}, \quad Br = PrEc$$

$$Nu_o = -2\left(1 + \frac{EcPr}{2}\right), \quad Nu_H = 2\left(1 - \frac{EcPr}{2}\right)$$

$$\frac{T(y) - T_o}{T_H - T_o} = \frac{y}{H} + \frac{EcPr}{2}\left(\frac{y}{H} - \frac{y^2}{H^2}\right)$$

Heat transfer in plane Couette flow

Isothermal top and adiabatic bottom walls

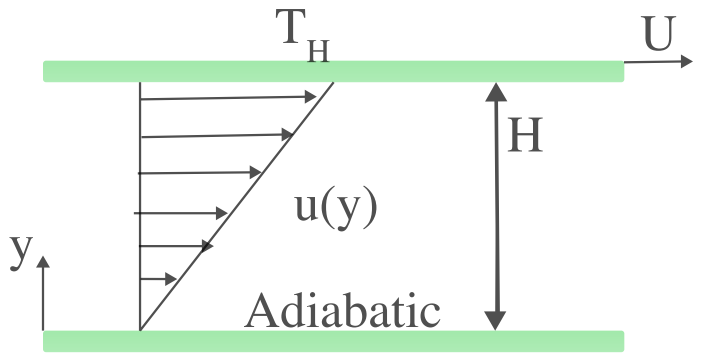


Figure 5: Illustration of problem

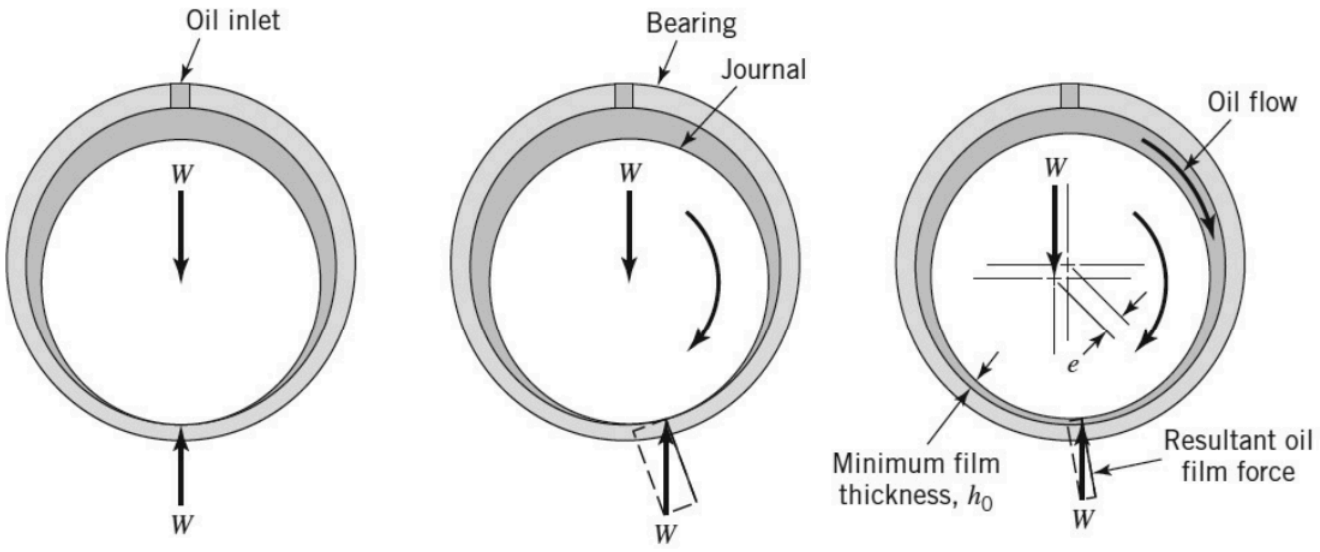
Nusselt number, $Nu = -4$ for the top wall

No heat transfer from the bottom wall, temperature of which is given by

$$T_o = T_H + \frac{\mu U^2}{2\kappa}$$

Problem 3

A 100-mm diameter journal rotates at 3000 rpm in a bearing with a 0.5-mm oil film of engine oil (μ=0.1 Pa·s, k=0.13 W/m·K, $\rho = 870$ kg/m^3). The shaft is maintained at 80°C, and the bearing is well insulated. Calculate the maximum temperature in the oil film due to viscous heating.



(a)
At rest

(b)
Slow rotation
(boundary lubrication)

(c)
Fast rotation
(hydrodynamic lubrication)

Problem 4

Consider the flow of a constant-property fluid at a mass flow rate $\dot{\mathrm{m}}$ in an electrically heated tube of diameter d and length L. The heat flux to the fluid along the length of the tube is given as

$$q_w'' = q_0'' \sin \frac{\pi x}{L}$$

where q_0'' is a given constant. Determine the variation of the tube surface temperature along the length of the tube. Assume that the heat transfer coefficient is constant and known.

Problem 5

Consider the laminar flow of an oil inside a duct with a Reynolds number of 1000. The length of the duct is 2.5 m and the diameter is 2 cm. The duct is heated electrically by the use of its walls as an electrical resistance. Properties of the oil at the average oil temperature are $\rho = 870kg/m^3$, $c_p = 1.959kJ/kg.K$, $\mu = 0.004Pa.s$ and $\kappa = 0.128W/m.K$. Obtain the local Nusselt number at the end of the duct.

Problem 6

Air at 1 atm and 20°C is to be heated at a rate of 0.04 kg/min in a circular pipe of 5 cm ID and 6 m length by maintaining a constant heat flux at the pipe wall. What is the required wall heat flux if the maximum local difference between the pipe wall and mean fluid temperatures is to be equal or less than 10°C? Also, determine the exit air temperature.

Problem 7

Consider the fully developed flow of a very viscous fluid in a circular pipe of radius, r_0. Obtain an expression for the Nusselt number if the boundary condition is given as

$$\text{at } r = r_0\text{: } T = T_w < T_m$$

where T_m is the mean fluid temperature

Problem 8

Consider the fully developed flow of a very viscous fluid between two parallel isothermal plates separated by a distance of $2d$. If the temperatures of the plates are T_H, T_C for hot an cold plates respectively. Determine Nusselt number for both the plates. Also derive the relation for the limiting case when $T_H = T_C$.

Thank You!