

In[\*]:=  $v = v0[y] (x / \delta[y]) (1 - x / \delta[y])^2$

Out[\*]= 
$$\frac{x v0[y] \left(1 - \frac{x}{\delta[y]}\right)^2}{\delta[y]}$$

In[\*]:=  $T0 = T00 + \gamma y$

Out[\*]= 
$$T00 + y \gamma$$

In[\*]:=  $T = T0 + (Tw - T0) (1 - x / \delta[y])^2$

Out[\*]= 
$$T00 + y \gamma + (-T00 + Tw - y \gamma) \left(1 - \frac{x}{\delta[y]}\right)^2$$

In[\*]:=  $momInt = \partial_y \int_0^{\delta[y]} v^2 dx == -v (\partial_x v /. x \rightarrow 0) + \int_0^{\delta[y]} g \beta (T - T0) dx$

Out[\*]= 
$$\frac{2}{105} v0[y] \times \delta[y] v0'[y] + \frac{1}{105} v0[y]^2 \delta'[y] == -\frac{v v0[y]}{\delta[y]} - \frac{1}{3} g \beta (T00 - Tw + y \gamma) \delta[y]$$

In[\*]:=  $egyInt = \partial_y \int_0^{\delta[y]} v (T - T0) dx == -\alpha (\partial_x T /. x \rightarrow 0)$

Out[\*]= 
$$-\frac{1}{30} \gamma v0[y] \times \delta[y] - \frac{1}{30} (T00 - Tw + y \gamma) \delta[y] v0'[y] - \frac{1}{30} (T00 - Tw + y \gamma) v0[y] \delta'[y] == \frac{2 \alpha (-T00 + Tw - y \gamma)}{\delta[y]}$$

In[\*]:= **nonDimRules** = {  
 $y \rightarrow ytH,$   
 $\delta[y] \rightarrow \delta t[yt] H RaH^{(-1/4)},$   
 $\delta'[y] \rightarrow \delta t'[yt] RaH^{(-1/4)},$   
 $v0[y] \rightarrow (\alpha / H) RaH^{(1/2)} v0t[yt],$   
 $v0'[y] \rightarrow (\alpha / H^2) RaH^{(1/2)} v0t'[yt],$   
 $\gamma \rightarrow b (Tw - T00) / H,$   
 $g \rightarrow RaH \gamma \alpha / (\beta (Tw - T00) H^3);$   
**simplifyRules** = {  
 $Tw - T00 > 0,$   
 $\alpha > 0, H > 0, RaH > 0, Pr > 0};$

In[\*]:= **momNonDim** = **momInt** /. **nonDimRules** // # /.  $v \rightarrow \alpha Pr$  & // **Simplify**[#, **simplifyRules**] &

Out[\*]= 
$$35 Pr (-1 + b yt) \delta t[yt] + v0t[yt] \left( \frac{105 Pr}{\delta t[yt]} + 2 \delta t[yt] v0t'[yt] \right) + v0t[yt]^2 \delta t'[yt] == 0$$

In[\*]:= **momPrInf** = **momNonDim** // #[[1]] / Pr & // **Limit**[#, Pr  $\rightarrow \infty$ ] & // # /.  $b \rightarrow 1$  & // # == 0 &

Out[\*]= 
$$\frac{105 v0t[yt]}{\delta t[yt]} + 35 (-1 + yt) \delta t[yt] == 0$$

```
In[*]:= v0tSol = Solve[momPrInf, v0t[yt]] [[1]] [[1]] [[2]]
```

```
Out[*]:=
```

$$-\frac{1}{3} (-1 + yt) \delta t[yt]^2$$

```
In[*]:= egyNonDim = egyInt /. nonDimRules // # /. v -> \alpha Pr & // Simplify[#, simplifyRules] &
```

```
Out[*]:=
```

$$\frac{(-1 + b yt) (-60 + \delta t[yt]^2 v0t'[yt])}{\delta t[yt]} + v0t[yt] (b \delta t[yt] + (-1 + b yt) \delta t'[yt]) = 0$$

```
In[*]:= problemDetails = {b -> 1, v0t[yt] -> v0tSol, v0t'[yt] -> D[v0tSol, yt]}
```

```
Out[*]:=
```

$$\left\{ b \rightarrow 1, v0t[yt] \rightarrow -\frac{1}{3} (-1 + yt) \delta t[yt]^2, \right. \\ \left. v0t'[yt] \rightarrow -\frac{1}{3} \delta t[yt]^2 - \frac{2}{3} (-1 + yt) \delta t[yt] \delta t'[yt] \right\}$$

```
In[*]:= egyPrInf = egyNonDim // # /. problemDetails & // Simplify[#, {\delta t[yt] > 0}] &
```

```
Out[*]:=
```

$$(-1 + yt) (180 + 2 \delta t[yt]^4 + 3 (-1 + yt) \delta t[yt]^3 \delta t'[yt]) = 0$$

```
In[*]:= transformationRules = {\delta t[yt] -> \Delta[yt] ^ (1 / 4), \delta t'[yt] -> \Delta'[yt] \times \Delta[yt] ^ (-3 / 4) / 4}
```

```
Out[*]:=
```

$$\left\{ \delta t[yt] \rightarrow \Delta[yt]^{1/4}, \delta t'[yt] \rightarrow \frac{\Delta'[yt]}{4 \Delta[yt]^{3/4}} \right\}$$

```
In[*]:= \Delta P = egyPrInf /. transformationRules // FullSimplify[#, {\Delta[yt] > 0, yt < 1}] &
```

```
Out[*]:=
```

$$720 + 8 \Delta[yt] + 3 (-1 + yt) \Delta'[yt] = 0$$

```
In[*]:= Solve[\Delta P, \Delta'[yt]]
```

```
Out[*]:=
```

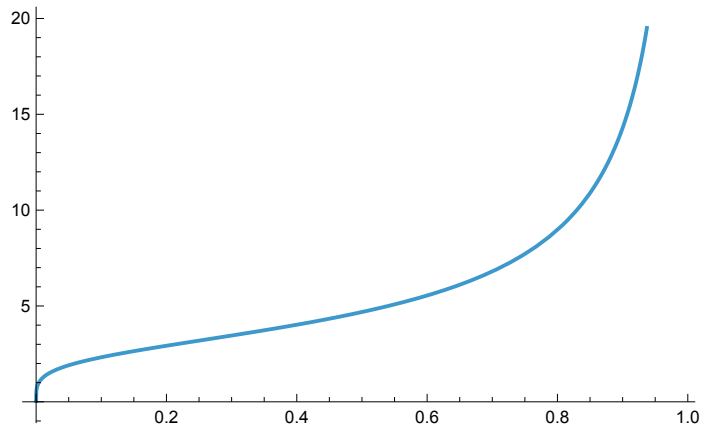
$$\left\{ \left\{ \Delta'[yt] \rightarrow -\frac{8 (90 + \Delta[yt])}{3 (-1 + yt)} \right\} \right\}$$

```
In[*]:= sol = NDSolve[{\Delta P, \Delta[0] == 0}, \Delta[yt], {yt, 0, 0.99}]
```

```
Out[*]:=
```

$$\left\{ \left\{ \Delta[yt] \rightarrow \text{InterpolatingFunction} \left[ \begin{array}{|c|} \hline \text{Domain: } \{0., 0.99\} \\ \hline \text{Output: scalar} \end{array} \right] [yt] \right\} \right\}$$

```
In[*]:= Plot[Evaluate[Δ[yt]^(1/4) /. sol], {yt, 0, 0.99}]
Out[*]=
```



```
In[*]:= hy = ((-κ D[T, x] /. x → 0) / (Tw - T00)) // # /. nonDimRules & //
          # /. transformationRules & // Simplify[#, Tw > T00] & // # /. problemDetails &
```

```
Out[*]=
```

$$-\frac{2 \text{RaH}^{1/4} (-1 + \text{yt}) \kappa}{H \Delta[\text{yt}]^{1/4}}$$

```
In[*]:= hyNonDim = hy H / (κ RaH^(1/4))
```

```
Out[*]=
```

$$-\frac{2 (-1 + \text{yt})}{\Delta[\text{yt}]^{1/4}}$$

```
In[*]:= hMeanNonDim = NIntegrate[hyNonDim /. sol, {yt, 0, 1}]
```

```
Out[*]=
```

$$\{0.324668\}$$

```
In[*]:= NuMean = hMeanNonDim[[1]] RaH^(1/4)
```

```
Out[*]=
```

$$0.324668 \text{RaH}^{1/4}$$

```
In[*]:= NuLiterature = 0.337 RaH^(1/4)
```

```
Out[*]=
```

$$0.337 \text{RaH}^{1/4}$$

```
In[*]:= deviation = (1 - NuMean / NuLiterature)
```

```
Out[*]=
```

$$0.0365936$$