EM 314 – NUMERICAL METHODS
ASSIGNMENT 01

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Question 1

① $\sin x \simeq x$

Taylor series,

$$f(x+h) = f(x) + \frac{f'(x)h}{1} + \frac{f''(x)h^2}{2!} + \frac{f'''(x)h^3}{3!} + \cdots$$

$$\sin(x_0+x_0) = f(x_0) + f'(x_0)x + f''(x_0)\frac{x^2}{2!} + f'''(x_0)\frac{x^3}{3!}$$

$$+ f''''(x_0)\frac{x^4}{4!} + f'''''(x_0)\frac{x^5}{5!} + \cdots$$

then $x_0 = 0$.

$$\sin(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!}$$

$$+ f''''(0)\frac{x^4}{4!} + f'''''(0)\frac{x^5}{5!}$$

$f(x) = \sin x$.

$f'(x) = \cos x$.

$f''(x) = -\sin x$

$f'''(x) = -\cos x$

$f''''(x) = \sin x$

$f'''''(x) = \cos x$

$$\sin(x) = 0 + x + 0 - \frac{x^3}{3!} + 0 + \frac{x^5}{5!}$$

$$\sin(x) = x \underbrace{- \frac{x^3}{3!} + \frac{x^5}{5!}}_{\text{error}}.$$

error $< 10^{-6}$

$$-\frac{x^3}{3!} + \frac{x^5}{5!} < 10^{-6}$$

$$\frac{x^5}{120} - \frac{x^3}{6} < 10^{-6}$$

$$x^5 - 20x^3 - 12\times10^{-5} < 0$$

$x_{min} = -4.472136$

$x_{max} = +4.472136$

So range $-4.472136 < x < 4.472136$

Question 2

② $\mathbb{F} \cdot (\beta, t, L, U)$, $\mathbb{F} \subset \mathbb{R}$

General form of floating pointer numbers:-

$$x_* = (-1)^s \pm (.\alpha_1 \alpha_2 --- \alpha_t)\, \beta^{m}, \quad m \in [L, U]$$

There are two sign bits.

α_1 can differ $(\beta - 1)$ times.

~~α_2, α_3~~

Other elements in mantissa can vary β times

so mantisa can vary $(\beta-1)\beta^{t-1}$ times.

The exponent is going to vary $(U - L + 1)$ times

Then we can say,

$\mathbb{F}$ contains precisely $2(\beta-1)\beta^{t-1}(U-L+1)$

Question 3

(3). Taylor series

$$f(x+h) = f(x) + f'(x)\,h + f''(\xi)\frac{h^2}{2!}$$

$x < \xi < x+h$.

Then,

$$f'(x)\,h = f(x+h) - f(x) - f''(\xi)\frac{h^2}{2!}$$

$$f'(x) = \underbrace{\frac{f(x+h) - f(x)}{h}} - \frac{f''(\xi)\,h}{2!}$$

$$f'(x) = f'_h(x) - \frac{f''(\xi)\,h}{2!}$$

$$\underbrace{f'_h(x) - f'(x)} = \frac{f''(\xi)\,h}{2!}$$

$$E_h(x) = \frac{f''(\xi)\,h}{2!}$$

$$E_h(x) = \underbrace{\left[\frac{f''(\xi)}{2!}\right]}_{\text{constant} = k} h$$

Therefore,

$$\lim_{h \to 0} \frac{|E_h(x)|}{|h|} = k$$

Then we can say.

$$E_h(x) = O(h).$$

Question 4

Editor - C:\Users\Prasad-PC\Desktop\q4.m

```matlab
val=500:500:5000;   %create a vector with matrix size values
size=length(val);   %get the length of val vector
t=(size);

for i=1:size
    t1=cputime();         %initial time when start up the process
    A=rand(val(i));       %find the random matrix
    deter=det(A);         %find determinent of matrix A
    t(i)=cputime()-t1;    %calculate the time taken to complete the process
end

hold on
loglog(log(val),log(t),'-*') %plott a graph log(val) Vs log(t)
title('Log(t) against Log(n) ','FontSize',14);
xlabel('log(n)');              %labelling
ylabel('log(t)');
p=polyfit(log(val),log(t),1); %find the best fit line's cofficients
p1=polyval(p,log(val));        % find best fit line's y coordinates
plot(log(val),p1,'r')          %plot a graph log(val) Vs p1

alpha=(log(t(7)-log(t(4))))/(log(val(7))-log(val(4))); %find the gradient of the graph
fprintf('alpha =%.4f\n',alpha); %display the value of alpha
```

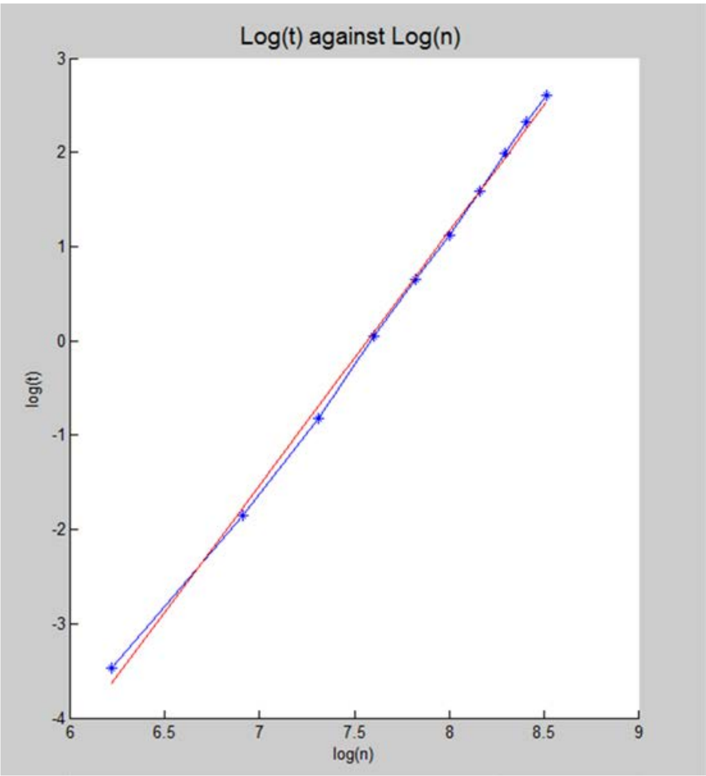
Command Window

File Edit Debug Desktop Window Help

```
>> clear
alpha =2.8139
```

(c)

(d)



(f) When using computer software for calculations we have to use variables. Sometimes there may be memory overflow cases. Then we get wrong answers. That's why experimental values differ from theoretical values.

Question 5

When N=10

Editor - C:\Users\Prasad-PC\Desktop\q5.m

```
N=10;              %set vector size
x=3;               %set constant value for x
k=1:N;             %find k vector
h=1./(2.^(k));     %find h vector
f=length(h);
E=length(h);

for i=1:length(h)
    f(i)=(log(x+h(i))-log(x))*1/h(i); %find f'h(x)
    E(i)=abs((1/x)-f(i));             %find E(x)
    fprintf('%d\t%f\t%f\t%f\n',k(i),h(i),f(i),E(i)); %displaying values
end

G=log(E(2)/E(1))/log(h(2)/h(1));     %find value for Gama
fprintf('Gama valus is %.4f\n',G);   % display Gema

loglog(h,E,'r')              %plot a graph (h) Vs (E)
title('Ehk against hk','FontSize',14);
xlabel('hk');
ylabel('Ehk');
```

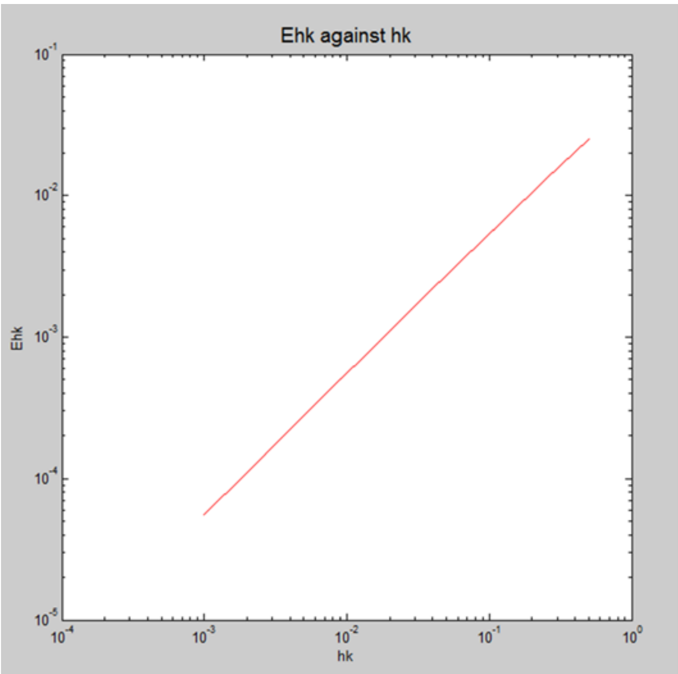
(a)

Command Window

k	h_k	$f'_{hk}(x)$	E_{hk}
1	0.500000	0.308301	0.025032
2	0.250000	0.320171	0.013163
3	0.125000	0.326576	0.006757
4	0.062500	0.329909	0.003425
5	0.031250	0.331609	0.001724
6	0.015625	0.332468	0.000865
7	0.007813	0.332900	0.000433
8	0.003906	0.333117	0.000217
9	0.001953	0.333225	0.000108
10	0.000977	0.333279	0.000054

Gama valus is 0.9273

(b)



(c) When N=40,

Editor - C:\Users\Prasad-PC\Desktop\q5.m

```
N=40;           %set vector size
x=3;            %set constant value for x
k=1:N;          %find k vector
h=1./(2.^(k));  %find h vector
f=length(h);
E=length(h);

for i=1:length(h)
    f(i)=(log(x+h(i))-log(x))*1/h(i); %find f'h(x)
    E(i)=abs((1/x)-f(i));             %find E(x)
    fprintf('%d\t%f\t%f\t%f\n',k(i),h(i),f(i),E(i)); %displaying values
end

G=log(E(2)/E(1))/log(h(2)/h(1));     %find value for Gama
fprintf('Gama valus is %.4f\n',G);   % display Gema

loglog(h,E,'r')             %plot a graph (h) Vs (E)
title('Ehk against hk','FontSize',14);
xlabel('hk');
ylabel('Ehk');
```

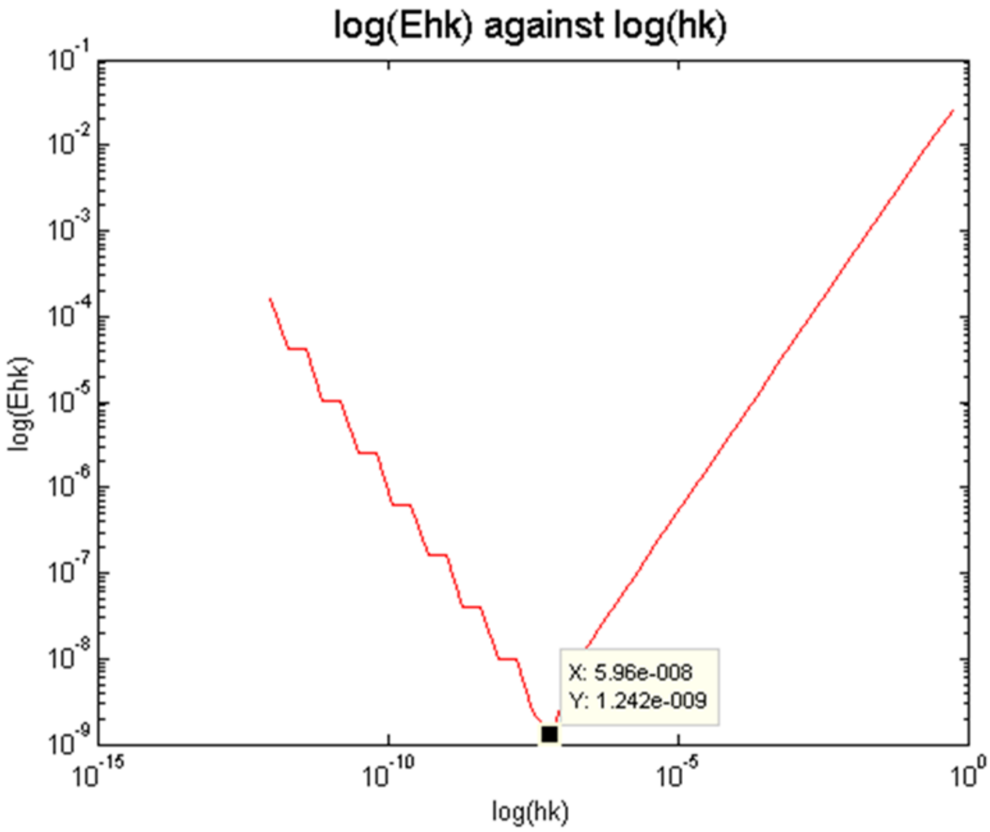
Command Window

k	h_k	$f'_{hk}(x)$	E_{hk}
1	0.500000	0.308301	0.025032
2	0.250000	0.320171	0.013163
3	0.125000	0.326576	0.006757
4	0.062500	0.329909	0.003425
5	0.031250	0.331609	0.001724
6	0.015625	0.332468	0.000865
7	0.007813	0.332900	0.000433
8	0.003906	0.333117	0.000217
9	0.001953	0.333225	0.000108
10	0.000977	0.333279	0.000054
11	0.000488	0.333306	0.000027
12	0.000244	0.333320	0.000014
13	0.000122	0.333327	0.000007
14	0.000061	0.333330	0.000003
15	0.000031	0.333332	0.000002
16	0.000015	0.333332	0.000001
17	0.000008	0.333333	0.000000
18	0.000004	0.333333	0.000000
19	0.000002	0.333333	0.000000
20	0.000001	0.333333	0.000000
21	0.000000	0.333333	0.000000
22	0.000000	0.333333	0.000000
23	0.000000	0.333333	0.000000
24	0.000000	0.333333	0.000000
25	0.000000	0.333333	0.000000
26	0.000000	0.333333	0.000000
27	0.000000	0.333333	0.000000
28	0.000000	0.333333	0.000000
29	0.000000	0.333333	0.000000
30	0.000000	0.333333	0.000000
31	0.000000	0.333333	0.000000
32	0.000000	0.333334	0.000001
33	0.000000	0.333334	0.000001
34	0.000000	0.333336	0.000003
35	0.000000	0.333336	0.000003
36	0.000000	0.333344	0.000010
37	0.000000	0.333344	0.000010
38	0.000000	0.333374	0.000041
39	0.000000	0.333374	0.000041
40	0.000000	0.333496	0.000163

```
Gama valus is 0.9273
fx >>
```

(d) The value of $f'_{hk}(x)$ is going to increase in 32nd step. As we can see minimum value for derivative of ln(x) exists between k=31 and 32. So minimum value of error we can see at this point and after that point error is going to increase. The reason for this thing is there are limits in Arithmetic of computer.

(e)



$\ln(h_{min})=5.96 \times 10^{-8}$

$hmin=e^{5.96 \times 10^{\wedge}(-8)}$