EM314 – NUMERICAL METHODS

ASSIGNMENT 03

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(1)

① · Third order $\longrightarrow$ 4 points.

x	1	2	3	4
$\ln(x)$	$\ln(1)$	$\ln(2)$	$\ln(3)$	$\ln(4)$

$$p(x) = \ln(1)\left(\frac{x-x_2}{x_1-x_2}\right)\left(\frac{x-x_3}{x_1-x_3}\right)\left(\frac{x-x_4}{x_1-x_4}\right) + \ln(2)\left(\frac{x-x_3}{x_2-x_3}\right)\left(\frac{x-x_4}{x_2-x_4}\right)\left(\frac{x-x_1}{x_2-x_1}\right)$$

$$+ \ln(3)\left(\frac{x-x_1}{x_3-x_1}\right)\left(\frac{x-x_2}{x_3-x_2}\right)\left(\frac{x-x_4}{x_3-x_4}\right) + \ln(4)\left(\frac{x-x_1}{x_4-x_1}\right)\left(\frac{x-x_2}{x_4-x_2}\right)\left(\frac{x-x_3}{x_4-x_3}\right)$$

$$= \ln(1)\left(\frac{x-2}{1-2}\right)\left(\frac{x-3}{1-3}\right)\left(\frac{x-4}{1-4}\right) + \ln(2)\left(\frac{x-3}{2-3}\right)\left(\frac{x-4}{2-4}\right)\left(\frac{x-1}{2-1}\right)$$

$$+ \ln(3)\left(\frac{x-1}{3-1}\right)\left(\frac{x-2}{3-2}\right)\left(\frac{x-4}{3-4}\right) + \ln(4)\left(\frac{x-1}{4-1}\right)\left(\frac{x-2}{4-2}\right)\left(\frac{x-3}{4-3}\right)$$

$$= \ln(1)\frac{(x-2)(x-3)(x-4)}{-6} + \ln(2)\frac{(x-3)(x-4)(x-1)}{2}$$

$$+ \ln(3)\frac{(x-1)(x-2)(x-4)}{-2} + \ln(4)\frac{(x-1)(x-2)(x-3)}{6}$$

$$= \frac{\ln(1)}{-6}(x^3-9x^2+26x-24) + \frac{\ln(2)}{2}(x^3-8x^2+19x-12)$$

$$+ \frac{\ln(3)}{-2}(x^3-7x^2+14x-8) + \frac{\ln(4)}{6}(x^3-6x^2+11x-6)$$

$$= x^3\left(\frac{\ln(1)}{-6} + \frac{\ln(2)}{2} + \frac{\ln(3)}{-2} + \frac{\ln(4)}{6}\right) + x^2\left(\frac{3}{2}\ln(1) - 4\ln(2) + \frac{7}{2}\ln(3) - \ln(4)\right)$$

$$+ x\left(-\frac{13}{3}\ln(1) + \frac{19}{2}\ln(2) + -7\ln(3) + \frac{11}{6}\ln 4\right) + \left[4\ln(1) - 6\ln(2) + 4\ln(3) - \ln 4\right]$$

$$= 0.0283\,x^3 - 0.3137\,x^2 + 1.436\,x - 1.1507$$

(2)

② $l_i(x) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \left(\frac{x - x_j}{x_i - x_j}\right)$

Basis function $l_i(x)$ depend only on the nodes. (points)

let $f(x) = k$

When we interpolate the function $f(x) = k$, the interpolated polynomial can be written as,

$$p(x) = \sum_{i=0}^{n} f(x_i)\, l_i(x)$$

For any $x_1, x_2 \dots x_n$ data are perfectly interpolated by the zeroth order polynomial $p(x) = f(x) = k$.

There fore,

$$k = \sum_{i=0}^{n} k\, l_i(x)$$

$$k = k \sum_{i=0}^{n} l_i(x)$$

$$1 = \sum_{i=0}^{n} l_i(x)$$

$$\therefore \sum_{i=0}^{n} l_i(x) = 1 \quad \text{for all } x$$

(3)

a)

Editor - C:\Users\Prasad-PC\LagrangeInterpolant.m

```
function sum = LagrangeInterpolant(X,Y)
   x=sym('x');
   mul=1; sum=0;

    for i=1:length(X)
        for j=1:length(X)
          if i==j
              continue;
          end
             k=(x-X(j))/(X(i)-X(j));
             mul=mul*k;
        end

        sum=sum+mul*Y(i);
        mul=1;
    end
    sum=simplify(sum);
end
```

b)

Editor - C:\Users\Prasad-PC\TestLagrange.m

```
x=[0 1/2 1];
y=[0 1/4 1];

p=LagrangeInterpolant(x,y);
fprintf('Polynomial is ');
disp(p);


 ezplot(p)
 hold on
 plot(x,y,'o')
```

kepler.m | testKepler.m | bisection.m | Gas.m | LagrangeInterpolant.m | TestLagrange.m | Thermocline.m

Command Window

```
Polynomial is x^2

>>
```

Yes. We obtain the expected answer.

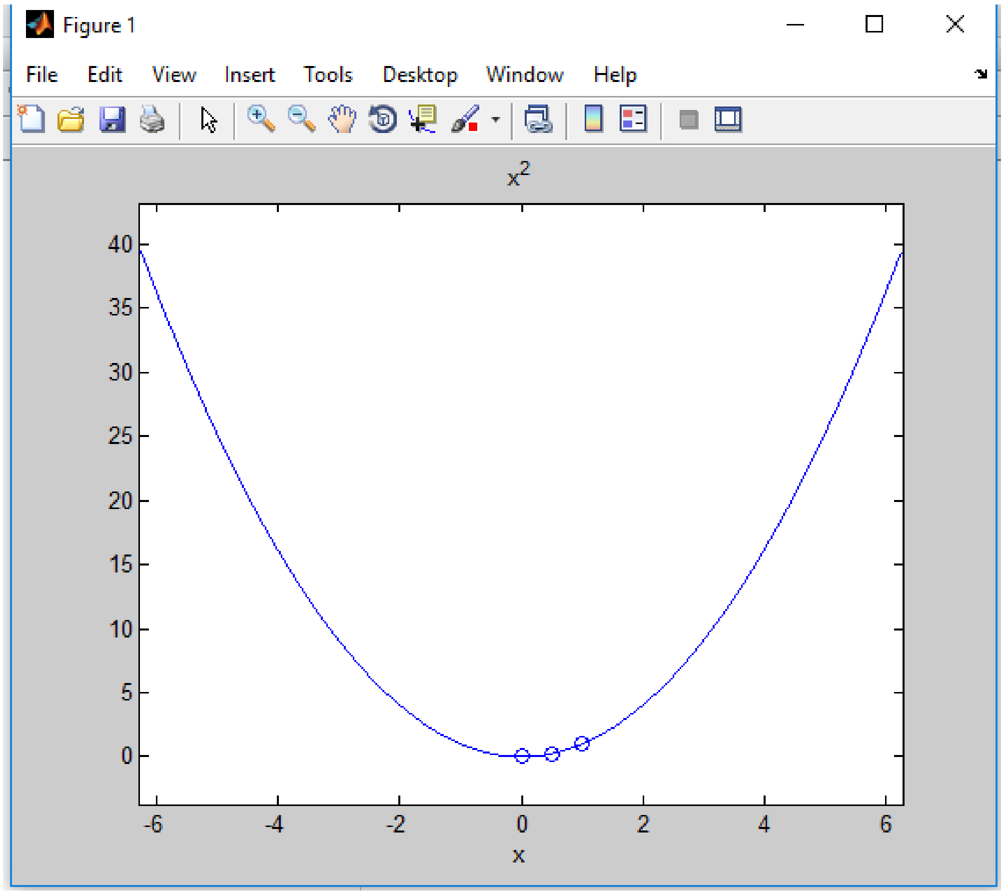
Figure 1
File Edit View Insert Tools Desktop Window Help
x2
40
35
30
25
20
15
10
5
0
-6
-4
-2
0
2
4
6
x

(4) a) and b)

```
z=[0 -2 -4 -6 -8 -10];
T=[29.1 29 28.7 28.2 20.7 19.1];

p=LagrangeInterpolant(z,T);
fprintf('Polynomial is ');
disp(p);

fprintf('The tenperature at depth 7m = %f\n',subs(p,-7));
```

kepler.m | testKepler.m | bisection.m | Gas.m | LagrangeInterpolant.m | TestLagrange.m | Thermocline.m

Command Window

```
Polynomial is - (53*x^5)/7680 - (299*x^4)/1920 - (2263*x^3)/1920 - (1711*x^2)/480 - (7*x)/2 + 291/10

The tenperature at depth 7m = 25.291016
>>
```

Answer is valid according to the table. Because when depth is 6m temperature is 28.2 and when depth is 8m temperature is 20.7. For 7m depth we got 25.29 as temperature. And it lies between 6m and 8m temperature values. So the answer is valid.

c)

Editor - C:\Users\Prasad-PC\Thermocline.m

```
z=[0 -2 -4 -6 -8 -10];
T=[29.1 29 28.7 28.2 20.7 19.1];

p=LagrangeInterpolant(z,T);
fprintf('Polynomial is ');
disp(p);

%plotting
ezplot(p)
hold on
plot(z,T,'o')

fprintf('The tenperature at depth 7m = %f\n',subs(p,-7));

x=diff(diff(p)); %second order differential equation

y=solve(x); %solve it when equation equals to zero

M=max(double([y])); %get the maximum
fprintf('Maximum =');
disp(M);
```

```
Command Window
Polynomial is - (53*x^5)/7680 - (299*x^4)/1920 - (2263*x^3)/1920 - (1711*x^2)/480 - (7*x)/2 + 291/10

Maximum =  -7.8519 - 0.0000i

fx >> 
```