<u>2020 PP</u>

(01)

a). even $\Rightarrow f(x) = \cos x$ where $(-\frac{\pi}{2} \leq x \leq \frac{\pi}{2})$

odd $\Rightarrow f(x) = \sin x$ where $(-\frac{\pi}{2} \leq x \leq \frac{\pi}{2})$

b). i. $f(x) = 2\sin x + 3\sin x \cos x$

$= 2\sin x + \frac{3}{2}\sin 2x$ //

$a_0 = 0$, $a_n = 0$, $a_1 = 2$, $a_2 = \frac{3}{2}$

ii. $g(x) = \sin^2 x$

$= \frac{1 - \cos 2x}{2}$

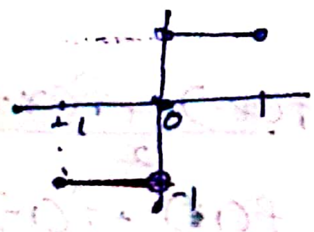
$= \frac{1}{2} - \frac{1}{2}\cos 2x$

$\therefore a_0 = \frac{1}{2}$, $a_2 = -\frac{1}{2}$, $b_n = 0$

so fourier series of $g(x) \Rightarrow f(x) = \frac{1}{2} - \frac{1}{2}\cos 2x$ //

c) fourier series and the $f(x)$ will be equel when $f(x)$ is continuous other wise f.s will be the average value of the upper and the lower limits of the function.

So, a) f.s and $f(x)$ converges to each other for all $x \in \mathbb{R}$, since $f(x)$ is continuous

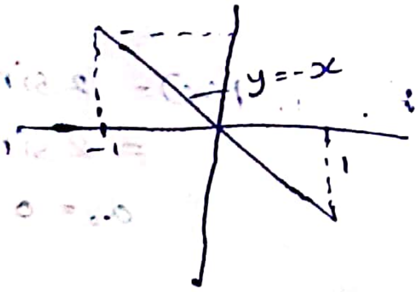
b) 

f.s and $f(x)$ converges to eachother for all $x \in \mathbb{R}$ except $x = (k-1)$ where $k \in \mathbb{Z}$

(02) $f(x) = -x$

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{p} x + b_n \sin \frac{n\pi}{p} x \right)$$

since $f(x) = -f(-x)$, function is odd.



so, $a_0 = 0$, $a_n = 0$,

$$b_n = \frac{2}{p} \int_0^p f(x) \sin \frac{n\pi}{p} x \, dx$$

$$= 2 \int_0^1 -x \sin(n\pi) x \, dx$$

$$= 2 \left[-\frac{x \cos(n\pi) x}{n\pi} \right]_0^1 - \underbrace{\int_0^1 \frac{\cos n\pi x}{n\pi} \times (-1)}_{0}$$

$$b_n = -2 \left[\frac{\cos n\pi}{n\pi} \right] \Rightarrow \frac{-2(-1)^n}{n\pi}$$

$$\Downarrow$$

f.s of $f(x) = -\frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos n\pi}{n} \sin n\pi x$ //

(b) when $x = -1 \Rightarrow \frac{f(x-) + f(x+)}{2}$

$$\Rightarrow \frac{f(-1-) + f(-1+)}{2}$$

$$\Rightarrow \frac{-1+1}{2}$$

$= 0$ $\therefore$ f.s converges to 0 @ $x = -1$ //

when $x = 1 \Rightarrow \frac{f(x-) + f(x+)}{2}$

$$\frac{f(1+) + f(1-)}{2}$$

$$= \frac{-1+1}{2}$$

$= 0$ $\therefore$ f.s converges to 0 @ $x = 1$ //

(03)

a. $x[n] = \left(\frac{2}{3}\right)^n$

$$Z(x(n)) = \sum_{n=0}^{\infty} x(n)\, z^{-n}$$

$$= \left(\frac{2}{3}\right)^0 z^0 + \left(\frac{2}{3}\right) z^{-1} + \left(\frac{2}{3}\right)^2 z^{-2} + \left(\frac{2}{3}\right)^3 z^{-3} + \dots$$

$$= 1 + \frac{2}{3} z^{-1} + \frac{4}{9} z^{-2} + \frac{8}{27} z^{-3} + \dots$$

$$= \frac{z}{\left(z - \frac{2}{3}\right)} = \frac{3z}{(3z-2)} \quad //$$

b. $f(n) - f(n+1) = (-1)^n \,,\; f(0) = 0$

$$F(z) - zf(z) - z\,f(0) = \frac{z}{z+1}$$

$$f(z)(1-z) = \frac{z}{z+1}$$

$1 = Az + A + Bz - B$

$A - B = 1$

$A + B = 0$

$2A = 1$

$A = \frac{1}{2}$ $B = -\frac{1}{2}$

$$f(z) = -\frac{z}{(z^2-1)(z+1)}$$

$$= -z\left(\frac{1}{2(z-1)} - \frac{1}{2(z+1)}\right]$$

$$z^{-1}\; f(z) = \left(\frac{z}{2(z+1)} - \frac{z}{2(z-1)}\right) z^{-1}$$

$$= \frac{1}{2} \times (-1)^n - \frac{1}{2} \times 1$$

$$f(n) = \frac{1}{2}\left((-1)^n - 1\right) \quad //$$

(04)

(a) $f(z) = \dfrac{z}{(z+2)(z+5)}$

$1 = Az + 5A + Bz + 2B$

$A + B = 0$

$5A + 2B = 1$

$3A = 1$

$A = \frac{1}{3}$ $B - \frac{1}{3}$

$= z\left(\dfrac{A}{(z+2)} + \dfrac{B}{(z+5)}\right)$

$= z\left(\dfrac{1}{3(z+2)} - \dfrac{1}{3(z+5)}\right)$

$\mathcal{Z}^{-1} f(z) = \mathcal{Z}^{-1}\left(\dfrac{z}{3(z+2)} - \dfrac{z}{3(z+5)}\right)$

$= \frac{1}{3} \times (-2)^n - \frac{1}{3} \times (-5)^n$

$f(n) = \frac{1}{3}\left((-2)^n - (-5)^n\right)$ //

(b) $f(z) = \dfrac{z}{(z+2)^2}$

$g(z) = z^{n-1} f(z)$

$= \dfrac{z^{n-1} \times z}{(z+2)^2} = \dfrac{z^n}{(z+2)^2}$

pole of order 2 @ $z = -2$

$\therefore R_{z=a} = \dfrac{d^{m-1}}{dz^{m-1}} \dfrac{(z-a)^m g(z)}{(m-1)!}$

$R_{-2} = \dfrac{d}{dz} \dfrac{\cancel{(z+2)^2} z^n}{\cancel{(z+2)^2} \times 1!}$

$R_{-2} = \dfrac{d}{dz}(z^n)$

$= n z^{(n-1)} \Big|_{z=-2}$

$R_{-2} = f(n) = n(-2)^{n-1}$ //

(5)

(a) $t * 2t$

$f(t) = t$
$g(t) = 2t$

$$(f*g)t = \int_0^t f(t-\tau)g(\tau)\,d\tau \quad , t \geq 0$$

$$= \int_0^t (t-\tau)\, 2\tau \, d\tau$$

$$= \int_0^t 2t\tau \, d\tau - \int_0^t 2\tau^2 \, d\tau$$

$$= 2t\left[\frac{\tau^2}{2}\right]_0^t - 2\left[\frac{\tau^3}{3}\right]_0^t$$

$$= \frac{2t \times t^2}{2} - \frac{2 \times t^3}{3}$$

$$= t^3 - \frac{2t^3}{3}$$

$$= \frac{t^3}{3} \,//$$

(b) (i) $$f(s) = \frac{1}{s^2+1} + \frac{2s+1}{(s-1)(s+2)}$$

$$2s+1 = As+2A+Bs-B$$

$$A+B = 2$$

$$2A-B = 1$$

$$3A = 3 \qquad B = 1 \,//$$

$$A = 1 \,//$$

$$= \frac{1}{s^2+1} + \frac{1}{(s-1)} + \frac{1}{(s+2)}$$

$$f(t) = \sin(t) + e^{t} + e^{-2t} \,//$$

(ii) $$f(s) = \frac{2}{s^2-2s+5}$$

$$= \frac{2}{(s-1)^2+4} = \frac{2}{(s-1)^2+2^2}$$

$$f(t) = e^{t}\sin 2t \,//$$

(06) $y'' + y = e^t \quad , y(0) = 0 \quad , y'(0) = 0$

$$L(y'' + y) = L(e^t)$$

$$s^2 L(y) - s(y(0)) - y'(0) + L(y) = \frac{1}{s-1}$$

$$L(y)(s^2+1) = \frac{1}{s-1}$$

$$L(y) = \frac{1}{(s-1)(s^2+1)}$$

$$= \frac{A}{(s-1)} + \frac{Bs + C}{(s^2+1)}$$

$$1 = As^2 + A + Bs^2 + Cs - Bs - C$$

$A + B = 0$

$A - C = 1$

$C - B = 0$

$A + C = 0$

$A = \frac{1}{2} \quad C = -\frac{1}{2}$

$B = -\frac{1}{2}$

$$L(y) = \frac{1}{2(s-1)} + \frac{(-s) + (-1)}{2(s^2+1)}$$

$$= \frac{1}{2(s-1)} - \frac{(s+1)}{2(s^2+1)}$$

$$= \frac{1}{2(s-1)} - \frac{s}{2(s^2+1)} - \frac{1}{2(s^2+1)}$$

$$f(t) = \frac{1}{2}e^t - \frac{1}{2}\cos(t) - \frac{1}{2}\sin t \quad //$$