INTRODUCTION TO PROBABILITY

INTRODUCTION

Before we turn our focus to probability, it is important to understand some basic tools that form essential building blocks to probability.

Counting principles, also known as counting techniques or counting methods, are mathematical methods used to determine the number of possible outcomes, arrangements, or combinations in various situations. One of the key things that you need to master is the idea of the **two major counting principles – permutations and combinations**. Knowing these two concepts will enable you to calculate the probability for a given scenario or events that you are interested in.

We will start by understanding the concept of permutations.

PERMUTATIONS

A permutation is a way of arranging a select group of objects in such a way that the **order is of significance**.

For example, imagine you have a group of five friends, and you want to know how many ways they can sit in a row of five chairs for a group photo. You would use permutations to calculate all the possible seating arrangements.

The following list shows some other examples where permutation is used to count the number of ways in which a particular sequence of events can occur:

- Finding all possible four-letter words that can be formed using the alphabets R, E, A and D
- Finding all possible ways that a group of 10 people can be seated in a row in a cinema hall, and so on

Generally speaking, if there are **n** 'objects' that are to be arranged among **r** available 'spaces', then the number of ways in which this task can be completed is **n!/(n-r)!**. If there are **n** 'spaces' as well, then the number of ways would be just **n!**.

$$n! = n*(n-1)*(n-2)....*3*2*1$$

Question 1: Find the number of ways in which the letters of the word 'MOSAIC' can be rearranged to form different six-letter words.

- 30
- 100
- 720
- 320

Question 2: Again, using the letters of the word 'MOSAIC', can you find out how many three-letter words can be formed?

- 100
- 300
- 130
- 120

COMBINATIONS

The second important counting principle that you need to be aware of is the method of using **combinations**. In the case of permutations, you had considered the 'order' to be an important factor. Now, **in the case of combinations, you need not take the order into account while finding the number of ways to arrange a group of objects.**

When you have to **choose** some objects from a larger set and **the order is of no significance**, then the rule of counting that you use is called **combination**.

Some examples of combinations are as follows.

- The number of ways in which you can pick three letters from the word 'ONEPIECE.'
- The number of ways a team can win three matches in a league of five matches.
- The number of ways in which you can choose 13 cards from a deck of 52 cards, and so on.

The formula for counting the number of ways to choose r objects out of a set of n objects is as follows:

$$\boxed{{}^{n}C_{r} = \frac{n!}{r!(n-r)!}}$$

HINT: A helpful hint here would be to **look for a keyword** in the given scenario to know which method is needed. If the problem requires you to **order/arrange** a group of objects, then you would most probably use the method of **permutations**. Else, if you are told to **pick/choose** a group of objects, then often you would be using the formula for **combinations**.

PROBABILITY: DEFINITION AND PROPERTIES:

Now that we have understood the two fundamental rules of counting, we will go ahead and finally establish the formal definition of probability.

Definition

Probability is a numerical measure that represents the likelihood of a particular event or outcome occurring/happening within a set of possible events. It is typically expressed as a value between 0 and 1.

$$\boxed{Probability = \frac{No\ of\ desired\ outcomes}{Total\ number\ of\ possible\ outcomes.}}$$

Properties

Probability values have two major properties:

- Probability values always lie in the range of 0 to 1. The value is 0 in the case of an impossible event (like the probability of you being in Delhi and Mumbai at the same time) and 1 in the case of a sure event (like the probability of the sun rising in the east tomorrow).
- The probabilities of all outcomes for an experiment always sum up to 1. For example, in a coin toss, there can be two outcomes, heads or tails. The probability of both outcomes is 0.5 each. Hence, the sum of the probabilities turns out to be 0.5 + 0.5 = 1.

Now, let us see few more definitions which are crucial in understanding probability.

Experiment

Any scenario for which you want to compute the probabilities is considered to be an experiment. It is of the following two types:

Deterministic:

A deterministic experiment is one in which the **outcome is entirely certain and can be predicted with precision**, given the initial conditions and a set of rules or equations. In deterministic experiments, **there is no randomness or uncertainty involved.**

- **Examples**
- Projectile Motion: Imagine you launch a projectile (like a ball) into the air at a certain angle and velocity. The path of the projectile can be determined exactly using the laws of physics, such as the equations of motion. The outcome, which includes the trajectory, position, and time of impact, is entirely predictable and follows deterministic rules.
- Simple Pendulum

Random:

A random experiment is one in which the **outcome is subject to chance, and it cannot be predicted with certainty**. Instead, we describe the possible outcomes and their associated probabilities. **Random experiments involve uncertainty.**

- **Examples**: Random Experiment
- Rolling a Loaded Die: Consider rolling a loaded six-sided die where one side has been weighted to make it more likely to land on a specific number, but you don't know which number that is. When you roll the die, the outcome is uncertain because you cannot predict with certainty which number will appear face up. However, you can assign probabilities to each outcome based on your knowledge of the loaded die.
- Drawing a Card from a Shuffled Deck
- Weather forecasting

In summary:

Deterministic experiments have certain, predictable outcomes based on known rules or equations.

Random experiments involve chance and uncertainty, and their outcomes are described in terms of probabilities.

Sample Space

A sample space is nothing but the list of all possible outcomes of a random experiment. It is denoted by S = {all the possible outcomes}. For example, in the coin toss example, the sample space is S = {H, T}, where H = heads and T = tails.

Event

It is a subset, i.e., a part of the sample space that you want to be true for your probability experiment. For example, if in a coin toss you want heads to be the desired outcome, then the event becomes {H}. As you can see clearly, {H} is a part of {H, T}. There are mainly two types of events-

1. Independent: Independent events are events where the occurrence of one event does not affect the occurrence of another event. For example, the chances of rain in Bengaluru on a particular day has no effect on the chances of rain in Mumbai 10 days later. Hence, these two events are independent of each other. Mathematically, two events A and B are independent if and only if:

$$P(A \cap B) = P(A) * P(B)$$

2. Mutually exclusive or Disjoint:

   Now, two or more events are said to be mutually exclusive when they do not occur at the same time, i.e., when one event occurs, the rest of the events do not occur. For example, if a student has been assigned grade C for a particular subject in an exam, he or she cannot be awarded grade B for the same subject in the same exam. So, the events in which a student gets a grade of B or C for the same subject in the same exam are mutually exclusive or disjoint.

   Mathematically, two events A and B are mutually exclusive if and only if:

   $A \cap B = \emptyset$ (the intersection of A and B is an empty set)

Two or more events cannot be independent and disjoint simultaneously. In independence, the occurrence of one event does not affect the other event, meaning there can be overlapping outcomes between A and B. In mutual exclusivity, there are no overlapping outcomes between A and B; they cannot occur together.

COMPLIMENTARY RULE FOR PROBABILITY

It states that if A and A' are two events which are mutually exclusive/disjoint and are complementary/in negation of each other (you can read A' as 'not A'), then:

$$P(A) + P(A') = 1$$

This rule is basically an extension from the basic rule of probability – **the sum of probabilities for all events always adds up to 1**. Here are some examples where you can use this rule to find the probability of the complement of an event.

- If the probability that a customer buys a product is 0.4, then the probability that he/she does not buy the product is 0.6.
- If the probability that you win the lottery is 33%, then the probability that you do not win the lottery is 67%, and so on.

RULES OF PROBABILITY

1. ADDITION RULE

If A and B are two events such that, P(A) and P(B) denotes the probability of the events, the addition rules states that that the probability of the event that either A or B will occur is given by:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Where,

$P(A \cup B)$ denotes the probability that either event A or B occurs.

P(A) denotes the probability that only event A occurs.

P(B) denotes the probability that only event B occurs.

P(A∩B) denotes the probability that both events A and B occur simultaneously.

2. MULTIPLICATION RULE

If P(A) and P(B) are the probabilities of two mutually independent events A and B, multiplication rule allows us to compute the probabilities of both occurring simultaneously, which is given as:

P(A and B) = P(A)*P(B)

This rule can be extended to multiple independent events where all you need to do is multiply the respective probabilities of all the events to get the final probability of all of them occurring simultaneously. For example, if you have four independent events A, B, C and D, then:

P(A and B and C and D) = P(A)*P(B)*P(C)*P(D)

3. COMPARISION BETWEEN ADDITION AND MULTIPLICATION RULES

Both the addition rule and the multiplication rule allow you to compute the probabilities of the occurrence of multiple events. However, there is a key difference between the two.

- The **addition rule** is generally used to find the probability of multiple events when either of the events can occur at that instance. For example, when you want to compute the probability of picking a face card or a heart card from a deck of 52 cards, a successful outcome occurs when either of the two events is true. This includes either getting a face card, a heart card, or even both a face and a heart card. This rule works for all types of events.
- The **multiplication rule** is used to find the probability of multiple events when all the events need to occur simultaneously. For example, in a coin toss experiment where you toss the coin three times and you need to find the probability of getting three heads at the end of the experiment, a successful outcome occurs when you get a head in the first, second and third toss as well. This rule is used for independent events only.