EX 13.2

~~Ex 13~~

(1) $y = \frac{x^2}{2} + 3x$, $x = 2$ and $dx = 0.5$ find Δy, dy

~~Ans~~ sol,

$f(x) = \frac{x^2}{2} + 3x$

$\Delta y = f(x + \Delta x) - f(x)$

$= f(2 + 0.5) - f(2)$

$= \left\{ \frac{(2.5)^2}{2} + 3 \times 2.5 \right\} - \left(\frac{4}{2} + 6 \right)$

$= 2.625$

$d(y) = f'(x)\,dx$

$= f'(2) \times 0.5$

$= (2+3) \times 0.5$

$= 2.5$

$\therefore$ difference $= \Delta y - dy$

$= 2.625 - 2.5$

$= 0.125$ //

(2) find app change in vol of cube of side x m, change is 1%. find % in change in vol?

Answer:

$l = x\,m$

$\Delta dx = 1\%$ of $x = 0.01x$.

$d(v) = x^3$

$\frac{d(v)}{dx} = 3x^2$

$= 3x^2 \times dx$

$= 3x^2 \times 0.01x$

$= 0.03 \times x^3$

change in vol $= \frac{0.03x^3}{x^3} \times 100$

$= 3\%$.

Q.no.3) Use differentials to find app. change in x^3 as x changes from 5 to 5.01.

Answer:

Given,

$\Delta x = 0.01$

$x = 5$

Let $f(x) = y = x^3$

diff. b.s. w.r.to 'x' we get,

$\frac{d(y)}{d(x)} = 3x^2$

$= 3x^2 \times dx$

$= 3 \times 5^2 \times (0.01)$

$= 0.75$ //

Q.no.4) find an approximate change in $1/x$ as x changes from 1 to 0.98.

Answer:

$x = 1$ and $x + \Delta x = 0.98$

$1 + \Delta x = 0.98$

$\Delta x = 0.2$

Now,

$y = x^3$

diff. b.s. w.r. to 'x', we get,

$\frac{d(y)}{d(x)} = 3x^2$

$= 3x^2 d(x)$

$= (x)^{-1} \times 0.02$

$= 0.02$ //

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Q.No.9) A circular copper plate is heated so that its radius increases from 5cm to 5.06 cm. find the approximate increase in area and also the actual increase in area.

Answer Given,

$r = 5$ (where r is radius of circular plate)

$r + \Delta r = 5.06$

or, $5 + dr = 5.06$

or, $dr = 0.06$

Area of circle $(A) = \pi r^2$

$A = f(r) = \pi r^2$

diff. b.s. w.r. to 'x' we get;

$\frac{d(A)}{dr} = 2\pi r$

$= 2\pi r\, dr$

$= 2\pi (5)(0.06)$

$= 0.6\pi\ cm^2$

Again,

$\Delta(A) = f(r + \Delta r) - f(r)$

$= f(5.06) - f(5)$

$= \pi (5.66)^2 - \pi 5^2$

$= 0.6036\ \pi$

(6) Find the approximate increases in the surface area of a cube if the edge increases from 10 to 10.01 cm. Calculate % error.

Answer: Given,

length of cube $(x) = 10\,cm$

and $x + \Delta x = 10.01$

or, $10 + \Delta x = 10.01$

or, $\Delta x = 0.01$

Now,

T.S.A. of cube $(A) = 6x^2$

i.e. $A = f(x) = 6x^2$

for approximate change in area,

diff. b.s. w.r.to 'x' we get,

$$\frac{d(A)}{dx} = 12x$$

or, $dA = 12x \cdot dx$

$= 12 \times 10 \times 0.01$

$= 1.2\,cm^2$

Again,

$\Delta x = f(x+\Delta x) - f(x)$

$= f(10.01) - f(10)$

$= (1.2006)\,cm^2$

$$\text{Error\%} = \frac{\Delta A - dA}{A} \times 100\%$$

$$= \left(\frac{6 \times 10^{-4}}{600}\right)\%$$

$$= 1 \times 10^{-4}\%$$

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Q.No.7) find the approximate increase in the volume of a sphere when its radius increases from 2 to 2.1 find also the actual increase and compare the two values.

Answer: Given,

$x = 2, \quad \Delta x = 0.1$

$V = \frac{4}{3}\pi x^3$

$f(x) = V = \frac{4}{3}\pi x^3$

diff. b.s. w.r. to 'x' we get,

$V' = \frac{4}{\cancel{3}} \times \cancel{3} \times \pi \times x^2$

$= 4 \times \pi x^2 dx$

$= 4 \times \frac{22}{7} \times [2]^2 \times 0.1$

$= 1.6\pi,$

$\Delta A = f(x + \Delta x) - f(x)$

$= f(2 + 0.1) - f(2)$

$= \frac{5.044\pi}{3}$