Ex: 12.1

1. a) $\sinh 3x$.

Diff. both side with respect to 'x' we get,

$$\frac{dy}{dx} = \frac{d(\sinh 3x)}{d(x)} = 3\cosh 3x.$$

b) $y = \tanh \frac{x}{2}$

Diff. both side with respect to 'x', we get,

$$\frac{dy}{dx} = \frac{d(\tanh \frac{x}{2})}{d(\frac{x}{2})} \times \frac{d(\frac{x}{2})}{d(x)}$$

$$= \frac{1}{2} \operatorname{sech}^2 \frac{x}{2}$$

c) $y = \coth x^2$.

D.h.s.w.r to 'x' we get,

$$\frac{dy}{dx} = \frac{d(\coth x^2)}{d(x^2)} = -2x \operatorname{cosech}^2 x^2$$

Imp

d) $y = \operatorname{sech}^2 3x$.

D.h.s.w.r to 'x' we get,

$$\frac{dy}{dx} = \frac{d(\operatorname{sech}^2 3x)}{d(\operatorname{sech} 3x)} \times \frac{d(\operatorname{sech} 3x)}{d(3x)} \times \frac{d(3x)}{d(x)},$$

$$= 2 \operatorname{sech} 3x \times -3e$$

(e) sol, $y = \cosh^3 \frac{2x}{3}$

Diff b.s. w.r.to 'x', we get,

$$\frac{dy}{dx} = \frac{d(\cosh \frac{2x}{3})^3}{d(\cosh \frac{2x}{3})} \times \frac{d(\cosh \frac{2x}{3})}{d(\frac{2x}{3})} \times \frac{d(\frac{2x}{3})}{d(x)}$$

$$= 3\cosh^2 \frac{2x}{3} \cdot \sinh \frac{2x}{3} \cdot \frac{2}{3}$$

$$= \cosh \frac{2x}{3} \left(2 \sinh \frac{2x}{3} \cdot \cosh \frac{2x}{3} \right)$$

$$= \cosh \frac{2x}{3} \cdot \sinh \frac{4x}{3}$$

(f) $\cosh^{-1} \frac{x}{3}$

Diff b.s. w.r.to 'x', we get,

$$\frac{dy}{dx} = \frac{d(\cosh^{-1} \frac{x}{3})}{d(\frac{x}{3})} \times \frac{d(\frac{x}{3})}{d(x)}$$

$$= \frac{1}{\sqrt{\frac{x^2}{9} - 1}} \times \frac{1}{3}$$

$$= \frac{3}{\sqrt{x^2 - 9}} \times \frac{1}{3}$$

$$= \frac{1}{\sqrt{x^2 - 9}}$$ Ans

find derivatives of.

2.a)

$\sinh^2 x + \cosh^2 x$.

diff. b.both. w.r.to 'x', we get,

$$\frac{dy}{dx} = \frac{d(\cosh^2 x)}{d(\cosh x)} \cdot \frac{d(\cosh x)}{d(x)} + \frac{d(\sinh^2 x)}{d(\sinh x)} \times \frac{d(\sinh x)}{d(x)}$$

$= 2\cosh x \cdot \sinh x + 2\sinh x \cdot \cosh x$.

$= 4\sinh x \cdot \cosh x$.

$= 2 \cdot 2\sinh x \cdot \cosh x$.

$= 2\sinh 2x$ //

b)

$\coth x - \frac{1}{3}\coth^3 x$.

diff b.s.w.r to 'x' we get

$$\frac{dy}{d(x)} = \frac{d(\coth x)}{d(x)} - \frac{1}{3}\frac{d(\coth^3 x)}{d(x)}$$

$$= -\operatorname{cosech}^2 x - \frac{1}{3}\frac{d(\coth^3 x)}{d(\coth x)} \times \frac{d(\coth x)}{d(x)}$$

$= -\operatorname{cosech}^2 x + \frac{1}{3} \times 3 \coth^2 x \operatorname{cosech}^2 x$.

$= \operatorname{cosech}^2 x (\coth^2 x - 1)$.

$= \operatorname{cosech}^4 x$ //

(d) Sol, Let $y = \sqrt{\sinh x} + \frac{1}{\sqrt{\cosh x}}$

Diff. b.s.w.r to 'x', we get,

$$\frac{dy}{dx} = \frac{d(\sinh x)^{\frac{1}{2}}}{d(\sinh x)} \times \frac{d(\sinh x)}{d(x)} + \frac{d(\cosh x)^{-\frac{1}{2}}}{d(\cosh x)} \times \frac{d(\cosh x)}{d(x)}$$

$$= \frac{1}{2\sqrt{\sinh x}} \cdot \cosh x - \frac{1}{2} (\cosh x)^{-\frac{3}{2}} \cdot \sinh x.$$

$$= \frac{1}{2} \left(\frac{\cosh x}{\sqrt{\sinh x}} - \frac{\sinh x}{\sqrt{\cosh^3 x}} \right) //$$

(e) Let $y = \tanh x \coth x$.

Diff b.s.w.r to 'x', we get,

$$\frac{dy}{dx} = \frac{d(\tanh x \coth x)}{d(x)}$$

$$= \tanh x \frac{d(\coth x)}{d(x)} + \coth x \cdot \frac{d(\tanh x)}{d(x)}$$

$$= -\tanh x \operatorname{cosec}^2 x + \coth x \operatorname{sech}^2 x$$

$$= \coth x \operatorname{sech}^2 x - \tanh x \operatorname{cosec}^2 x. //$$

(f) Let $y = \frac{\sinh 2x}{x + \cosh 2x}$.

$$\frac{dy}{dx} = \frac{(x + \cosh 2x) 2\cosh 2x - \sinh 2x \cdot (1 + 2\sinh 2x)}{(x + \cosh 2x)^2}$$

$$= \frac{2x \cosh 2x + 2\cosh^2 2x - \sinh 2x - 2\sinh^2 2x.}{(x + \cosh 2x)^2}$$

$$= \frac{2x\cosh 2x - \sinh 2x + 2\cosh^2 2x - \sinh^2 2x.}{(x+\cosh 2x)^2}$$

$$= \frac{2x\cosh 2x - \sinh 2x + 2.}{(x+\cosh 2x)^2} \quad //$$

g)

(3) (a) $\operatorname{sech}(\tan^{-1}x)$

Diff. b.s.w.r to 'x', we get,

$$\frac{d(y)}{d(x)} = \frac{d\{\operatorname{sech}(\tan^{-1}x)\}}{d(\tan^{-1}x)} \times \frac{d(\tan^{-1}x)}{d(x)}$$

$$= \operatorname{sech}(\tan^{-1}x).\tanh(\tan^{-1}x).\frac{1}{1+x^2} \#$$

(b) $y = \operatorname{sech}^{-1}x - \cosh^{-1}x.$

Diff. b.s.w.r to 'x', we get,

$$\frac{dy}{d(x)} = \frac{d\{\operatorname{sech}^{-1}x - \cosh^{-1}x\}}{d(x)}$$

$$= -\frac{1}{x\sqrt{1-x^2}} - \frac{1}{\sqrt{x^2-1}} \#$$

(c) $y = \tan^{-1}(\sinh x)$

Diff. b.s.w.r to 'x' we get,

$$\frac{dy}{d(x)} = \frac{d\{\tan^{-1}(\sinh x)\}}{d(\sinh x)} \times \frac{d(\sinh x)}{d(x)}$$

$$= \frac{1}{1+\sinh^2 x}.\cosh x.$$

$$= \frac{\cosh x}{\cosh^2 x} \quad // \cosh^2 x - \sinh^2 x = 1 //$$

$$= \operatorname{sech} x \#$$

Tricky

(7) $y = 2\tan^{-1}\left(\tan\frac{1}{2}x\right)$

Diff. b.s.w.r.to 'x' we get,

$$\frac{dy}{d(x)} = 2\,\frac{d\{\tan^{-1}(\tan\frac{x}{2})\}}{d(\tan\frac{x}{2})} \times \frac{d(\tan\frac{x}{2})}{d(x)}$$

$$= 2 \cdot \frac{1}{1+\tan^2\frac{x}{2}} \cdot \sec^2\frac{x}{2} \cdot \frac{1}{2}$$

$$= \frac{1+\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} \quad // \sec^2x - \tan^2x = 1 //$$

$$= \frac{1}{\cos^2\frac{x}{2}} \quad // \cos2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} //$$

$$= \sec x \ \#$$

(8) $y = \sinh^{-1}(\cos 2x)$

Diff. b.s.w.r.to 'x' we get,

$$\frac{dy}{d(x)} = \frac{d\{\sinh^{-1}(\cos2x)\}}{d(\cos2x)} \times \frac{d(\cos2x)}{d(2x)} \times \frac{d(2x)}{d(x)}$$

$$= \frac{1}{\sqrt{1+\cos^2 2x}} \cdot \sin 2x \cdot 2$$

$$= \frac{2\sin 2x}{\sqrt{1+\cos^2 2x}} \ \#$$

$\frac{d(\sin x)^{x}}{d(x)}$ // not applicable //

(f). $y = \coth(\sin^{-1}x)$

Diff. b.s.w.r. to 'x' we get,

$$\frac{dy}{d(x)} = \frac{d\{\coth(\sin^{-1}x)\}}{d(\sin^{-1}x)} \times \frac{d(\sin^{-1}x)}{d(x)}$$

$$= \operatorname{cosech}^2(\sin^{-1}x) \cdot \frac{1}{\sqrt{1-x^2}} \#$$

(4).

a). $y = e^{\sin x \ln x}$

Diff. b.s.w.r. to 'x' we get,

$$\frac{dy}{dx} = \frac{d\{e^{\sin x \ln x}\}}{d(\sin x \ln x)} \times \frac{d(\sin x \ln x)}{d(x)}$$

$$= e^{\sin x \ln x} \cdot \cos x \ln x \#$$

(b) $y = e^{\tan h \frac{x}{2}}$

Diff. b.s.w.r to 'x' we get,

$$\frac{dy}{d(x)} = \frac{d\{e^{\tan h \frac{x}{2}}\}}{d\{\tan h \frac{x}{2}\}} \times \frac{d\{\tan h \frac{x}{2}\}}{d\{x\}}$$

$$= \frac{e^{\tan h \frac{x}{2}} \sec^2 h \frac{x}{2}}{2} \#$$

(c) $y = \log(\tanh x)$

Diff. b.s.w.r.to 'x', we get,

$$\frac{dy}{dx} = \frac{d\{\log(\tanh x)\}}{d\{\tanh x\}} \cdot \frac{d\{\tanh x\}}{d(x)}$$

$$= \frac{1}{\tanh x} \cdot \operatorname{sech}^2 x$$

Tricky

$$= \frac{\cosh x}{\sinh x} \cdot \frac{1}{\cosh^2 x}$$

$$= \frac{2}{2\sinh x \cdot \cosh x}$$

$$= 2\,\operatorname{cosech} 2x \quad \#$$

(d) $y = \log \sinh \frac{x}{a}$

Diff. b.s.w.r.to 'x' we get,

$$\frac{dy}{dx} = \frac{d\{\log \sinh \frac{x}{a}\}}{d\{\sinh \frac{x}{a}\}} \cdot \frac{d\{\sinh \frac{x}{a}\}}{d(x)}$$

$$= \frac{1}{\sinh \frac{x}{a}} \cosh \frac{x}{a} \cdot \frac{1}{a}$$

$$= \frac{1}{a} \coth \frac{x}{a} \quad \#$$

(e) $y = \log(\cosh x^2)$

Diff. b.s.w.r to 'x' we get,

$$\frac{dy}{d(x)} = \frac{d\{\log(\cosh x^2)\}}{d(\cosh x^2)} \times \frac{d(\cosh x^2)}{d(x^2)} \times \frac{d(x^2)}{d(x)}$$

$$= 2x \cdot \frac{1}{\cosh x^2} \cdot \sinh x^2$$

$$= 2x \tanh x^2 \quad \#$$

(5) 8. $y = x^{\cosh x/a}$

Diff. b.s.w.r to 'x' we get,

$\frac{dy}{dx}$ = logging on both sides we get,

$$\log(y) = \log\left(x^{\cosh \frac{x}{a}}\right)$$

or, $\log(y) = \cosh\frac{x}{a} \cdot \log(x)$

Diff. b.s.w.r to 'x' we get,

$$\frac{1}{y} \cdot \frac{dy}{d(x)} = \cosh\frac{x}{a} \cdot \frac{1}{x} + \log x \cdot \sinh\frac{x}{a} \cdot \frac{1}{a}$$

$$= y\left[\cosh\frac{x}{a} \cdot \frac{1}{x} + \frac{1}{a} \log x \cdot \sinh\frac{x}{a}\right]$$

$$= x^{\cosh\frac{x}{a}}\left(\frac{1}{x}\cosh\frac{x}{a} + \frac{1}{a}\log x \cdot \sinh\frac{x}{a}\right) \quad \#$$

(c) $y = x^{\cosh^2 \frac{x}{a}}$

logging on both sides we get,

$\log(y) = \cosh^2 \frac{x}{a} \cdot \log(x)$.

or, $\frac{d\{\log y\}}{d(x)} = d\left\{\cosh^2 \frac{x}{a} \cdot \log(x)\right\}$

or, $\frac{1}{y} \cdot \frac{dy}{d(x)} = \cosh^2 \frac{x}{a} \cdot \frac{d\{\log(x)\}}{d(x)} + \log(x) \cdot \frac{d\{\cosh^2 \frac{x}{a}\}}{d(x)}$.

or, $\frac{1}{y} \cdot \frac{dy}{d(x)} = \cosh^2 \frac{x}{a} \cdot \frac{1}{x} + \log x \cdot 2 \cosh \frac{x}{a} \cdot \sinh \frac{x}{a} \cdot \frac{1}{a}$.

or, $\frac{dy}{d(x)} = x^{\cosh^2 \frac{x}{a}} \left[\frac{1}{x} \cosh^2 \frac{x}{a} + \frac{1}{a} \log x \cdot \sinh \frac{2x}{a}\right]$

(d) $y = x^{\sin^2 \frac{x}{a}}$.

logging on both sides we get,

$\log(y) = \log\left(x^{\sin^2 \frac{x}{a}}\right)$

or, $\log(y) = \sin^2 \frac{x}{a} \cdot \log x$.

Diff. b.s. w.r to 'x', we get,

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or, $\frac{1}{y}\frac{\log(y)}{d(x)} = \frac{d\{\sin\frac{x^2}{a} \cdot \log x\}}{d(x)}$

$= \sin\frac{x^2}{a} \cdot \frac{d\{\log x\}}{d(x)} + \log x \cdot \frac{d\{\sin\frac{x^2}{a}\}}{d(x)}$

$= \sin\frac{x^2}{a} \cdot \frac{1}{x} + \log x \cdot 2\sin\frac{x}{a} \cdot \cos\frac{x}{a} \cdot \frac{1}{a}$

$= y\left[\frac{1}{x}\sin\frac{x^2}{a} + \frac{1}{a}\log x \cdot \sin 2\frac{x}{a}\right]$

$= x^{\sin^2 x/a}\left[\frac{1}{x}\sin\frac{x^2}{a} + \frac{1}{a}\log x \cdot \sin 2\frac{x}{a}\right]$ A.

(d) $y = x^{\tan^{-1}\frac{x}{3}}$

logging on both side we get,

$\log(y) = \log(x)^{\tan^{-1}\frac{x}{3}}$

or, $\log(y) = \tan^{-1}\frac{x}{3} \cdot \log(x)$.

diff. b.s. w.r.to 'x' we get,

$\frac{1}{y}\cdot\frac{d(y)}{d(x)} = \tan^{-1}\frac{x}{3} \cdot \frac{d\{\log(x)\}}{d(x)} + \log x \cdot \frac{d\{\tan^{-1}\frac{x}{3}\}}{d(x)}$.

$= y\left[\tan^{-1}\frac{x}{3} \cdot \frac{1}{x} + \frac{\log x}{\sqrt{9-x^2}}\right]$

$= x^{\tan^{-1}\frac{x}{3}}\left[\frac{1}{x}\tanh^{-1}\frac{x}{3} + \frac{\log x}{\sqrt{9-x^2}}\right]$

6 (a)

$$y = \left(\sinh\frac{x}{a}\right)^{x^2}$$

taking log on both sides.

$$\log y = \log\left\{\sinh\frac{x}{a}\right\}^{x^2}$$

or $\log y = x^2 \cdot \log\left(\sinh\frac{x}{a}\right)$.

Diff. b.s. w.r.to 'x' we get,

$$\frac{d\{\log y\}}{d(y)} \times \frac{d(y)}{d(x)} = x^2 \frac{d\left\{\log\left(\sin\frac{x}{a}\right)\right\}}{d\left\{\sin\frac{x}{a}\right\}} \times \frac{d\left(\sin\frac{x}{a}\right)}{d(x)} + \log\left(\sinh\frac{x}{a}\right) \cdot \frac{d(x^2)}{d(x)}$$

or $\frac{dy}{dx} = y\left[\frac{x^2}{a}\cdot\coth\frac{x}{a} + 2x\cdot\log\left(\sinh\frac{x}{a}\right)\right]$

or $\frac{dy}{dx} = \left(\sinh\frac{x}{a}\right)^{x^2}\left[\frac{x^2}{a}\cdot\coth\frac{x}{2} + 2x\cdot\log\left(\sinh\frac{x}{a}\right)\right]$ #

(b) $y = \left(\cosh\frac{x}{a}\right)^{\log x}$.

taking log on both sides we get,

$\log y = \log\left\{\cosh\frac{x}{a}\right\}^{\log x}$.

or, $\log y = \log x \cdot \log\left(\cosh\frac{x}{a}\right)$

Diff b.s.w.r. to 'x' we get,

$\frac{1}{y}\frac{dy}{dx} = \log x \cdot \frac{d}{dx}\left\{\log\left(\cosh\frac{x}{a}\right)\right\} + \log\left(\cosh\frac{x}{a}\right)\cdot\frac{d\{\log x\}}{dx}$

or, $\frac{1}{y}\cdot\frac{dy}{dx} = \log x \cdot \frac{1}{\cosh\frac{x}{a}} \cdot \sinh\frac{x}{a} \cdot \frac{1}{a} + \log\left(\cosh\frac{x}{a}\right)\cdot\frac{1}{x}$.

$= \left(\cosh\frac{x}{a}\right)^{\log x}\left[\frac{1}{a}\log x \cdot \tanh\frac{x}{a} + \frac{1}{x}\log\left(\cosh\frac{x}{a}\right)\right]$ #

(c) $y = (\sinh x)^{\tanh x}$.

logging on both sides we get,

$\log y = \log\{(\sinh x)^{\tanh x}\}$

or, $\log y = \tanh x \cdot \log(\sinh x)$

diff. b.s. w.r. to 'x' we get,

$$\frac{1}{y}\cdot\frac{dy}{dx} = \tanh x \cdot \frac{d\{\log(\sinh x)\}}{d(x)} + \log(\sinh x)\cdot\frac{d\{\tanh x\}}{d(x)}$$

or, $\frac{1}{y}\cdot\frac{dy}{dx} = \tanh x \cdot \frac{1}{\sinh x}\cdot\cosh x + \log(\sinh x)\cdot \operatorname{sech}^2 x$.

or, $\frac{dy}{dx} = [(\sinh x)^{\tanh x}]\{\tanh x \cdot \coth x + \log(\sinh x)\cdot \operatorname{sech}^2 x\}$

or, $\frac{dy}{dx} = [(\sinh x)^{\tanh x}]\{1 + \log(\sinh x)\cdot \operatorname{sech}^2 x\}$ #

(d) $y = (\log x)^{\sinh x}$.

logging on both sides we get,

$\frac{dy}{dx} = d\{(\log x)^{\sinh x}\}$

(d) $y = (\log x)^{\sinh x}$.

Logging on both sides we get,

$\log y = \log \{\log x\}^{\sinh x}$.

or, $\log y = \sinh x \cdot \log \{\log x\}$

Diff. b.s. w.r.to 'x' we get,

$\frac{1}{y}\frac{d(y)}{d(x)} = \sinh x \cdot \frac{d\{\log(\log x)\}}{d(x)} + \log\{\log x\} \cdot \frac{d\{\sinh x\}}{d(x)}$.

or $\frac{1}{y}\frac{d(y)}{d(x)} = \sinh x \cdot \frac{1}{\log x \cdot x} + \log(\log x) \cdot \cosh x$

or, $\frac{d(y)}{d(x)} = y\left[\frac{\sinh x}{x \cdot \log x} + \log(\log x) \cdot \cosh x\right]$

$= (\log x)^{\sinh x}\left[\frac{\sinh x}{x \cdot \log x} + \log(\log x) \cdot \cosh x\right]$ //

(e) $y = (\cosh x)^{\sinh^{-1} x}$.

Logging on both sides, we get,

$\log y = \log \{\cosh x\}^{\sinh^{-1} x}$.

or, $\log y = \sinh^{-1} x \cdot \log \{\cosh x\}$.

Diff. b.s.w.r.to 'x' we get,

$$\frac{1}{y}\frac{d(y)}{d(x)} = \sinh^{-1}x \cdot \frac{d\{\log(\cosh x)\}}{d(x)} + \log(\cosh x)\cdot\frac{d(\sinh^{-1}x)}{d(x)}$$

or, $$\frac{1}{y}\frac{d(y)}{d(x)} = \sinh^{-1}x \cdot \frac{1}{\cosh x}\sinh x + \log(\cosh x)\cdot\frac{1}{\sqrt{1+x^2}}$$

or, $$\frac{d(y)}{d(x)} = y\left[\sinh^{-1}x \cdot \tanh x + \frac{\log(\cosh x)}{\sqrt{1+x^2}}\right]$$

$$= (\log x)^{\sin \sinh^{-1}x}$$

$$= (\cosh x)^{\left[\sinh^{-1}x\cdot\tanh x + \frac{\log(\cosh x)}{\sqrt{1+x^2}}\right]} \#$$

(f) $y = (\cosh^{-1}x)^{\sinh x}$.

logging on both sides we get,

$\log(y) = \log$

$$\frac{d(y)}{d(x)} = \frac{d\{(\cosh^{-1}x)^{\sinh x}\}}{d(x)}$$

or,

(f) $y = (\cosh^{-1}x)^{\sinh x}$

logging on both sides we get,

$\log y = \log(\cosh^{-1}x)^{\sinh x}$

or, $\log y = \sinh x \cdot \log(\cosh^{-1}x)$

Diff. b.s.w.r.to 'x' we get,

$\frac{1}{y}\cdot\frac{d(y)}{d(x)} = \sinh x \cdot \frac{d\{\log(\cosh^{-1}x)\}}{d(x)} + \log(\cosh^{-1}x)\frac{d(\sinh x)}{d(x)}$

or, $\frac{1}{y}\frac{d(y)}{d(x)} = \sinh x \cdot \frac{1}{\cosh^{-1}x}\frac{1}{\sqrt{x^2-1}} + \log(\cosh^{-1}x)\cdot\cosh x$

or, $\frac{d(y)}{d(x)} = y\left[\frac{\sinh x}{\cosh^{-1}x\sqrt{x^2-1}} + \log(\cosh^{-1}x)\cdot\cosh x\right]$

$= \left[(\cosh^{-1}x)^{\sinh x}\right]\left[\frac{\sinh x}{\cosh^{-1}x\sqrt{x^2-1}} + \log(\cosh^{-1}x)\cdot\cosh x\right]$ #

(g) $y = (\tan x)^{\cosh^{-1}3x}$.

logging on both sides, we get,

$\log y = \log(\tan x)^{\cosh^{-1}3x}$.

or, $\log y = \cosh^{-1}3x \cdot \log(\tan x)$

Diff. b.s.w.r.to 'x' we get,

$\frac{1}{y}\cdot\frac{d(y)}{d(x)} = \cosh^{-1}3x \cdot \frac{d\{\log(\tan x)\}}{d(x)} + \log(\tan x)\cdot\frac{d\{\cosh^{-1}3x\}}{d(x)}$

$$\frac{1}{y}\frac{d(y)}{d(x)} = \cosh^{-1}3x \cdot \frac{1}{\tan x}\sec^2 x + \log(\tan x)\frac{1}{\sqrt{9x^2-1}}$$

or, $$\frac{d(y)}{d(x)} = y\left[\frac{\cosh^{-1}3x \cdot \sec^2 x}{\tan x} + \frac{3\log(\tan x)}{\sqrt{9x^2-1}}\right]$$

$$= (\tan x)^{\cosh^{-1}3x}\left(\frac{2\cosh^{-1}3x}{\sin x \cdot \cos x} + \frac{3\log(\tan x)}{\sqrt{9x^2-1}}\right)$$

(h) $$y = \left(\sin\frac{x}{a} + \cosh\frac{x}{a}\right)^{nx}.$$

Diff. ~~b.s.w.r.to~~ logging on both sides, we get,

$$\log y = \log\left\{\sin\frac{x}{a} + \cosh\frac{x}{a}\right\}^{nx}.$$

or, $$\log y = nx \cdot \log\left\{\sin\frac{x}{a} + \cosh\frac{x}{a}\right\}$$

Diff. b.s.w.r. to 'x' we get,

$$\frac{1}{y}\cdot\frac{d(y)}{d(x)} = nx \cdot d\left\{\log\left(\sin\frac{x}{a} + \cosh\frac{x}{a}\right)\right\} + \log\left(\sin\frac{x}{a} + \cosh\frac{x}{a}\right)\cdot\frac{d\{nx\}}{d(x)}$$

or, $$\frac{1}{y}\cdot\frac{d(y)}{d(x)} = \frac{nx \cdot \left(\cos\frac{x}{a}\cdot\frac{1}{a} + \sinh\frac{x}{a}\cdot\frac{1}{a}\right)}{\sin\frac{x}{a} + \cosh\frac{x}{a}} + \log\left(\sin\frac{x}{a} + \cosh\frac{x}{a}\right)\underbrace{n}_{\text{constant}}$$

or, $$\frac{d(y)}{d(x)} = \left(\sin\frac{x}{a} + \cos\frac{x}{a}\right)^{nx}\left\{\frac{\frac{nx}{a}\left(\cos\frac{x}{a}\cdot\sinh\frac{x}{a}\right)}{\left(\sin\frac{x}{a}\cdot\cosh\frac{x}{a}\right)} + n\log\left|\sin\frac{x}{a} + \cosh\frac{x}{a}\right|\right\}$$

Ex: 12.1

1. a) $\sinh 3x$.

Diff. both side with respect to 'x' we get,

$$\frac{dy}{dx} = \frac{d(\sinh 3x)}{d(x)} = 3\cosh 3x.$$

b) $y = \tanh \frac{x}{2}$

Diff. both side with respect to 'x', we get,

$$\frac{dy}{dx} = \frac{d(\tanh \frac{x}{2})}{d(\frac{x}{2})} \times \frac{d(\frac{x}{2})}{d(x)}$$

$$= \frac{1}{2} \operatorname{sech}^2 \frac{x}{2}$$

c) $y = \coth x^2$.

D.h.s.w.r to 'x' we get,

$$\frac{dy}{dx} = \frac{d(\coth x^2)}{d(x^2)} = -2x \operatorname{cosech}^2 x^2$$

Imp

d) $y = \operatorname{sech}^2 3x$.

D.h.s.w.r to 'x' we get,

$$\frac{dy}{dx} = \frac{d(\operatorname{sech}^2 3x)}{d(\operatorname{sech} 3x)} \times \frac{d(\operatorname{sech} 3x)}{d(3x)} \times \frac{d(3x)}{d(x)},$$

$$= 2 \operatorname{sech} 3x \times -3e$$

(e) sol, $y = \cosh^3 \frac{2x}{3}$

Diff b.s. w.r.to 'x', we get,

$$\frac{dy}{dx} = \frac{d(\cosh \frac{2x}{3})^3}{d(\cosh \frac{2x}{3})} \times \frac{d(\cosh \frac{2x}{3})}{d(\frac{2x}{3})} \times \frac{d(\frac{2x}{3})}{d(x)}$$

$$= 3\cosh^2 \frac{2x}{3} \cdot \sinh \frac{2x}{3} \cdot \frac{2}{3}$$

$$= \cosh \frac{2x}{3} \left(2 \sinh \frac{2x}{3} \cdot \cosh \frac{2x}{3} \right)$$

$$= \cosh \frac{2x}{3} \cdot \sinh \frac{4x}{3}$$

(f) $\cosh^{-1} \frac{x}{3}$

Diff b.s. w.r.to 'x', we get,

$$\frac{dy}{dx} = \frac{d(\cosh^{-1} \frac{x}{3})}{d(\frac{x}{3})} \times \frac{d(\frac{x}{3})}{d(x)}$$

$$= \frac{1}{\sqrt{\frac{x^2}{9} - 1}} \times \frac{1}{3}$$

$$= \frac{3}{\sqrt{x^2 - 9}} \times \frac{1}{3}$$

$$= \frac{1}{\sqrt{x^2 - 9}}$$ Ans