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classmate Date Page

Rotational Dynamics

Group 'B'

Q.No.11a. What do you mean by moment of inertia?

Answer: Moment of inertia of a particle is mathematically defined as the product of it's mass and square of distance from the axis of rotation.

b. State the principle of conservation of angular momentum.

Answer: The principle of conservation of angular momentum states that "the total angular momentum of a system remains constant if ~~not~~ net external torque acting on it is zero."

c. Calculate the new revolution per minute if some of mass 0.02 kg dropped gently on the disc. 0.08 m. from the

Answer: Given,

$I_1 = 5 \times 10^{-4}\ kgm^2$

$f_1 = 40\ rpm = \frac{2}{3}\ rps$

mass $(m) = 0.02\ kg$

$r = 0.08\ m$

$I_2 = I_1 + I_m$

$= 5 \times 10^{-4} * 0.02(0.08)^2$

We know:

$I_1 \omega_1 = I_2 \omega_2$

$5 \times 10^{-4} \times 2\pi \times \frac{2}{3} = (5 \times 10^{-4} + 0.02(0.08)^2 .$

$\therefore f_2 = 0.53\ rps$ //

Q.NO.3.b) If no internal torque acts on a body, will its angular velocity remain constant? Give reason.

Answer:

When internal torque is absent, angular velocity will not remain constant.

when, torque $(\tau) = 0$

$$\frac{dL}{dt} = 0$$

$$\Rightarrow L = \text{Constant} \quad // \text{ where, } L \rightarrow \text{angular momentum}$$

$$Lw = \text{constant}$$

$$\Rightarrow w \propto \frac{1}{I}$$

c) The kinetic energy of a shaft rotating at a constant angular velocity of 22 $rads^{-1}$ is 7.80 J. find its angular momentum.

Answer: Given,

$K.E. = 7.8\,J$

$w = 22\ rads^{-1}$

Angular momentum $(L) = ?$

we know,

$$K.E = \frac{1}{2} I w^2$$

$$\text{or, } K.E = \frac{1}{2} wL \quad (L = Iw)$$

$$\therefore L = 0.708\ kg\,m^2 s^{-1} //$$

Q.No 7.a) What is angular momentum? Write it's SI units and dimensions.

Answer: The moment of momentum about an axis of rotation is called angular momentum.

It's SI unit is, $kg\ m^2 s^{-1}$.

$[L] = [ML^2T^{-1}]$

b) Does the moment of inertia become zero as soon as it stops rotating? Explain.

Answer: No, the moment of inertia doesn't become zero as soon as it stops as it is purely a function of how mass is distributed.

Q.No 10.a) Why is it that a long jumper keeps his body long and straight before the jump, but a diver keeps his body slightly curled up?

Answer: Because it decreases moment of inertia and the diver spins faster in mid air and when a long jumper keeps his body long, it increases it's moment of inertia and thus decrease its angular speed which results in a safer landing.

b) Derive relation between torque applied and angular acceleration produced in a rigid body and hence define moment of inertia.

Answer:

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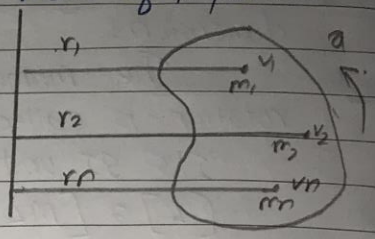
Let us consider a rigid body made of n-particle.
of masses $m_1, m_2 \dots m_n$

Let distances $r_1, r_2, \dots r_n$.



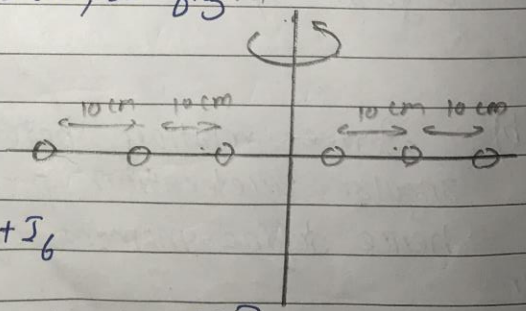
we know,
Torque = sum of torques of acting on all particles.

$$\tau = (m_1 r_1 \alpha) r_1 + (m_2 r_2 \alpha) r_2 + \dots + (m_n r_n \alpha) r_n$$
$$= m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + \dots + m_n r_n^2 \alpha$$
$$= \alpha (m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2)$$
$$= \alpha \Sigma m r^2$$
$$= \alpha I \quad \text{// where } I = \Sigma m r^2 \text{ is moment of inertia. //}$$

$$\therefore \tau = I\alpha$$

Long Question Answer

Q.No.1) Six small washers are spaced 10 cm apart on a rod of negligible mass and 0.5 m in length. The mass of each washer is 20g. The rod rotates about an axis located at 25 cm, as fig.



8. What is the moment of inertia of the system?

Answer.

$$I = I_1 + I_2 + I_3 + I_4 + I_5 + I_6$$

$$= \frac{20}{1000} \times 2 \left\{ \left(\frac{3}{100}\right)^2 + \left(\frac{15}{100}\right)^2 + \left(\frac{25}{100}\right)^2 \right\}$$

$$= 0.0035 \text{ kg m}^2 \text{ //}$$

b. If the system two washers closest to the axis are removed, what is the moment of inertia of the remaining four washers?

Answer:

$$I = 0.04 \times \left[\left(\frac{15}{100}\right)^2 + \left(\frac{25}{100}\right)^2 \right]$$

$$= 0.034 \text{ kgm}^2 //$$

C. If the system with six washers rotates at 5 rev/s. What is its rotational kinetic energy?

Answer:

$$f = 5 \text{ revs}^{-1}$$

$$K.E = \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} \times 1 \times (2\pi 5)^2 \quad // \omega = 2\pi f //$$

$$= 1.73 \text{ J} //$$

Q.NO.2.C) A horizontal disc rotating freely about a vertical axis makes 90 rev/min. A small piece of putty of mass 2×10^{-2} kg falls vertically on to the disc and sticks to it at a distance of 6×10^{-2} m from the axis. Calculated M.I of disc.

Answer:

$$f_1 = 90 \text{ rev/min} = \frac{9}{6} \text{ rps}$$

$$f_2 = 80 \text{ rev/min} = \frac{8}{6} \text{ rps}$$

$$m = 2 \times 10^{-2} \text{ kg}, \quad r = 6 \times 10^{-2} \text{ m}$$

we know,

$$I_1 \omega_1 = I_2 \omega_2$$

$$I_1 \times 2\pi \frac{9}{6} = \left(I_1 + 2 \times 10^{-2} \times (6 \times 10^{-2})^2 \right) 2\pi \frac{8}{6}$$

$$\therefore I_1 = 5.76 \times 10^{-4} \text{ kgm}^2 //$$

Numerical (NPP)

Q.No.4.b) Calculate the moment of inertia of a rod of length 10cm and mass 200gram about to perpendicular axis passing through its centre.

Answer: Given,

$m = 0.2\,kg$

$l = 0.1\,m$

$I = ?$

$$I = \frac{1}{2} ml^2$$

$$= \frac{1}{2} \times 0.2 \times (0.1)^2$$

$$= 1.67 \times 10^{-4}\ kgm^2$$

for k

$$k = \frac{\sqrt{3}.l}{6}$$

$$= \frac{\sqrt{3} \times 0.01}{6}$$

$$= 0.0288\ m$$

Numerical Problem

Q.No.1). A disc of radius 1m and mass 5 kg is rolling along a horizontal plane. Its moment of inertia about its centre is $2.5\ kgm^2$. If its velocity along the plane is $2\ ms^{-1}$, find its angular velocity and the total energy

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Answer: Given,

$r = 1m$, $m = 5kg$, $I = 2.5\ kgm^2$.

$\omega = ?$

Total K.E = ?

$V = r\omega$

or, $2 = 1 \times \omega$

$\omega = 2\ rads^{-1}$

Total K.E $= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

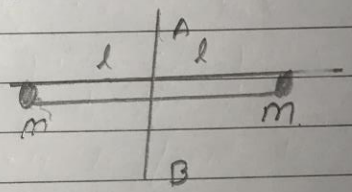
$= \frac{1}{2} \times (5) \times 2^2 + \frac{1}{2} \times (2.5) \cdot 2^2$.

$= 15J$ //

Q.no.2). find the moment of inertia of the system. about AB.

Answer.

If 'I' be the total moment of inertia.



$I = ml^2 + ml^2 + \frac{1}{2}m(4l)^2$

$= \frac{7}{3}ml^2$

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$$\text{or, } K.E = \frac{1}{2} \omega L \quad (L = I\omega)$$

$$\therefore L = 0.708\ kg\,m^2s^{-1} //$$