Ex: 13.4

(Tangent and Normal).

Q.No 1) A particles moves in straight line. It's eqn is.

$s = 2t^2 + 5t - 4$

Answer: diff. b.s. w.r to 't' we get,

$\frac{d(s)}{d(t)} = 2 \times 2 \times t + 5 - 4$

$= 2 \times 2 \times 6 + 5 - 4$

$= 29\ m/s$

for accl^2

$\frac{d(v)}{d(t)} = 4t + 5 - 4$

$= 4\ m/s^2$

Q.No 5) Displacement of particle varies with time by relation $x = -15t^2 + 20t + 30$. find velocity and accl^2.

Answer: diff. b.s. w.r. to 't' we get,

$\frac{d(x)}{d(t)} = -15 \times 2 \times t - 20.$

$= -15 \times 2 \times \frac{1}{2} - 20$

$= .$

for accl^2

$\frac{d(v)}{d(t)} = \frac{d(-15 \times 2 \times t - 20)}{dt}$

$= -30\ m/s^2$ //

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2.a) Side of square sheet is increasing at rate 5 cm/min find rate of increase in area when l is 12 cm.

⟹ we know that,

$$A = l^2$$

or, $\dfrac{d(A)}{d(t)} = 2l \cdot \dfrac{d(l)}{d(t)}$

or, $\dfrac{5x}{100\times 60} = 2\times\left(\dfrac{12}{100}\right)\times\dfrac{d(l)}{d(t)}$

or, $\dfrac{d(l)}{d(t)} = 120\ cm^2/min$

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b) A stone thrown into a pond produces a circular ripples which expands from point of impact. find ~~a~~ rate of area increase?

⟹ we know that,

$$A = \pi r^2$$

~~a~~ Diff. b.s. w.r to 't' we get,

$$\frac{d(A)}{d(t)} = \cancel{\ } \pi 2r \cdot \frac{d(r)}{d(t)}$$

or $= \dfrac{22}{7}\times 2\times 15\times 3.5$

$= 330\ cm^2/sec$.

3\. a) from a cylindrical drum and leaking rate 2 cm/min. find rate of volume decreased.

→

$$V = \pi r^2 h$$

Diff. b.s. w.r to 't' we get,

$$\frac{d(V)}{d(t)} = \pi 2r^2 \cdot \frac{d(h)}{d(t)}$$ // Here radius is constant //

$$= \frac{22}{7} \times 2 \times 10.5 \times 10.5 \times \frac{2}{60}$$

$$= 693 \text{ cm}^3/\text{min}$$

b) Water is poured into a right cirIular cylinder of radius 8 cm at the rate of 18 cu.cm/min. find rate of increase in water level.

→

$$V = \pi r^2 h$$

Diff. b.s. w.r. to 't' we get,

$$\frac{d(V)}{d(t)} = \pi r^2 \frac{d(h)}{d(t)}$$

or, $$18 = \frac{22}{7} \times 8 \times 8 \times \frac{d(h)}{d(t)}$$

or, $$\frac{d(h)}{d(t)} = \frac{9}{32\pi} \text{ cm/min}$$

c) Gasoline is pumped into a vertical cylinderical tank at rate 24.cu.cm/min. |r = 9cm|. find rate surface rising.

Answer:

$$V = \pi r^2 h$$

diff. b.s. w.r to 't' we get,

$$\frac{d(V)}{d(t)} = \pi r^2 \frac{d(h)}{d(t)}$$

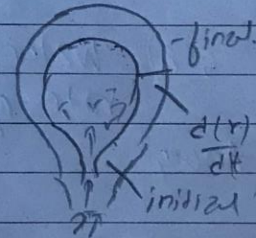
or, $$24 = \pi (9)^2 \frac{d(h)}{d(t)}$$

or, $$\frac{d(h)}{d(t)} = \frac{8}{27\pi} \text{ cm/min}$$

4.a) A spherical balloon is inflated at the rate of 18 cu.cm/sec. At what rate is radius increasing?

Answer:

$$V = \frac{4}{3}\pi r^3$$



or, d'diff. b.s. w.r. to 't' we get,

$$\frac{d(V)}{d(t)} = \frac{4}{3}\pi\, 3r^2 \frac{d(r)}{d(t)}$$

Or, $$18 = \frac{4}{3}\pi\, 3 \cdot \times 8 \times 8 \cdot \frac{d(r)}{d(t)}$$

Or, $$\frac{d(r)}{d(t)} = \frac{9}{128\pi} \text{ cm/sec.}$$

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b) A spherical ball of salt is dissolving so that rate volume ∝ Area. prove radius is decreasing at constant rate.

Answer: We know that,

$$-\frac{d(v)}{d(t)} \propto A$$

Since volume is decreasing

or, $\frac{d(v)}{d(t)} = -kA$ (proportional constant)

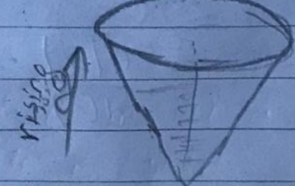
or, $\frac{d\left(\frac{4}{3}\pi r^3\right)}{d(t)} = -k.4\pi r^2$

or, $\frac{4}{3}\cancel{\pi}\cdot 3\cancel{r^2}\frac{dr}{dt} = -k\cdot\cancel{4}\cdot\cancel{\pi r^2}$

or, $\frac{dr}{dt} = k$ (since all the value are constant)

5.a) Water flows into an inverted conical tank at the rate of 4.4 cu.cm/sec. find rate of height increased.

Answer, $r = \frac{1}{3}h$ ~~or~~



$V = \cancel{\frac{1}{3}\pi r^2 h}\ \frac{1}{3}\pi r^2 h$

$\cancel{\frac{d(v)}{d(t)} = \frac{1}{3}\pi\, 2r\cdot\frac{d(r)}{d(t)}}$ (note: here radius is not constant so we convert it to get 1 variable only)

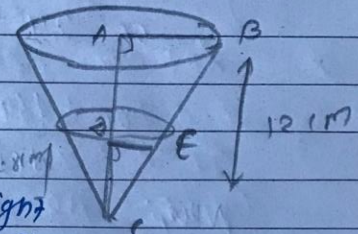
$\frac{d(v)}{d(t)} = \frac{1}{3}\frac{\pi h^3}{\times 9}$

$\frac{d(v)}{d(t)} = \frac{1}{3}\frac{\pi}{9}\times 3\cdot(3.5)(3.5)\cdot\frac{d(h)}{d(t)}$

or, $4.4 = \frac{1}{3} \times \frac{22}{7} \times \frac{3}{9} \times 3.5 \cdot 3.5 \cdot \frac{d(h)}{d(t)}$

or, $\frac{d(h)}{d(t)} = \frac{36}{35}$ cm/sec.

(b) Water flows into an inverted conical tank. Calculated level rising of water? Assume $h = 12$cm and radius of top is 6cm.



In ΔABC and DCE since the Δ are simillar we can write,

$\frac{AC}{CD} = \frac{AB}{DE}$ //Here radius and height both are not constant so we did it to make one variable//

or, $\frac{12}{h} = \frac{6}{r}$

or, $r = h/2$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \frac{h^2}{4} h$

Diff. b.s. w.r.to 't' we get,

$\frac{d(V)}{d(t)} = \frac{1}{3}\pi \frac{3h^2}{4} \frac{d(h)}{d(t)}$

or, $\frac{d(h)}{d(t)} = \frac{24 \times 3 \times 4 \times 7}{22 \times 3 \times 8 \times 8}$

$= \frac{21}{8\pi}$ cm/sec

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6. If the volume of the expanding cube is increasing at the rate of 24 cm^3/min. how fast its T.S.A increasing?

Answer: we know that,

$$v = l^3$$

or, diff. b.s. w.r to 't' we get,

$$\frac{d(v)}{d(t)} = 3l^2 \frac{d(l)}{d(t)}$$

$$\text{or, } 24 = 3 \times 6 \times 6 \times \frac{d(l)}{d(t)}$$

$$\text{or, } \frac{d(l)}{d(t)} = \frac{2}{9} \text{ cm/min}$$

Now,

$$\therefore T.S.A\ (A) = 6l^2$$

$$\text{or, } 216 = 6l^2$$

$$\text{or, } l^2 = 36$$

$$\text{or, } l = 6 \text{ cm}$$

Again,

$$A = 6l^2$$

diff. b.s. w.r to 't' we get,

$$\frac{d(A)}{d(t)} = 6.2.l.\frac{d(l)}{d(t)}$$

$$= 12 \times (6) \times \frac{2}{9}$$

$$= 16 \text{ cm}^2/\text{min}.$$

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7) Two concentric circles are expanding in such a way that the radius of the inner circle is increasing at the rate of 10 cm/sec. ~~rate~~ find rate of area between them (copy from book).

7) ~~Two concentric circles are expanding in such a way~~

Let r_1 and r_2 be the radius of circle at any time t.

Given,

$$r_1 = 24\ cm,\ r_2 = 30\ cm.$$

$$\frac{dr_1}{d(t)} = 10\ cm/sec,\ \frac{dr_2}{d(t)} = 7\ cm/sec.$$

Area between the circle $(A) = \pi r_2^2 - \pi r_1^2$

$$= \pi(r_2^2 - r_1^2)$$

diff. b.s. w.r. to 't' we get,

$$\frac{d(A)}{d(t)} = \pi\left[2r_2 \cdot \frac{dr_2}{d(t)} - 2r_1 \frac{dr_1}{d(t)}\right]$$

$$= \pi\left[2 \cdot 30 \cdot 7 - 2 \times 24 \times 10\right]$$

$$= -60\pi\ cm^2/sec$$

negative represents the area is decreasing

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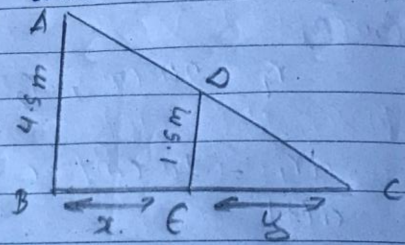
Q.No.8). A man of height 1.5m walks away from a lamp post of height 4.5m at rate 20 cm/sec. How fast is the shadow lengthening when the man is 42 cm from the post?



Answer:

To find : $\frac{d(y)}{d(t)}$

Let x and y be the distance between lamp. post and a length of ~~sa~~ shadow at time (t) respectively.

Given, $AB = 4.5m$

$DE = 1.5\,m$

$BE = 42\,cm$

$\triangle ABC \sim \triangle DEC$

$\frac{AB}{DE} = \frac{BC}{EC}$

or, $\frac{\cancel{450}^{3}}{\cancel{150}} = \frac{x+y}{y}$

or, $3y = x+y$

or, $2y = x.$

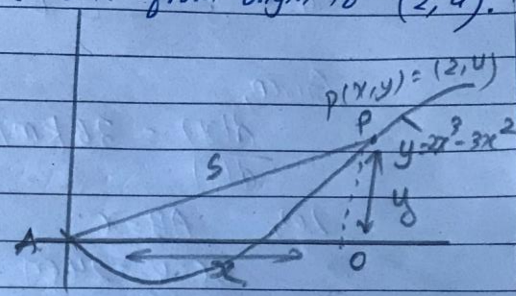
Diff. b.s. w. r. to 't' we get,

$2 \cdot \frac{d(y)}{d(t)} = \frac{d(x)}{d(t)}.$

or, $\frac{d(y)}{d(t)} = \frac{\cancel{20}^{10}}{\cancel{2}} = 10\ cm/sec$ //

[very hard]

Q.NO.9) A point is moving along the curve $y = 2x^3 - 3x^2$ in such a way that its x-cordinate is increasing at the rate of 2cm/sec. find rate distance from origin to (2, 4).



To find :

$$\frac{d(s)}{d(t)} = ?$$

Here, In ΔAOP

$$s^2 = x^2 + y^2$$

or, $s^2 = 2^2 + 4^2$

or, $s = 2\sqrt{5}$

In right angled ΔAOP

$$s^2 = x^2 + y^2$$

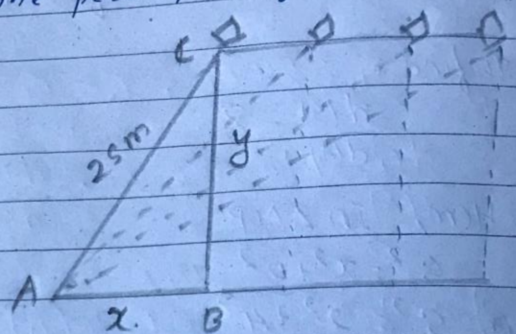
or, $s^2 = x^2 + (2x^3 - 3x^2)^2$

or, $2s \cdot \frac{d(s)}{d(t)} = 2x \cdot \frac{d(x)}{d(t)} + 2(2x^3 - 3x^2)(6x^2 - 6x)\frac{dx}{dt}$

or, $2 \times 2\sqrt{5} \frac{d(s)}{d(t)} = 2(2) \cdot 2 + 2(2 \cdot (2)^3 - 3 \cdot 2^2)(6 \cdot 2^2 - 6 \cdot 2) 2$

or, $\frac{d(s)}{d(t)} = 10\sqrt{5}$ cm/sec //

Q. No. 10 a) A kite is 24 m high and there are 25 m of cord out. If the kite moves horizontally at the rate of 36 km/hr directly away from the person, how fast is cord out.



Given,

$$\frac{d(x)}{d(t)} = 36\ km/hr$$

Let, $AB = x$

$BC = y = 24\ m$

$AC = 25\ m = s$

In ΔABC,

$$AC^2 = AB^2 + BC^2$$

$$(25)^2 = (AB)^2 + (24)^2$$

or, $AB = 7\ m$ //

In ΔABC,

$$AC^2 = AB^2 + BC^2$$

or, $s^2 = x^2 + y^2$

or, $2s \cdot \frac{d(s)}{d(t)} = 2x \cdot \frac{dx}{d(t)}$ // since vertical height is not changing, y is constant //

or, $\frac{d(s)}{d(t)} = \frac{2 \times 7 \times 36}{25}$

or, $= 10.08\ km/hr$ //

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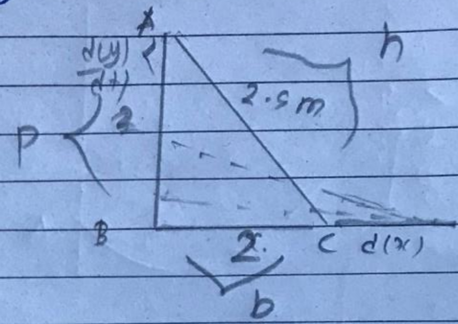
b) A 2.5 m ladder leans against a vertical wall. If the top slides downwards at the rate of 12 cm/sec. find speed of lower end when it is 2m from the wall.



In ΔABC.

$h^2 = p^2 + b^2$

or, $(2.5)^2 = (2)^2 + p^2$

or, $6.25 - 4 = p^2$

or, $p = 1.5$.

Now, we know that,

Since length of ladder is constant — $h^2 = p^2 + b^2$

diff. b.s. w. r. to. 't' we get,

$$0 = 2p \cdot \frac{d(p)}{d(t)} + 2 \cdot b \cdot \frac{d(b)}{d(t)}$$

or, $$-2 \times 1.5 \cdot 12 = 2 \cdot 2 \cdot \frac{d(b)}{d(t)}$$

or, $$\frac{d(b)}{d(t)} = 9 \text{ cm/sec}$$