Chapter 13 (Ex: 13.1)

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Application of Derivatives

1. Using L Hospital find,

a) $\lim_{x \to 2} \frac{x^3-8}{x^2-4}$ ($\frac{0}{0}$ form)

$= \lim_{x \to 2} \frac{3x^{\cancel{2}}}{2\cancel{x}}$

$= \frac{3 \times \cancel{2}}{\cancel{2}} = 3$ //

b)

$\lim_{x \to 1} \frac{x^4-3x^3+2}{2x^3-5x^2+3x}$ ($\frac{0}{0}$ form)

$= \lim_{x \to 1} \frac{4x^3-9x^2 \cancel{+2)}}{6x^2-10x+3}$

$= \frac{4(1)^3-9(1)^2}{6(1)^2-10 \times 1+3}$

$= \frac{4-9}{6-10+3}$

$= \frac{\cancel{-}5}{\cancel{-1}}$

$= 5$ //

c) $\lim_{x \to 0} \frac{e^x - x - 1}{x^2}$ $\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x \to 0} \frac{e^x - 1}{2x}$ $\left(0/0 \text{ form}\right)$

$= \lim_{x \to 0} \frac{e^x}{2}$

$= \frac{1}{2}$ //

d) $\lim_{x \to 0} \frac{e^x + e^{-x} - 2\cos x}{\sin^2 x}$ $\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x \to 0} \frac{e^x - e^{-x} + 2\sin x}{2\sin x \cdot \cos x}$ $\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x \to 0} \frac{e^x + e^{-x} + 2\cos x}{2\cos 2x}$

$= \frac{1 + 1 + 2}{2 \times 1}$

$= \frac{\cancel{4}\,2}{\cancel{2}}$

$= 2$ //

For L Hospital Rule
either $\frac{0}{0}$ form or $\frac{\infty}{\infty}$ form

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(e) $\lim_{x \to 1} \frac{1 - 2x + x^2}{1 + \log x - x}$ $\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x \to 1} \frac{-2 + 2x}{x^{-1} \frac{1}{x} - 1}$ $\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x \to 1} \frac{2}{-x^{-2}}$

$= -2(1)^2$

$= -2$ //

f) $\lim_{x \to 0} \frac{\tan ax}{\tan bx}$ $\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x \to 0} \frac{a \sec^2 ax}{b \sec^2 bx}$ $\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x \to 0} \frac{a \cos^2 bx}{b \cos^2 ax}$

$= \frac{a}{b}$ //

g) $\lim_{x \to 0} \frac{x^2 - \sin^2 x}{x \sin x}$ ($\frac{0}{0}$ form)

$= \lim_{x \to 0} \frac{2x - 2\sin x \cdot \cos x}{x\cos x + \sin x}$ ($\frac{0}{0}$ form)

$= \lim_{x \to 0} \frac{2x - \sin 2x}{x\cos x + \sin x}$ ($\frac{0}{0}$ form)

$= \lim_{x \to 0} \frac{2 - 2\cos 2x}{-x\sin x + \cos x}$

$= \frac{0}{2} = 0$

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h) $\lim_{x \to 0} \frac{\tan x - x}{x - \sin x}$ ($\frac{0}{0}$ form)

$= \lim_{x \to 0} \frac{\sec^2 x - 1}{1 - \cos x}$ ($\frac{0}{0}$ form)

$= \lim_{x \to 0} \frac{\tan^2 x}{1 - \cos x}$ ($\frac{0}{0}$ form)

$= \lim_{x \to 0} \frac{2\tan x . \sec^2 x}{\sin x.}$

$= \lim_{x \to 0} \frac{2 . \cancel{\sin x}.}{\cos x . \cancel{\sin x} \cos^2 x}$

$= 2$

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i) $$\lim_{x \to 0} \frac{x - \sin x \cdot \cos x}{x^3} \quad \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x \to 0} \frac{\cancel{1} - \cancel{2}\cos \cancel{2}x}{\cancel{2x^2}} \quad \frac{1 + \sin x \cdot \cos x}{3x^2}$$

$$= \lim_{x \to 0} - \cos x \cdot s$$

i) $$\lim_{x \to 0} \frac{x - \sin x \cos x}{x^3} \quad \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x \to 0} \frac{1 - \{-\sin x \cdot \sin x + \cos x \cdot \cos x\}}{3x^2} \quad \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x \to 0} \frac{1 - \cos 2x}{3x^2}$$ // Concept Here we converted into $\cos\theta$. since, $\cos 0$ is one //

$$= \lim_{x \to 0} \frac{\cancel{2} \sin 2x}{\underset{3}{\cancel{6}} x} \quad \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x \to 0} \frac{2 \cos 2x}{3}$$

$$= \frac{2}{3}$$ //

j) $\lim_{x\to 0} \frac{(e^x-1)\tan x}{x^2}$ $\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x\to 0} \frac{(e^x-1)\sec^2 x + \tan x\{e^x\}}{2x}$ $\left(\frac{0}{0} \text{ form}\right)$

(2.) find the limiting values.

a) sol,

$\lim_{x\to\infty} \frac{2x^2+3x}{1+5x^2}$ $\left(\frac{\infty}{\infty} \text{ form}\right)$

$= \lim_{x\to\infty} \frac{4x+3}{10x}$ $\left(\frac{\infty}{\infty} \text{ form}\right)$

$= \lim_{x\to\infty} \frac{4}{10}$

$= \frac{2}{5}$ #

b) $\lim_{x \to \infty} \frac{3x^2-5}{2x^3+4x+3}$ $\left(\frac{\infty}{\infty} \text{ form}\right)$

$= \lim_{x \to \infty} \frac{6x}{3x^2+4}$ $\left(\frac{\infty}{\infty} \text{ form}\right)$

concept

$= \lim_{x \to \infty} \frac{\cancel{6}}{\cancel{6}x}$ $\left(\text{Here } \frac{1}{\infty} = 0\right)$

$= 0$ #

c)

$\lim_{x \to \pi/2} \frac{\sec 7x}{\sec 5x}$ $\left(\frac{\infty}{\infty} \text{ form}\right)$

$= \lim_{x \to \pi/2} \frac{\cos 5x}{\cos 7x}$ $\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x \to \pi/2} \frac{\cancel{+}5 \sin 5x}{\cancel{+}7 \sin 7x}$

$= \frac{1 \times 5}{(-1) \times 7}$ concept // Here value of $\sin 630$ is -1 //

$= \frac{-5}{7}$ //

d) $\lim_{x \to 0} \frac{\log \sin x}{\cot x}$ $\left(\frac{\infty}{\infty} \text{ form}\right)$

$= \lim_{x \to 0} \frac{\frac{1}{\sin x} \cdot \cos x}{-\operatorname{cosec}^2 x}$ $\left(\frac{\infty}{\infty} \text{ form}\right)$

$= \lim_{x \to 0} -\sin x \cdot \cos x$

$= 0$ //

e) $\lim_{x \to 0} \frac{\log x}{\log \cot x}$

$= \lim_{x \to 0} \frac{\frac{1}{x} \cdot \cot x}{-\operatorname{cosec}^2 x}$ (concept Here chain rule is in use)

$= \lim_{x \to 0} \frac{1 \cdot \cos x \cdot \sin^2 x}{x \cdot \sin x}$

$= \lim_{x \to 0} \frac{\sin 2x}{2x}$

$= \lim_{x \to 0} \frac{2 \cos 2x}{2}$

$= 1$ //

f) $$\lim_{x \to \infty} \frac{x^4}{e^x} \left(\frac{\infty}{\infty} \text{ form}\right)$$

$$= \lim_{x \to \infty} \frac{4x^3}{e^x} \left(\frac{\infty}{\infty} \text{ form}\right)$$

$$= \lim_{x \to \infty} \frac{12x^2}{e^x} \left(\frac{\infty}{\infty} \text{ form}\right)$$

$$= \lim_{x \to \infty} \frac{24x}{e^x} \left(\frac{\infty}{\infty} \text{ form}\right)$$

$$= \lim_{x \to \infty} \frac{24}{e^x}$$

$$= \frac{24}{\infty}$$

$$= 0 \;//$$

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g) $$\lim_{x \to \pi/2} \frac{\tan 5x}{\tan x} \left(\frac{\infty}{\infty}\right)$$

$$= \lim_{x \to \pi/2} \frac{5\sec^2 5x}{\sec^2 x} \left(\frac{\infty}{\infty}\right)$$

$$= \lim_{x \to \pi/2} \frac{5\cos^2 x}{\cos^2 5x} \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x \to \pi/2} \frac{\cancel{5} \times \cancel{2} \cos x \cdot (-\sin x)}{\cancel{5} \times \cancel{2} \cdot \cos 5x\,(-\sin 5x)} \left(\frac{0}{0} \text{ form}\right)$$

concept
// do not put 2 //

$$= \lim_{x \to \pi/2} \frac{2\sin 2x}{\sin 10x}$$

$$= \lim_{x \to \pi/2} \frac{2\cos 2x}{10\cos 10x}$$

$$= \frac{1}{5} \#$$