NUMERICAL ANALYSIS

Q.NO 1) A particle executing SHM has maximum velocity of 40cms⁻¹ and a maximum acceleration of 50cms⁻². find its amplitude and period of oscilliation.

Answer: Given,

$v_{max} = 40\ cms^{-1} = 40 \times 10^{-2}\ m/s$

$a_{max} = 50\ cms^{-2} = 50 \times 10^{-2}\ m/s^2$

$a_{max} = \omega^2 A$

or, $50 \times 10^{-2} = \omega^2 A \quad \cdots (i)$

$v_{max} = \omega A \quad \cdots (ii)$

equating we get,

$\frac{50 \times 10^{-2}}{40 \times 10^{-2}} = \frac{\omega^2 A}{\omega A}$

or, $\omega = 1.25$ m

Time of oscillation (T) =

$\omega = 2\pi T$

or, $\frac{7 \times 1.25}{2 \times 22} = T$

or, $T = 5.02$ seconds

(Amplitude) $a_{max} = \omega^2 A$

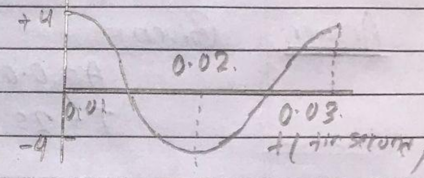
or, $\frac{50 \times 10^{-2}}{(1.25)^2} = A$

or, $A = 0.32\ m$

Q.No.2) The velocity-time diagram of a harmonic oscillator is shown. find frequency.

Answer:

By observation, we know that.



To complete one oscillation it takes 0.04 sec.

$\therefore$ So, the time period $(T) = 0.04$ sec

$\therefore$ The frequency is $(f) = \frac{1}{T} = \frac{1}{0.04} = 25$ HZ

Q.No.3) Some springs are combined in series and parallel arrangement. find the ratio of frequencies.

Answer:

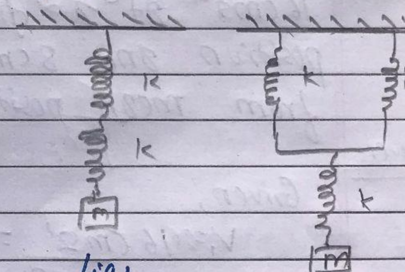


fig 1 fig 2

from fig 1.

$$T_1 = 2\pi\sqrt{\frac{2m}{K}}$$

from fig 2.

$$T_2 = 2\pi\sqrt{\frac{3m}{2K}}$$

Now,

Equating ratios, we get,

$$\frac{T_1}{T_2} = \sqrt{\frac{2\times2}{3}} = \frac{2}{\sqrt{3}}$$

$$\therefore \frac{f_1}{f_2} = \frac{\sqrt{3}}{2}$$

So, the ratio is $\sqrt{3} : 2$

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Q.no.4) In a pendulum the amplitude is 0.05 m and period is 2s. find V_{max}.

Answer: Given,

$A = 0.05 m$

$T = 2s$

$\omega = \frac{2\pi}{T} = \frac{2 \times 22}{7 \times 2} = 3.1428$

$V_{max} = \omega A = 3.1428 \times 0.05 = 0.1571 ms^{-1}$!

Q.no.5) The velocity of a particle describing SHM is $16 cms^{-1}$ at a distance of 8 cm from mean position and $8 cms^{-1}$ at a distance of 12 cm from mean position. Calculate its amplitude.

Answer:

Given,

$V_1 = 16 cms^{-1} = 16 \times 10^{-2} m/s$

$x_1 = 8 \times 10^{-2} m$

$V_2 = 8 cms^{-1} = 8 \times 10^{-2} m/s$

$x_2 = 12 \times 10^{-2} m$

from first parameters. or reading.

$V_1 = \omega \sqrt{A^2 - (8 \times 10^{-2})^2}$

or, $16 \times 10^{-2} = \omega \sqrt{A^2 - (8 \times 10^{-2})^2}$ --- (i)

from second we get,

$V_2 = \omega \sqrt{A^2 - (12 \times 10^{-2})^2}$

or, $8 \times 10^{-2} = \omega \sqrt{A^2 - (12 \times 10^{-2})^2}$ ... (ii)

Equating (i) and (ii) we get,

$$2\ \frac{\cancel{16\times10^{-2}}}{\cancel{8\times10^{-2}}} = \frac{\cancel{\omega}\sqrt{A^2-(8\times10^{-2})^2}}{\cancel{\omega}\sqrt{A^2-(12\times10^{-2})^2}}$$

Or, $2A^2 - 2(12\times10^{-2})^2 = A^2 - (8\times10^{-2})^2$

Or, $A^2 = (8\times10^{-2})^2 + 2(12\times10^{-2})^2$

Or, $A = \sqrt{0.0352}$

$= 18.76\ cm$

Q.No.6) A particle executes SHM of period 8 seconds. After what time of its passing through mean position will the energy be half kinetic and half potential?

Answer:

$$\omega = \frac{2\pi}{8} = \frac{\pi}{4}\ rads^{-1}$$

Let 't' be the time after which the energy will be half and half potential.

Since, motion starts from mean position, $y = A\sin\omega t$

$$\frac{dy}{dt} = v = A\cos\omega t \cdot \omega$$

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m.A^2\omega^2\cos^2\omega t.$$

/Here the value of this expression must be half of total energy//.

$\therefore \frac{1}{2}.\quad K.E = \frac{1}{2}T.E$

Or, $\frac{1}{2}mA^2\omega^2\cos^2\omega t = \frac{1}{2}\left(\frac{1}{2}m\omega^2A^2\right)$

$\therefore \cos\omega t = \frac{1}{\sqrt{2}}$ Or, $\omega t = \frac{\pi}{4}$ $\therefore t = \frac{\pi}{4}.\omega$

Or, $\cancel{\omega t} = \frac{1}{\sqrt{2}}$.

$= \left(\frac{\pi}{4}\right)\times\left(\frac{4}{\pi}\right)$

$= 1\ sec$

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Q.No.7) The time period of oscillation of a mass m suspended by an ideal spring is 2s. If an additional mass of 2kg be suspended, the time period is increased by 1s. find the value of m.

Answer:

Time period of a mass $(t_1) = 2 sec.$

Time period after adding additional mass $(t_2) = 3 sec.$

We know that,

$$T = 2\pi \sqrt{\frac{m}{K}}$$

$T \propto \sqrt{M}$ (since it is directly proportional)

$$\frac{T_1}{\sqrt{m_1}} = \frac{T_2}{\sqrt{m_2}}$$

or, $\frac{2}{\sqrt{m}} = \frac{3}{\sqrt{m+2}}$ since the mass is increased by 2 kg.

or, $4m + 8 = 9m$

or, $m = \frac{8}{5}$

or, $m = 1.6$ kg //

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Q.No.8) If the length of a pendulum is decreased by 2%. find the gain or loss in time per day. (shortcut method)

Answer:

$T \propto \sqrt{l}$

$\Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l} = \frac{0.02}{2} = 0.01$ (Here, since time is $\propto$ length. so, the change in ratio are also propertional)

$\Delta T = 0.01 T$

~~Loss~~) Gain in time per day $= 0.01 \times 24 \times 60 \times 60$
$= 864 \text{ sec}$

Q.No.9) If the acceleration due to gravity on the moon is one-sixth of that on the Earth, what will be the length of a second pendulum?

Answer: Given,
Since the pendulum is second ($T = 2$)

Now, $T = 2\pi\sqrt{\frac{l}{g}}$

or, $\cancel{2} = \cancel{2} \times \frac{22}{7} \times \sqrt{\frac{l}{g_{moon}}}$

or, $\left(\frac{7}{22}\right)^2 \times \frac{9.8}{6} = l$

or, $l = 0.165 \text{ m} = 0.165 \times 100 \text{ cm} = 16.5 \text{ cm}$

$\therefore$ ~~$l=2\pi$~~