Capacitated Vehicle Routing Problem: Exact Formulation

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Formulation with Subtour Elimination

The parameters are, for each edge $e \in E$, there is an associated edge cost C_e. For each node $v \in V$, there is an associated demand d_v. For each vehicle, there is an associated capacity q. In the formulation, $\delta^+(i)$ denote the forward star of vertex i, i.e. the set of all the vertices that can be reached from vertex i and the $\delta^-(i)$ denote the reverse star of vertex i, i.e. the set of all the vertices from which vertex i can be reached.

The decision variable,

$$X_{ij}^k = \begin{cases} 1 & \text{if the route } k \text{ uses the arc } (i,j) \\ 0 & \text{otherwise} \end{cases}$$

$$Min \sum_{k=1}^{T} \sum_{(i,j)\in E} C_{ij} X_{ij}^k \tag{1}$$

$$S.T. \sum_{k=1}^{T} \sum_{j\in\delta^+(i)} X_{ij}^k = 1 \ \ \forall i \in V \setminus \{0\} \tag{2}$$

$$\sum_{j\in\delta^+(i)} X_{ij}^k - \sum_{j\in\delta^-(i)} X_{ji}^k = 0 \ \ \forall i \in V \setminus \{0\}, k = 1, ..., T \tag{3}$$

$$\sum_{i\in\delta^-(0)} X_{i0}^k \leq 1 \ \ \forall k = 1, ..., T \tag{4}$$

$$\sum_{i\in\delta^+(0)} X_{0i}^k \leq 1 \ \ \forall k = 1, ..., T \tag{5}$$

$$\sum_{i\in V} d_i * \sum_{j\in delta^+(i)} X_{ij}^k \leq q_k \ \ \forall k = 1, ..., T \tag{6}$$

$$\sum_{(i,j)\in E(S)} X_{ij}^k \leq |S| - 1 \ \ \forall S \subset V\{0\}, k = 1, ..., T \tag{7}$$

$$X_{ij}^k \in \{0, 1\} \tag{8}$$

Here, (2) denotes the constraints that all the vertices except the depot 0 has to be visited exactly once. (3) is the flow conservation constraint which says if a vehicle goes into a vertex, it must come out. (4) and (5) denote that each route or vehicle has to go into the depot and come out of the depot once. (6) denotes the sum of the demands of the nodes has to be less that the capacity of the vehicle visiting them. (7) denotes the subtour elimination constraints.