EE2703 A6: The Laplace Transform

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1 Introduction

In this assignment, we will look at how to analyze "Linear Time-invariant Systems" using the scipy.signal library in Python .We limit our analysis to systems with rational polynomial transfer functions. More specifically we consider 3 systems: A forced oscillatory system, A coupled system of Differential Equations and an RLC low pass filter

2 Assignment Questions

2.1 Time response of a spring system

Consider the forced oscillatory system given by the equation(with 0 initial conditions):

$$\ddot{x} + 2.25x = f(t) \tag{1}$$

where

$$f(t) = cos(1.5t)e^{-0.5t} * u(t) \tag{2}$$

Solving for $X(s)$ in Laplace domain we get,

$$X(s) = \frac{s + 0.5}{(s + 0.5^2) + 2.25)(s^2 + 2.25)} \tag{3}$$

Use the impulse response of $X(s)$ to get its inverse Laplace transform.

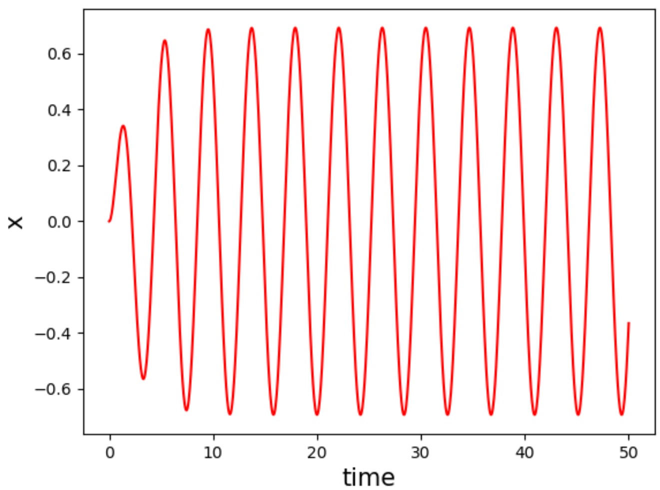

Figure 1: System Response with Decay = 0.5

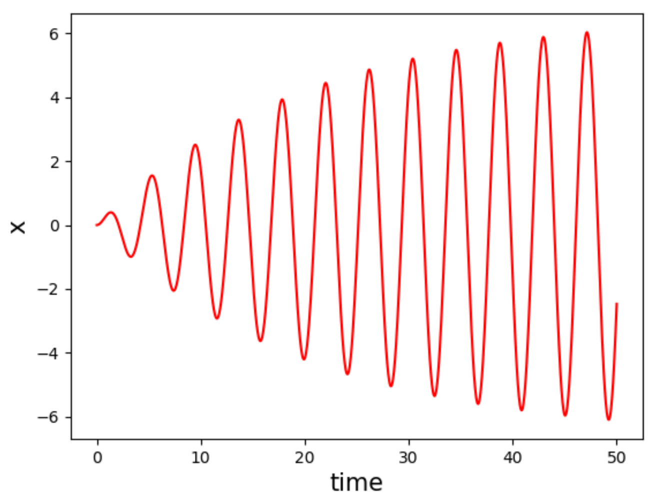

Figure 2: System Response with Decay = 0.5

2.2 Response over different frequencies

Model the system as an LTI system,the following graphs are obtained by varying the frequency of the force $f(t)$. From the given equation, we can see that the

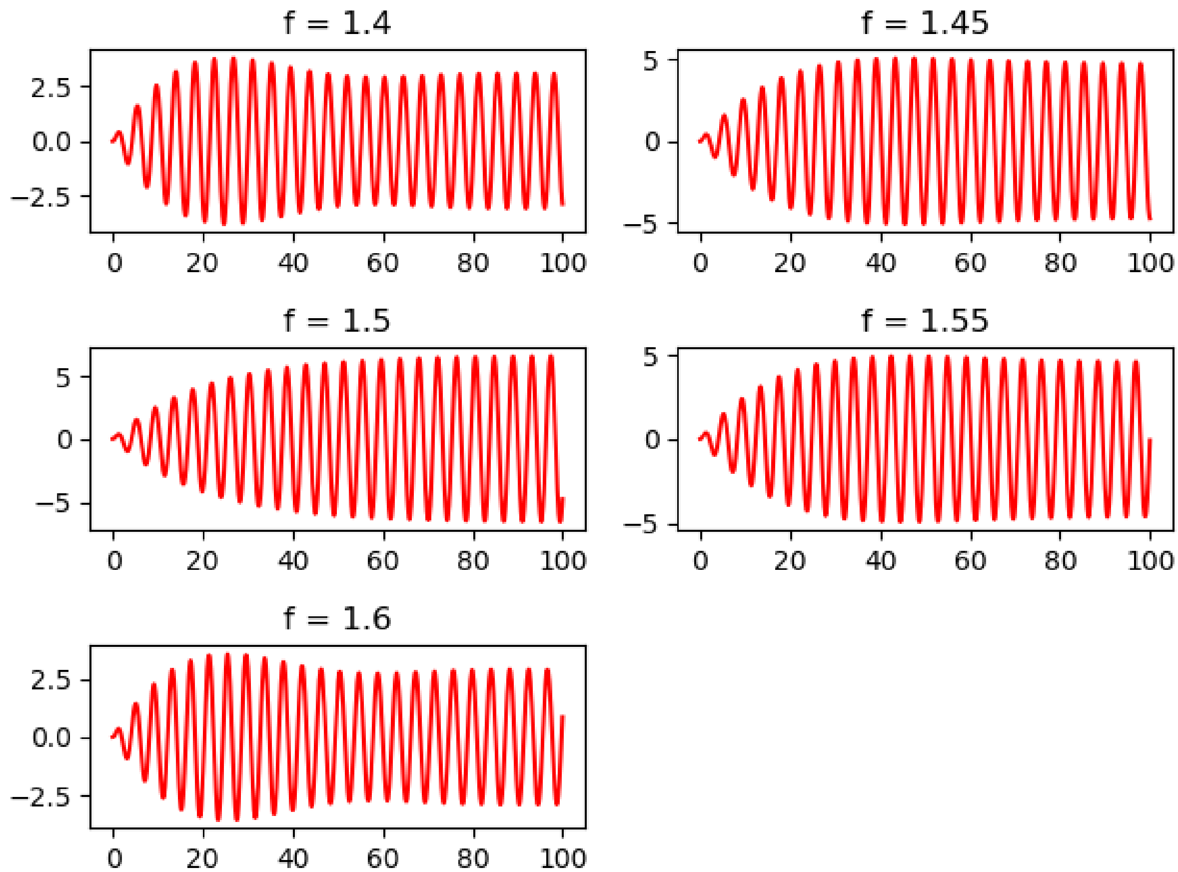

Figure 3: System Response with frequencies from 1.4 to 1.6

natural response of the system has the frequency w = 1.5 rad/s . Thus, as expected the maximum amplitude of oscillation is obtained when the frequency of $f(t)$ is 1.5 rad/s, due to resonance.

2.3 The coupled spring problem

We now consider a coupled Differential system

$$\ddot{x} + (x - y) = 0 \tag{4}$$

and

$$\ddot{y} + 2(y - x) = 0 \tag{5}$$

with the initial conditions: $\dot{x}(0) = 0, \dot{y}(0) = 0, x(0) = 1, y(0) = 0$. Taking Laplace Transform and solving for $X(s)$ and $Y(s)$, We get:

$$X(s) = \frac{s^2 + 2}{s^3 + 3s} \tag{6}$$

$$Y(s) = \frac{2}{s^3 + 3s} \tag{7}$$

We notice that the outputs of this system are 2 sinusoids which are out of phase. This system can be realized by creating an undamped single spring double mass system.

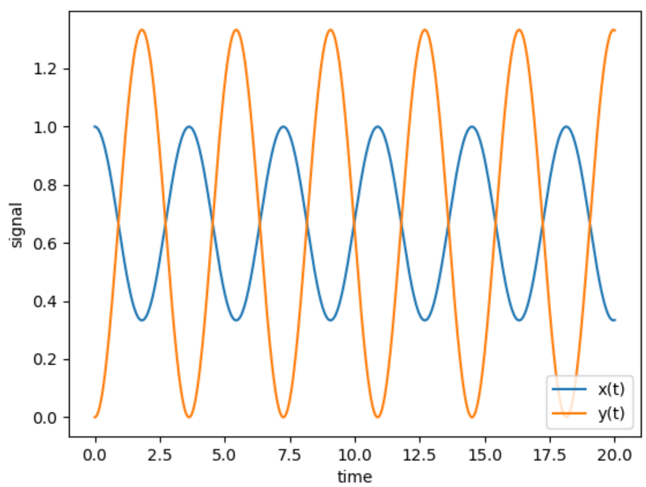

Figure 4: Coupled Oscillations

2.4 The Two-Port Network

The Steady-State transfer function of the given circuit is given by

$$H(s) = \frac{10^6}{s^2 + 100s + 10^6} \tag{8}$$

The magnitude and phase response are as follows:

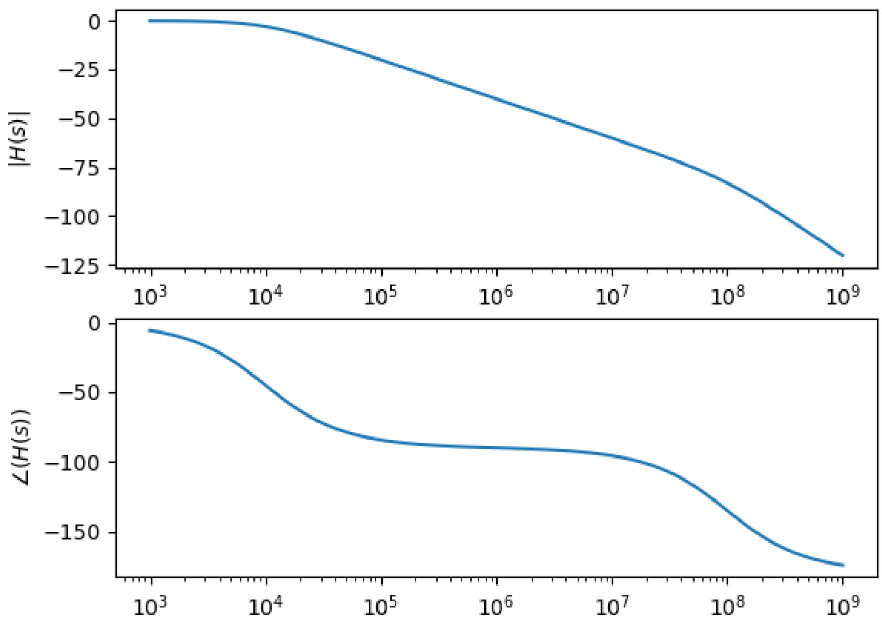

Figure 5: Bode Plots For RLC Low pass filter

Plot the response of the low pass filter to the input:

$$V_i(t) = (cos(10^3 t) - cos(10^6 t))u(t)$$

for $0 < t < 30\mu s$ and $0 < t < 30ms$

From the Bode plot of $H(s)$ we can see that the system acts like a low pass filter. It provides unity gain for frequency less than 10^3 rad/s. Thus the low frequency component remains same. On the other hand, the system dampens the high frequency component with $|H(s)|_{dB}$ approximately 0.

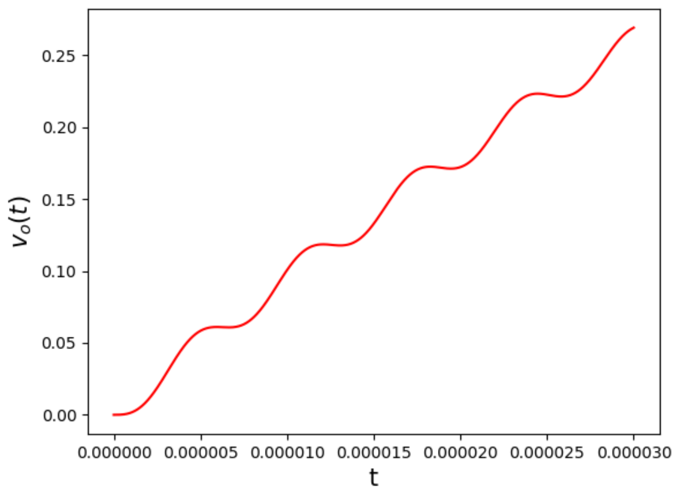

Figure 6: System response for t < 30us

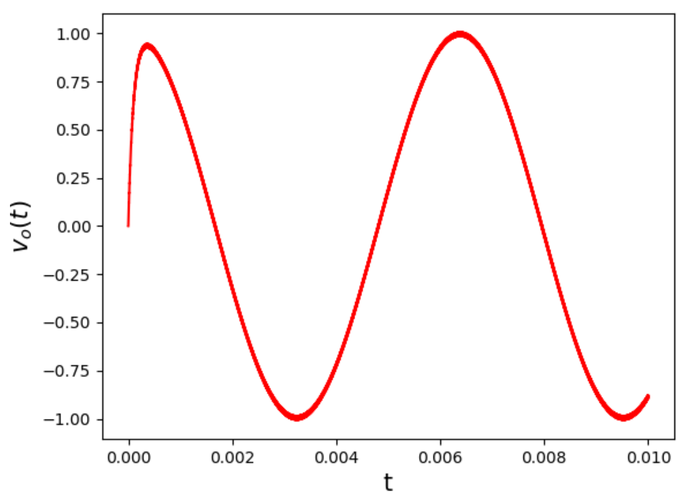

Figure 7: System response for t < 10ms

3 Conclusion

The scipy.signal library provides a useful toolkit of functions for circuit analysis. The toolkit was used for the analysis of LTI systems in various domains.
The forced response of a simple spring body system was obtained over various frequencies of the applied force, and highest amplitude was observed at resonant frequency.
A coupled spring problem was solved using the sp.impulse function to obtain two sinusoids of the same frequency.
A two-port network, functioning as a low-pass filter was analysed and the output was obtained for a mixed frequency input.