Assignment - 2

Prasanna Kumar R - SM21MTECH14001

Problem

Perpendiculars PL , PM are drawn from P (h, k) on the axes OL,OM. Show that the length of the perpendicular from P on the LM is $hk\sin^2\omega/\sqrt{h^2+k^2+2hk\cos\omega}$ and that the equation of the perpendicular is $h(x-h) = k(y-k)$

Solution

Let PQ be the perpendicular from P on LM, then Area of $\triangle$ PLM is given as ,

$$\frac{1}{2}\|\mathbf{L}-\mathbf{M}\|\|\mathbf{P}-\mathbf{Q}\| \tag{1}$$

Again, taking PM as base and L as vertex, Area of $\triangle$ PLM is given as ,

$$\frac{1}{2}\|\mathbf{P}-\mathbf{M}\|\|\mathbf{P}-\mathbf{L}\|\sin\measuredangle LPM \tag{2}$$

As $\measuredangle LPM = 180° - \omega$, by (1) and (2) we get,

$$\frac{1}{2}\|\mathbf{L}-\mathbf{M}\|\|\mathbf{P}-\mathbf{Q}\| = \frac{1}{2}\|\mathbf{P}-\mathbf{M}\|\|\mathbf{P}-\mathbf{L}\|\sin\measuredangle LPM \tag{3}$$

Here,

$$\|\mathbf{L}-\mathbf{M}\| = \sin\omega\sqrt{h^2+k^2+2hk\cos\omega}$$

$$\|\mathbf{P}-\mathbf{M}\| = k\sin\omega$$

$$\|\mathbf{P}-\mathbf{L}\| = h\sin\omega$$

$$\begin{aligned}(\sin\omega\sqrt{h^2+k^2+2hk\cos\omega})\|\mathbf{P}-\mathbf{Q}\| &= \\ k\sin\omega\ h\sin\omega\ \sin(180°-\omega) \\ \|\mathbf{P}-\mathbf{Q}\| &= \\ \frac{hk\sin^2\omega}{\sqrt{h^2+k^2+2hk\cos\omega}}\end{aligned} \tag{4}$$

Equation of LM is,

$$\begin{aligned}\frac{x}{\mathbf{OM}} + \frac{y}{\mathbf{OL}} &= 1 \\ \frac{x}{h+k\cos\omega} + \frac{y}{k+h\cos\omega} &= 1\end{aligned} \tag{5}$$

Equation of any line through (h, k) will be,

$$y - k = m(x-h) \tag{6}$$

As lines LM and PQ are perpendicular,

$$1 + \left(m - \frac{k+h\cos\omega}{h+k\cos\omega}\right)\cos\omega - m\left(\frac{k+h\cos\omega}{h+k\cos\omega}\right) = 0 \tag{7}$$

Solving the above equation,

$$m = \frac{h}{k}$$

Substituting m in (7), we get

$$\begin{aligned}(y-k) &= \frac{h}{k}(x-h) \\ \Rightarrow h(x-h) &= k(y-k)\end{aligned} \tag{8}$$

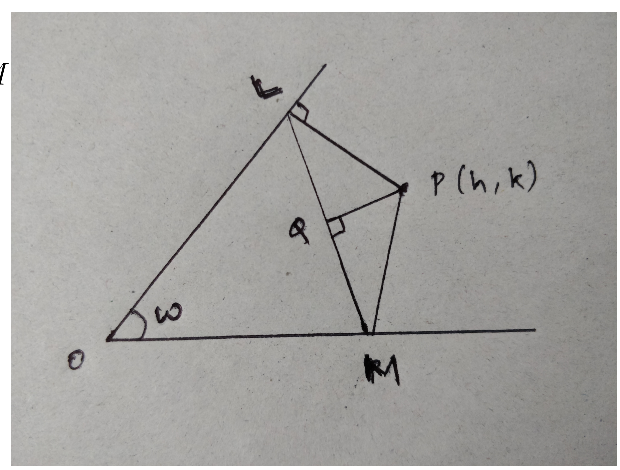

Fig. 1. Diagram