Assignment - 2

Prasanna Kumar R - SM21MTECH14001

PROBLEM

Perpendiculars PL , PM are drawn from P (h, k) on the axes OL,OM. Show that the length of the perpendicular from P on the LM is $hk\sin^2\omega/\sqrt{h^2+k^2+2hk\cos\omega}$ and that the equation of the perpendicular is $h(x-h) = k(y-k)$

SOLUTION

Let PQ be the perpendicular from P on LM, then Area of △ PLM is given as ,

$$\frac{1}{2}\|\mathbf{L}-\mathbf{M}\|\|\mathbf{P}-\mathbf{Q}\| \tag{1}$$

Again, taking PM as base and L as vertex, Area of △ PLM is given as ,

$$\frac{1}{2}\|\mathbf{P}-\mathbf{M}\|\|\mathbf{P}-\mathbf{L}\|\sin\measuredangle LPM \tag{2}$$

As $\measuredangle LPM = 180° - \omega$, by (1) and (2) we get,

$$\frac{1}{2}\|\mathbf{L}-\mathbf{M}\|\|\mathbf{P}-\mathbf{Q}\| = \frac{1}{2}\|\mathbf{P}-\mathbf{M}\|\|\mathbf{P}-\mathbf{L}\|\sin\measuredangle LPM \tag{3}$$

Here,

$$\|\mathbf{L}-\mathbf{M}\| = \sin\omega\sqrt{h^2+k^2+2hk\cos\omega}$$

$$\|\mathbf{P}-\mathbf{M}\| = k\sin\omega$$

$$\|\mathbf{P}-\mathbf{L}\| = h\sin\omega$$

$$(\sin\omega\sqrt{h^2+k^2+2hk\cos\omega})\|\mathbf{P}-\mathbf{Q}\| = k\sin\omega\ h\sin\omega\ \sin(180° - \omega)$$

$$\|\mathbf{P}-\mathbf{Q}\| = \frac{hk\sin^2\omega}{\sqrt{h^2+k^2+2hk\cos\omega}} \tag{4}$$

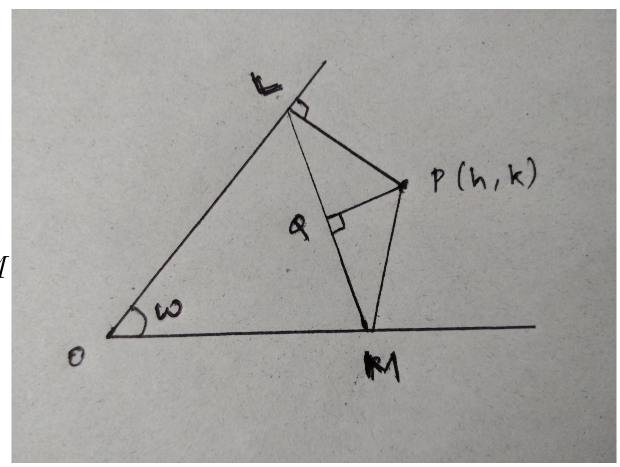

Fig. 1. Diagram