2D Composite Transformations

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Composite Transformation

- We can setup a sequence of transformations as a composite transformation matrix by calculating the product of the individual transformations

$$P' = M2 \cdot M1 \cdot P$$

$$= M \cdot P$$

2D Composite Transformations (cont.)

- Composite 2D Translations
  - If two successive translation are applied to a point P, then the final transformed location P' is calculated as

$$\mathbf{P}' = \mathbf{T}(t_{x_2}, t_{y_2}) \cdot \mathbf{T}(t_{x_1}, t_{y_1}) \cdot \mathbf{P} = \mathbf{T}(t_{x_1} + t_{x_2}, t_{y_1} + t_{y_2}) \cdot \mathbf{P}$$

$$\begin{bmatrix} 1 & 0 & t_{x2} \\ 0 & 1 & t_{y2} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & t_{x1} \\ 0 & 1 & t_{y1} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_{1x} + t_{2x} \\ 0 & 1 & t_{1y} + t_{2y} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}(t_{x_2}, t_{y_2}) \cdot \mathbf{T}(t_{x_1}, t_{y_1}) = T(t_{x_1} + t_{x_2}, t_{y_1} + t_{y_2})$$

2D Composite Transformations (cont.)

- Composite 2D Rotations

$$\mathbf{P}' = \mathbf{R}(\theta_1 + \theta_2) \cdot \mathbf{P}$$

$$\begin{bmatrix} \cos\Theta_2 & -\sin\Theta_2 & 0 \\ \sin\Theta_2 & \cos\Theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\Theta_1 & -\sin\Theta_1 & 0 \\ \sin\Theta_1 & \cos\Theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\Theta_1 + \Theta_2) & -\sin(\Theta_1 + \Theta_2) & 0 \\ \sin(\Theta_1 + \Theta_2) & \cos(\Theta_1 + \Theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

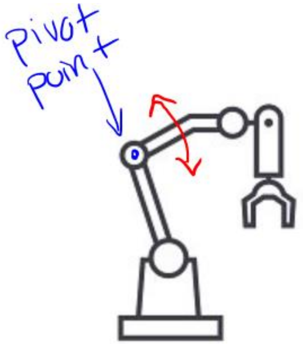
2D Composite Transformations (cont.)

- Composite 2D Scaling

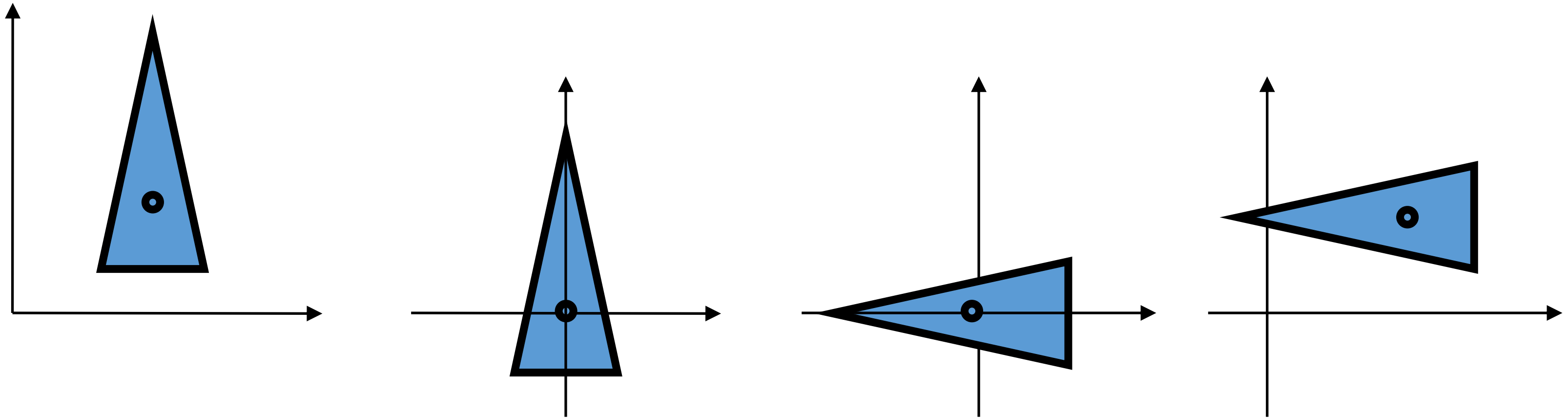
$$\mathbf{S}\left(s_{x_2}, s_{y_2}\right) \cdot \mathbf{S}\left(s_{x_1}, s_{y_1}\right) = \mathbf{S}\left(s_{x_1} \cdot s_{x_2},\ s_{y_1} \cdot s_{y_2}\right)$$

$$\begin{bmatrix} s_{2x} & 0 & 0 \\ 0 & s_{2y} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_{1x} & 0 & 0 \\ 0 & s_{1y} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} s_{1x} \cdot s_{2x} & 0 & 0 \\ 0 & s_{1y} \cdot s_{2y} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

General Pivot Point Rotation



- Steps:
  1. Translate the object so that the pivot point is moved to the coordinate origin.
  2. Rotate the object about the origin.
  3. Translate the object so that the pivot point is returned to its original position.



2D Composite Transformations (cont.)

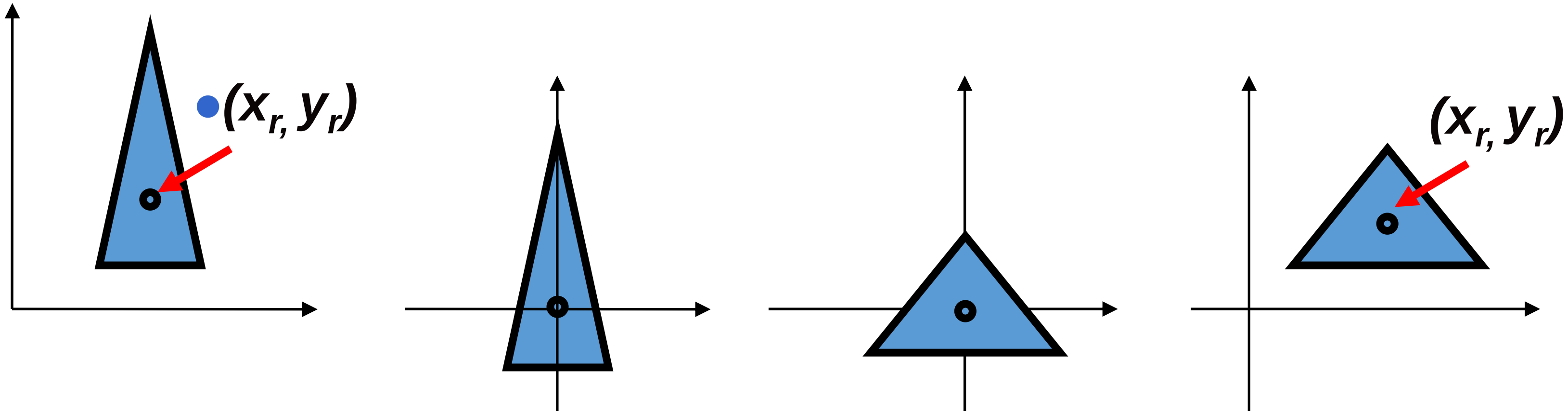
- General 2D Pivot-Point Rotation

$$\begin{bmatrix} 1 & 0 & x_r \\ 0 & 1 & y_r \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\Theta & -\sin\Theta & x_r(1-\cos\Theta) + y_r \sin\Theta \\ \sin\Theta & \cos\Theta & y_r(1-\cos\Theta) - x_r \sin\Theta \\ 0 & 0 & 1 \end{bmatrix}$$

General Fixed Point Scaling

- Steps:
  1. Translate the object so that the fixed point coincides with the coordinate origin.
  2. Scale the object about the origin.
  3. Translate the object so that the pivot point is returned to its original position.



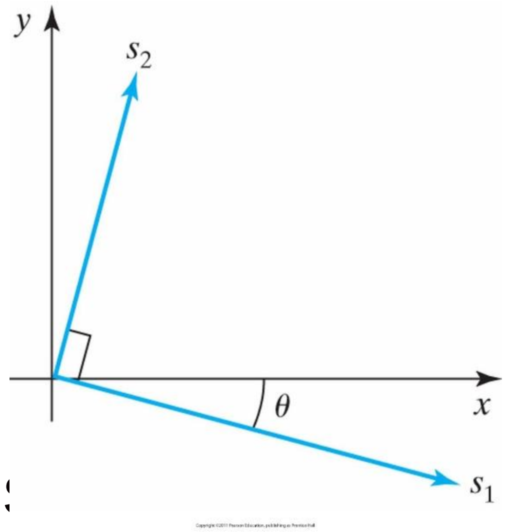
General Fixed Point Scaling (cont.)

- General 2D Fixed-Point Scaling:

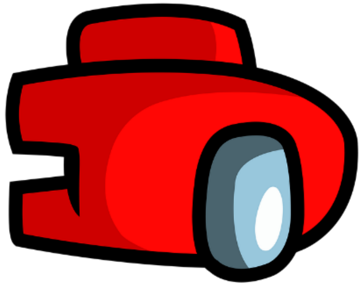
$$\begin{bmatrix} 1 & 0 & x_f \\ 0 & 1 & y_f \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 0 & -x_f \\ 0 & 1 & -y_f \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & x_f(1-s_x) \\ 0 & s_y & y_f(1-s_y) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}(x_f, y_f) \cdot \mathbf{S}(s_x, s_y) \cdot \mathbf{T}(-x_f, -y_f) = \mathbf{S}(x_f, y_f, s_x, s_y)$$

General 2D Scaling Directions



- General procedure:
  1. Rotate so that directions coincides with *x* and *y* axes
  2. Apply scaling transformation $S(s_1, s_2)$
  3. Rotate back



- The composite matrix:

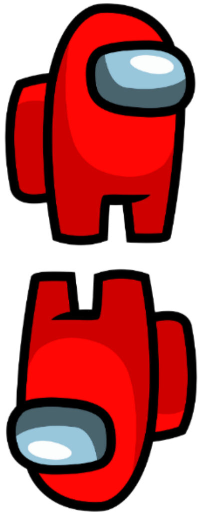
$$R^{-1}(\Theta) * S(s_1, s_2) * R(\Theta) = \begin{bmatrix} s_1 \cos^2 \Theta + s_2 \sin^2 \Theta & (s_2 - s_1) \cos \Theta \sin \Theta & 0 \\ (s_2 - s_1) \cos \Theta \sin \Theta & s_1 \sin^2 \Theta + s_2 \cos^2 \Theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrix Concatenation Properties

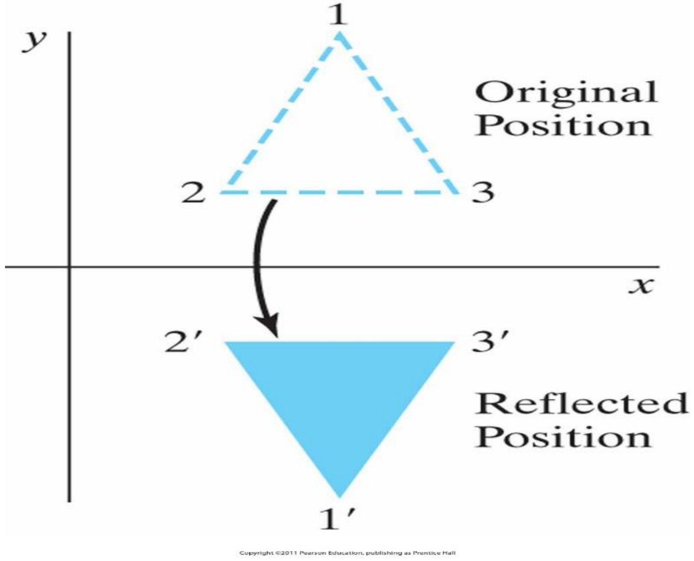
- Matrix multiplication is associative !
- $M3 \cdot M2 \cdot M1 = (M3 \cdot M2) \cdot M1 = M3 \cdot (M2 \cdot M1)$
- A composite matrix can be created by multiplying left-to-right (premultiplication) or right-to-left (postmultiplication)
- Matrix multiplication is not commutative !
  - $M2 \cdot M1 \neq M1 \cdot M2$
- But:
  - Two successive rotations
  - Two successive translations
  - Two successive scaling
- are commutative!

Other 2D Transformations

- Reflection
  - Transformation that produces a mirror image of an object
  - Image is generated relative to an axis of reflection by rotating the object 180° about the reflection axis
  - Reflection about the line y=0 (the x axis)

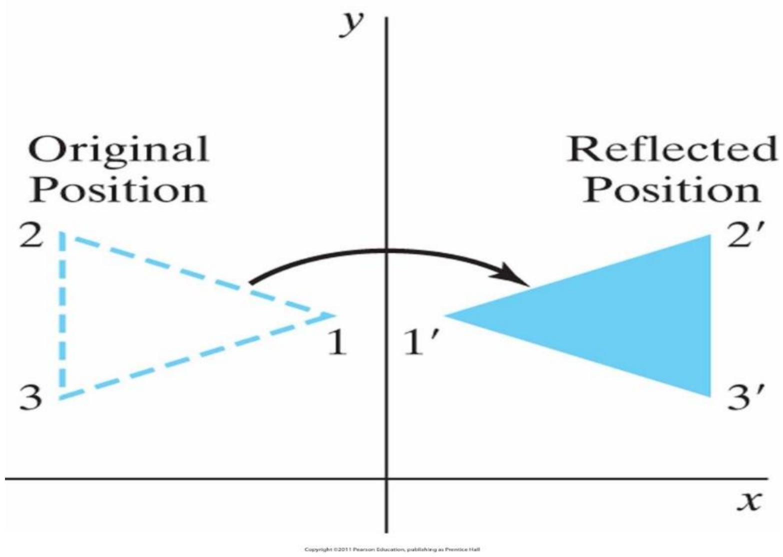


$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



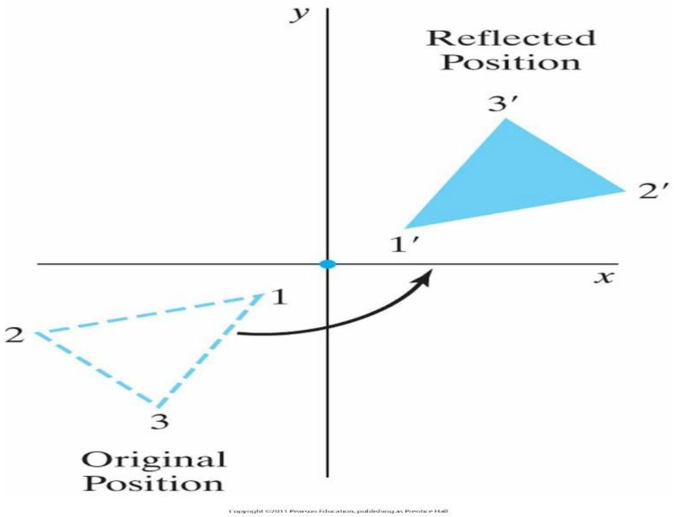
- Reflection about the line x=0 (the y axis)

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



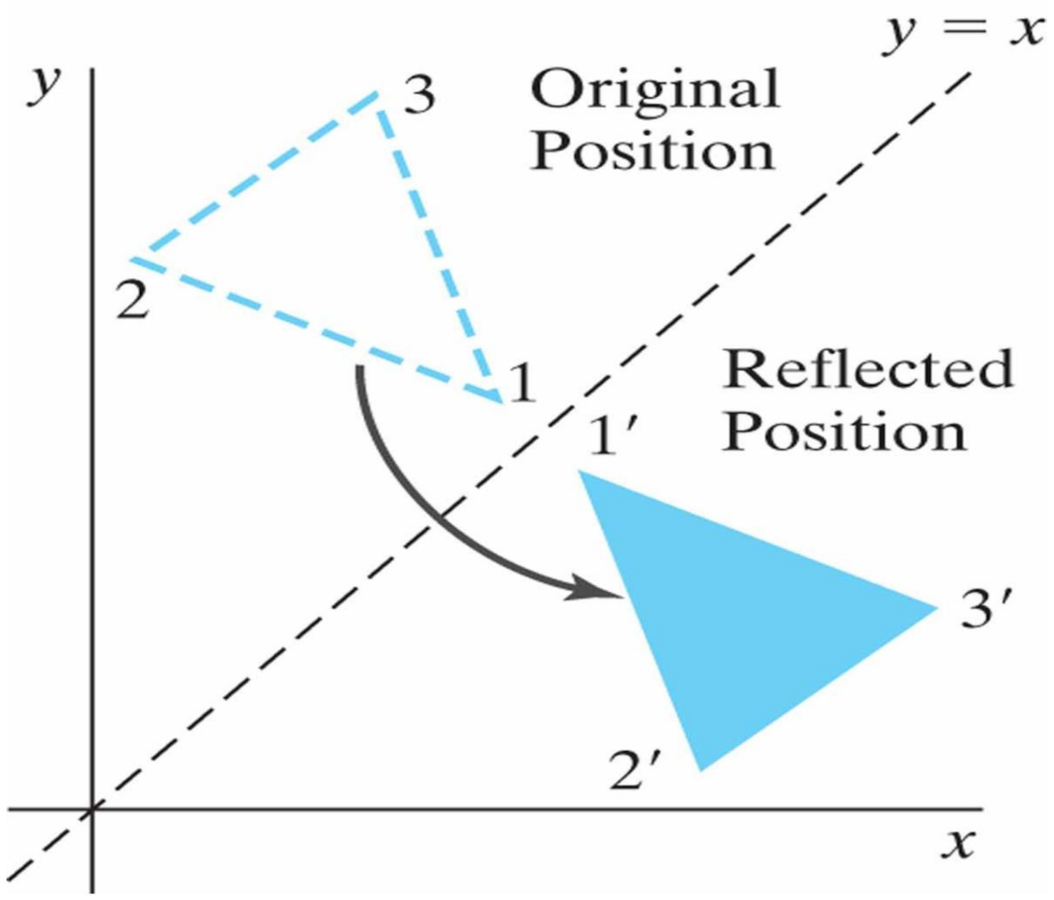
- Reflection about the origin

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Other 2D Transformations (cont.)

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



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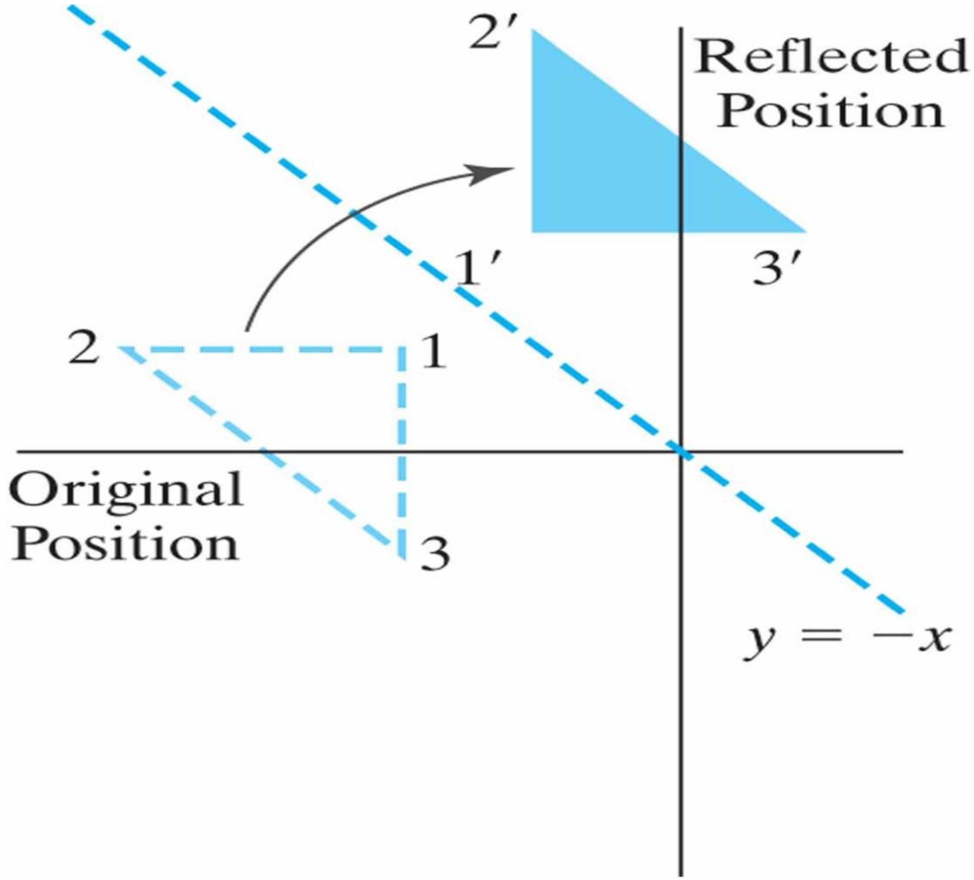
Other 2D Transformations (cont.)

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



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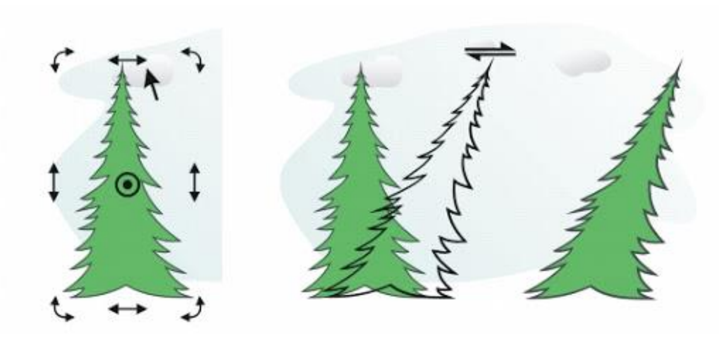
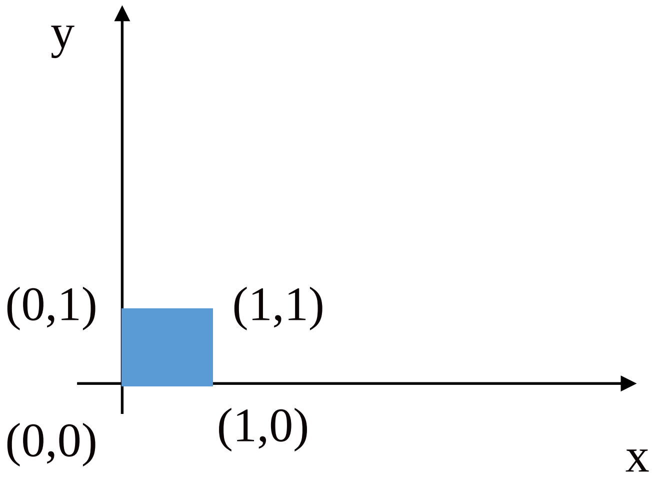
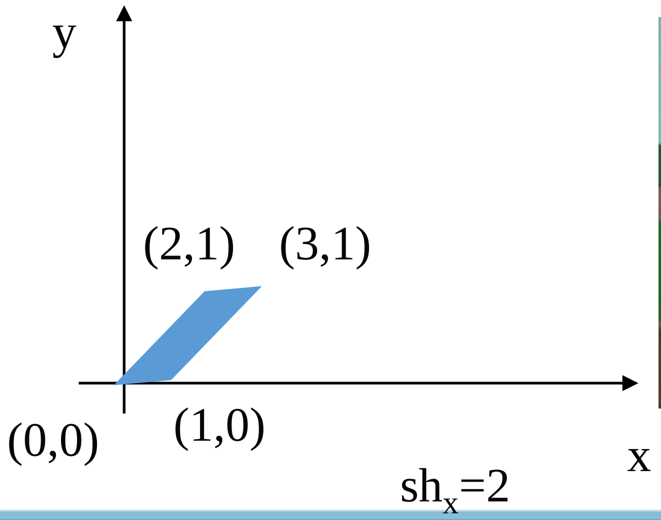
Other 2D Transformations (cont.)



- Shear
  - Transformation that distorts the shape of an object such that the transformed shape appears as the object was composed of internal layers that had been caused to slide over each other





Other 2D Transformations (cont.)

- Shear
  - An x-direction shear relative to the x axis

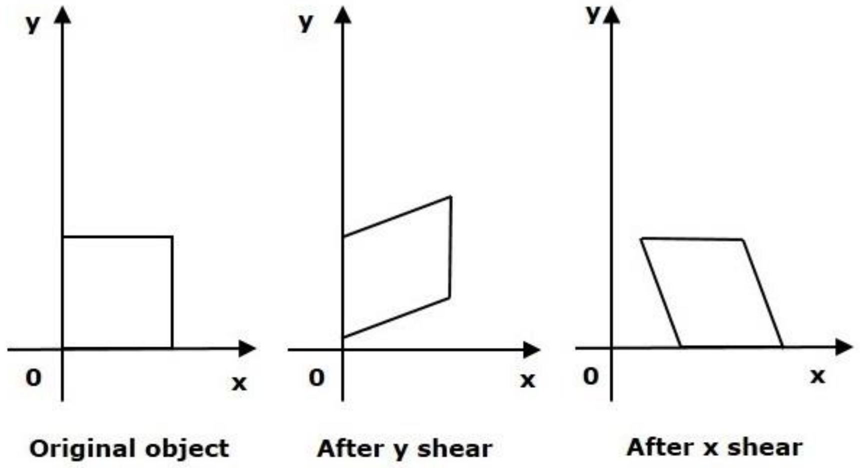
$$\begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x' = x + sh_x \cdot y$$

$$y' = y$$

  - An y-direction shear relative to the y axis

$$\begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Other 2D Transformations (cont.)

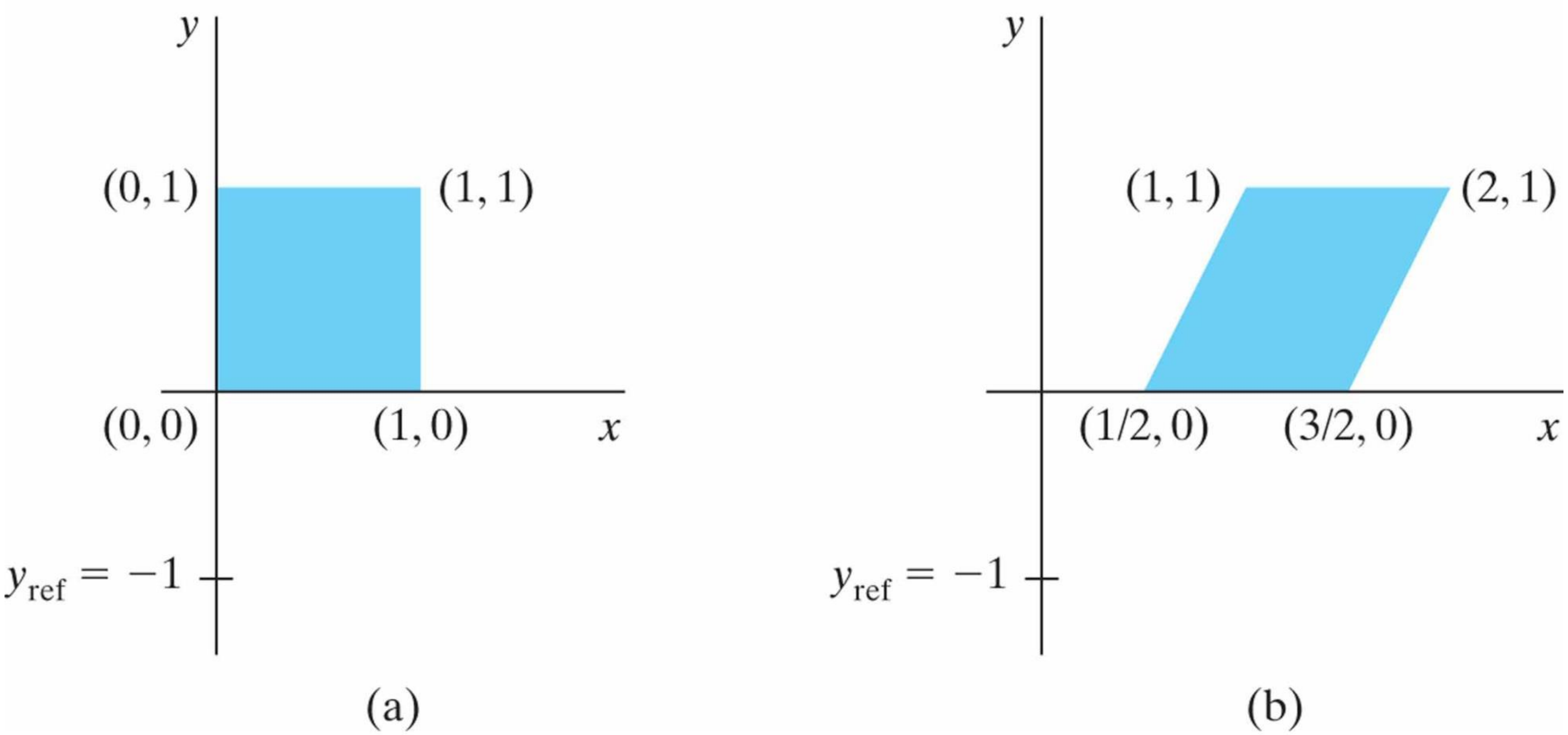
- Shear
  - x-direction shear relative to other reference lines

$$\begin{bmatrix} 1 & sh_x & -sh_x * y_{ref} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x' = x + sh_x * \left(y - y_{ref}\right)$$

$$y' = y$$

Example



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A unit square (a) is transformed to a shifted parallelogram (b) with $sh_x = 0.5$ and $y_{ref} = -1$ in the shear matrix

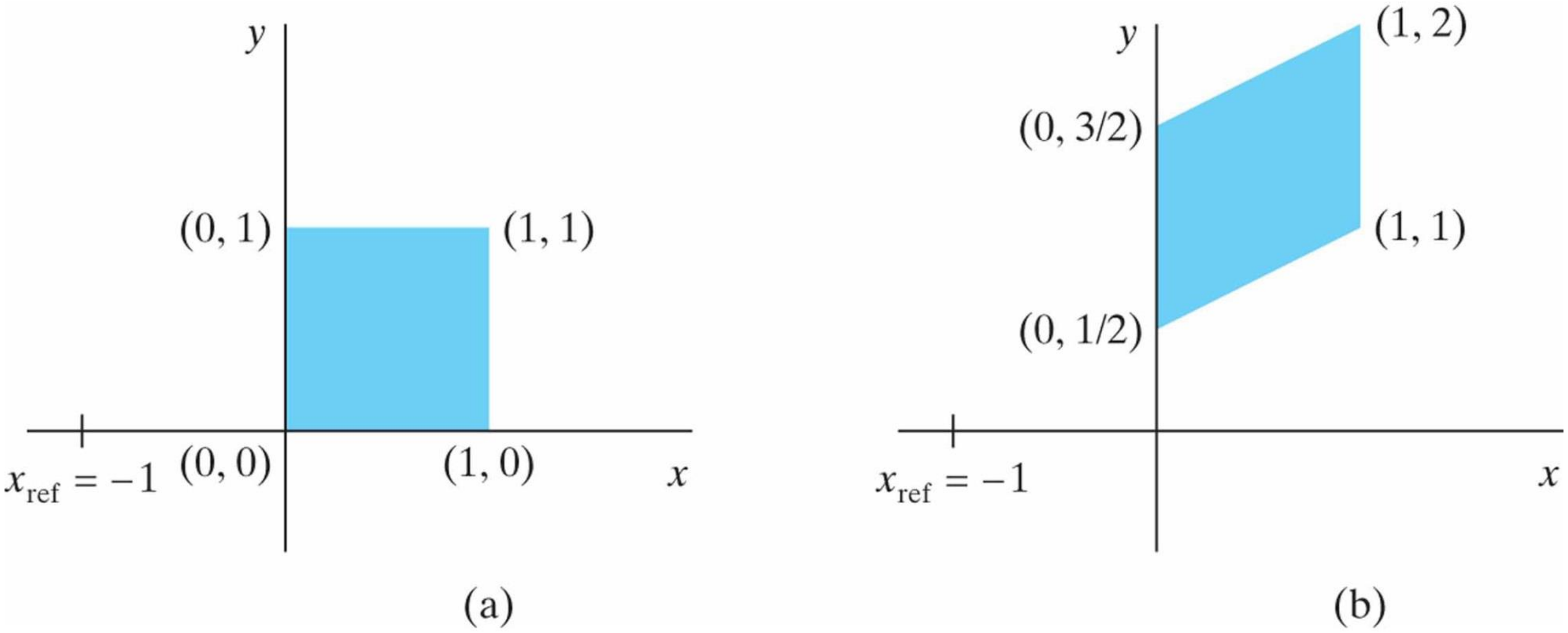
Other 2D Transformations (cont.)

- Shear
  - y-direction shear relative to the line $x = x_{ref}$

$$\begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & -sh_y * x_{ref} \\ 0 & 0 & 1 \end{bmatrix}$$

$$x' = x$$
$$y' = x + sh_y * \left(x - x_{ref}\right)$$

Example



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A unit square (a) is turned into a shifted parallelogram (b) with parameter values $sh_y = 0.5$ and $x_{ref} = -1$ in the y -direction shearing transformation

What is happening here?