Assignment-1

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Download all latex codes from:

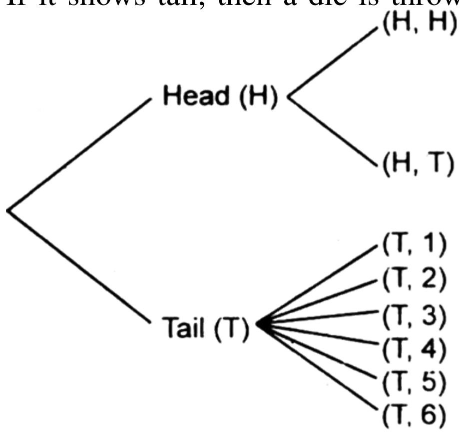
https://github.com/PrasannaLanka/Assignment1/blob/main/Assignment1/main.tex

Problem:

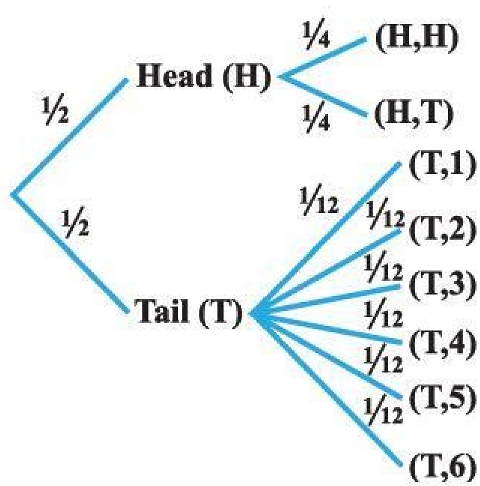
Consider the experiment of tossing a coin. If the coin shows head, toss it again but if it shows tail, then throw a die. Find the conditional probability of the event that "the die shows a number greater than 4" given that "there is at least one tail".

Solution:

Given that a coin is tossed.
If coin shows head, it is tossed again.
If it shows tail, then a die is thrown.



Different probabilities are given by



Probabilities	
Event	Probability
(H,H)	$\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$
(H,T)	$\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$
(T,1)	$\frac{1}{2}\cdot\frac{1}{6} = \frac{1}{12}$
(T,2)	$\frac{1}{2}\cdot\frac{1}{6} = \frac{1}{12}$
(T,3)	$\frac{1}{2}\cdot\frac{1}{6} = \frac{1}{12}$
(T,4)	$\frac{1}{2}\cdot\frac{1}{6} = \frac{1}{12}$
(T,5)	$\frac{1}{2}\cdot\frac{1}{6} = \frac{1}{12}$
(T,6)	$\frac{1}{2}\cdot\frac{1}{6} = \frac{1}{12}$

Let $X \in \{0, 1\}$ be the random variable such that 1 represents occurrence of tail,0 represents occurrence of head when coin is tossed.

TABLE I: Probability distribution for values of X

X	P(X)
1	$\frac{1}{2}$
0	$\frac{1}{2}$

Let Y denotes the getting a number on the die thrown, then the probability distribution is

TABLE II: Probability distribution for values of Y

Y	1	2	3	4	5	6
P(Y)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$\Pr(X = 1)$
$= \Pr(H, T) + \Pr(T, 1) + \Pr(T, 2) + \Pr(T, 3) + \Pr(T, 4) + \Pr(T, 5) + \Pr(T, 6)$
$= \frac{1}{4} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12}$
$= \frac{1}{4} + \frac{6}{12}$
$= \frac{3}{4}$

$\Pr(X = 1, Y > 4)$
$= \Pr(T, 5) + \Pr(T, 6)$
$= \frac{1}{12} + \frac{1}{12}$
$= \frac{1}{6}$

We need $\Pr(Y > 4|X = 1)$
We know that

$$\Pr(Y > 4|X = 1) = \frac{\Pr(Y > 4, X = 1)}{\Pr(X = 1)} \tag{0.0.1}$$

$$\Pr(Y > 4|X = 1) = \frac{\frac{1}{6}}{\frac{3}{4}} = \frac{2}{9} \tag{0.0.2}$$

$\therefore$ The required probability is $\boxed{\frac{2}{9}}$