Assignment-4

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Download all latex codes from:

https://github.com/PrasannaLanka/Assignment4/blob/main/Assignment4/codes/Assignment4.tex

and python codes from

https://github.com/PrasannaLanka/Assignment4/blob/main/Assignment4/codes/Assignment4.py

PROBLEM: CSIR UGC NET EXAM (JUNE 2013), Q.84

Let X_1, X_2, X_3, X_4, X_5 be independent and identically distributed random variables each following a uniform distribution on (0,1) and M denote their median. Then which of the following statements are true?

1) $\Pr\left(M < \frac{1}{3}\right) = \Pr\left(M > \frac{2}{3}\right)$

2) M is uniformly distributed on (0,1)

3) $E(M) = E(X_1)$

4) $V(M) = V(X_1)$

SOLUTION

Theorem 0.1. *A random variable X is said to be uniformly distributed in $a \le x \le b$ if its density function is*

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \le x \le b \\ 0 & \text{otherwise} \end{cases} \tag{0.0.1}$$

and the distribution is called uniform distribution. The mean and variance are respectively,

$$\mu = \frac{a+b}{2} \tag{0.0.2}$$

$$\sigma^2 = \frac{(b-a)^2}{12} \tag{0.0.3}$$

The given random variables are following standard uniform distribution where $a = 0$, $b = 1$.
Since X_1, X_2, X_3, X_4, X_5 are i.i.d random variables, they can be represented by a single random variable X.
Let

$$\{x_1, x_2, x_3, x_4, x_5\} \in X \tag{0.0.4}$$

From (0.0.1)

$$f(x) = \begin{cases} 1 & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases} \tag{0.0.5}$$

From (0.0.2) and (0.0.3)

$$E(X) = \frac{1}{2} \tag{0.0.6}$$

$$V(X) = \frac{1}{12} \tag{0.0.7}$$

Median M would be one among the given five random variables which means M is uniformly distributed on $(0, 1)$
Hence **Option 2 is true.**

$$\Pr\left(M < \frac{1}{3}\right) = \Pr\left(X < \frac{1}{3}\right) \tag{0.0.8}$$

$$= \int_{-\infty}^{\frac{1}{3}} f(x)dx \tag{0.0.9}$$

$$= \int_{0}^{\frac{1}{3}} dx \tag{0.0.10}$$

$$= \frac{1}{3} \tag{0.0.11}$$

$$\Pr\left(M > \frac{2}{3}\right) = \Pr\left(X > \frac{2}{3}\right) \tag{0.0.12}$$

$$= \int_{\frac{2}{3}}^{\infty} f(x)dx \tag{0.0.13}$$

$$= \int_{\frac{2}{3}}^{1} dx \tag{0.0.14}$$

$$= \frac{1}{3} \tag{0.0.15}$$

$$\Pr\left(M < \frac{1}{3}\right) = \Pr\left(M > \frac{2}{3}\right) \tag{0.0.16}$$

Hence **Option 1 is true.**

$$E(M) = E(X) = E(X_1) \tag{0.0.17}$$

Hence **Option 3 is true.**

$$V(M) = V(X) = V(X_1) \tag{0.0.18}$$

Hence **Option 4 is true.**