Magnetic Properties

The magnetic materials are of two types, namely, soft materials and hard materials. Soft magnetic materials are used in ac applications, since they are easily magnetised and demagnetised. However, hard magnetic materials are used in producing permanent magnets, since they retain magnetism on a permanent basis. Due to such properties, these materials are significantly used in information storage devices. In order to realise the operating principles of different magnetic devices, it is essential to understand the magnetic phenomena. So at first we define various terms, viz., intensity of magnetisation, magnetic susceptibility, relative permeability, etc. Magnetic flux density $\vec{B}$ and magnetic field strength $\vec{H}$ have already been discussed in deatil in chapter 10.

10.15 MAGNETIC FLUX DENSITY ($\vec{B}$)

When a magnetic material is placed in an external magnetic field, it gets magnetised. The magnetism thus produced in the material is known as induced magnetism and this phenomenon is referred to as magnetic induction. The magnetic lines of force inside such magnetised materials are called magnetic lines of induction. The number of magnetic lines of induction crossing unit area at right angles to the flux is called the magnetic flux density $\vec{B}$. Its unit is the Tesla which is equal to 1 Wb/m^2.

10.16 MAGNETIC FIELD STRENGTH ($\vec{H}$)

As mentioned earlier, a magnetic material becomes magnetised when placed in a magnetic field. The actual magnetic field inside the material is the sum of external field and the field due to its magnetisation.

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} \qquad \text{or} \qquad \vec{B} = \mu_0(\vec{H} + \vec{M})$$

Magnetic field strength at a point in a magnetic field is the magnitude of the force experienced by a unit pole situated at that point. The SI unit, corresponding to force of 1 Newton, is the A/m. The CGS unit, corresponding to a force of 1 dyne is the Oersted which is equal to 79.6 A/m.

18.1.1 Intensity of Magnetisation (I)

It is defined as the magnetic moment per unit volume of the magnetised substance

$$I = \frac{M}{V}$$

which is for a substance of length $2l$ and cross-sectional area a becomes

$$I = \frac{m \times 2l}{a \times l} = \frac{m}{a}$$

Thus, it can also be defined as pole-strength per unit area of cross-section. The intensity of magnetisation is sometimes represented by M. In that case, another symbol is used for the magnetic moment.

18.1.2 Magnetic Susceptibility (χ_m)

It is the ratio of the magnetic moment per unit volume (I) to the magnetic field strength (H) of the magnetising field.

$$\chi_m = \frac{I}{H}$$

It is positive for a paramagnetic material and negative for a diamagnetic one.

18.1.3 Relative Permeability (μ_r)

It is the ratio of the magnetic permeability (μ) of the substance to the permeability of the free space (μ_0).

$$\mu_r = \frac{\mu}{\mu_0}$$

This can also be defined as the ratio of the magnetic flux density produced in the medium to that which would be produced in a vacuum by the same magnetising force.

18.1.4 Relation between Permeability (μ_r) and Magnetic Susceptibility (χ_m)

As discussed earlier, the magnetic flux density B can be written in terms of the magnetic field strength H and the intensity of magnetisation I as

$$B = \mu_0 (H + I) \qquad \text{(i)}$$

$$\chi_m = \frac{I}{H} \qquad \text{(ii)}$$

Therefore,

$$B = \mu_0 (H + I) = \mu_0 H\left(1 + \frac{I}{H}\right) = \mu_0 H (1 + \chi_m)$$

$$\text{or} \quad \frac{B}{H} = \mu = \mu_0 (1 + \chi_m)$$

$$\therefore \quad \text{Relative permeability } \mu_r = \frac{\mu}{\mu_0} = 1 + \chi_m$$

where μ_0 is the magnetic permeability of free space.

18.3.1 Diamagnetic Materials

On placing in an external magnetic field, the materials which acquire feeble magnetism in the direction opposite to that of the applied field are called *diamagnetic materials*. This property is found in the substances whose outermost orbit has an even number of electrons. Since the electrons have spins opposite to each other, the net magnetic moment of each atom is zero. The magnetism of diamagnetic materials is called *diamagnetism*. If these materials are brought close to the pole of a powerful electromagnet, they are repelled away from the magnet.

Examples of diamagnetic materials are bismuth, zinc, copper, silver, gold, lead, water, etc.

18.3.2 Paramagnetic Materials

On placing in an external magnetic field, the materials which acquire feeble magnetism in the direction of the applied field are called *paramagnetic materials,* and their magnetism is known as *paramagnetism*. This property is found in the substances whose outermost orbit has an odd number of electrons. The source of paramagnetism is the permanent magnetic moment possessed by the atoms of paramagnetic materials. If these substances are brought close to a pole of a powerful electromagnet, they get attracted towards the magnet.

Examples of paramagnetic materials are aluminium, odium, platinum, manganese, copper chloride, liquid oxygen, etc.

18.3.3 Ferromagnetic Materials

On placing in an external magnetic field, the materials which acquire strong magnetism in the direction of the applied field are called *ferromagnetic materials* and their magnetism is called *ferromagnetism.* This property is found in the substances which are generally like paramagnetic materials. These are strongly attracted by magnets.

Examples of ferromagnetic materials are iron, nickel, cobalt, magnetite (Fe_3O_4), etc.

18.3.4 Anti-ferromagnetic Materials

18.3.4 Anti-ferromagnetic Materials

Anti-ferromagnetic substances are crystalline materials. In these materials, the dipole moments of the neighbouring dipoles are equal and opposite in orientation so that the net magnetisation vanishes. If they are placed in the magnetic field, they are feebly magnetised in the direction of the field. Such materials are called *anti-ferromagnetic materials* and their magnetism is called *anti-ferromagnetism*. Examples of anti-ferromagnetic materials are: MnO, FeO, CaO, NiO, MnO_4, MnS, etc. Susceptibility of these materials vary with temperature. It increases with increasing temperature and reaches a maximum at a particular temperature called the Neel temperature (T_N). Above this temperature, these materials behave like paramagnetic materials.

18.3.5 Ferrimagnetic Materials

If the spins of the atoms are such that there is a net magnetic moment in one direction, the materials are called *ferrimagnetic materials*. Examples of ferrimagnetic materials are ferrites which consist of mainly ferric oxide Fe_2O_3 combined with one or more oxides of divalent metals.

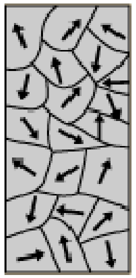
Ferromagnetic ↑ ↑ ↑ ↑ ↑ ↑ ↑	Below T_C, spins are aligned parallel in magnetic domains
Antiferromagnetic ↑ ↓ ↑ ↓ ↑ ↓ ↑	Below T_N, spins are aligned antiparallel in magnetic domains
Ferrimagnetic ↑ ↓ ↑ ↓ ↑ ↓ ↑	Below T_C, spins are aligned antiparallel but do not cancel
Paramagnetic ↑ ↓ ↑ ↑ ↓ ↑ ↓	Spins are randomly oriented (any of the others above T_C or T_N)

18.4 COMPARISON OF PROPERTIES OF PARAMAGNETIC, DIAMAGNETIC AND FERROMAGNETIC MATERIALS

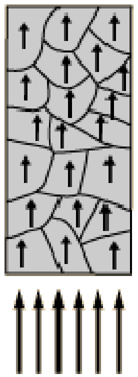
S.No.	Paramagnetic Materials	Diamagnetic Materials	Ferromagnetic Materials
1.	These materials show positive magnetic susceptibility (of the order of 10^{-6}).	These materials show negative susceptibility (of the order of 10^{-6}).	These materials show positive and high magnetic susceptibility ($\sim 10^{6}$).
2.	The relative permeability is slightly more than unity ($\mu_r > 1$).	μ_r is slightly less than unity ($\mu_r < 1$).	The μ_r for a ferro-magnetic material is of the order of few thousands.
3.	The magnetic susceptibility is strongly dependent on temperature and varies inversely with temperature.	The magnetic susceptibility of diamagnetic materials is practically independent of temperature.	The magnetic susceptibility decreases with increase in temperature.
4.	When a bar of a paramagnetic material is suspended between the poles of a magnet, it stays parallel to the lines of force.	When a bar of these materials is suspended between the poles of a magnet, it stays parallel to the magnetic field.	When a bar of these materials is suspended between the poles of a magnet, it behaves like a paramagnetic material.
5.	If these materials are placed in a non-uniform field, they are attracted towards the stronger field.	If these materials are placed in a non-uniform field, they are attracted towards the weaker field.	These materials behave like paramagnetic substances, if placed in a non-uniform field.

Domain Theory: Ferromagnetism

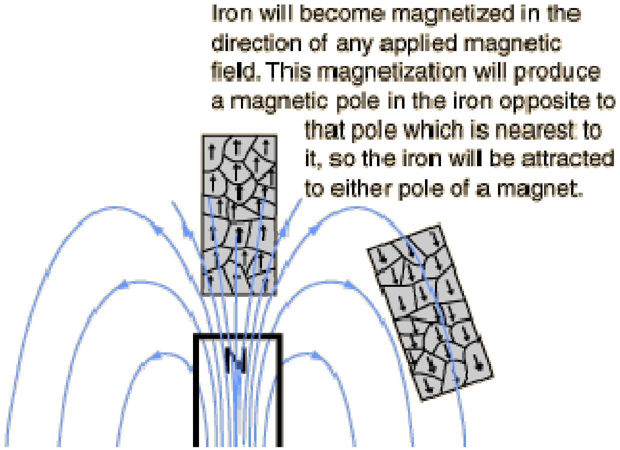
We know that each atom of ferromagnetic material such as iron, nickel, cobalt, etc. has a permanent magnetic moment like paramagnetic substances. The magnetic susceptibility of a ferromagnetic substance is a thousand times greater than that of a paramagnetic substance. In general, a specimen of a ferromagnetic substance contains a number of small regions called domains. These domains contain a large number of atoms, nearly 10^{17} to 10^{22}, and have the dimensions of about 10^{-6}cm^3 to 10^{-2}cm^3. Each domain consists of magnetic moments that are aligned, giving rise to a permanent net magnetic moment per domain. Each of these domains is separated from the rest by domain boundaries called *Bloch walls* having thickness about 100nm. Domains exist even in absence of external field. In a material that has never been exposed to a magnetic field, the individual domains have a random orientation. This type of arrangement represents the lowest free energy.

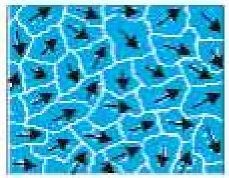


In bulk material the domains usually cancel, leaving the material unmagnetized.

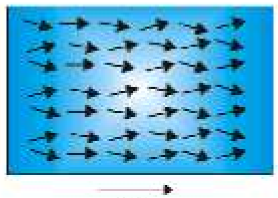


Externally applied magnetic field.



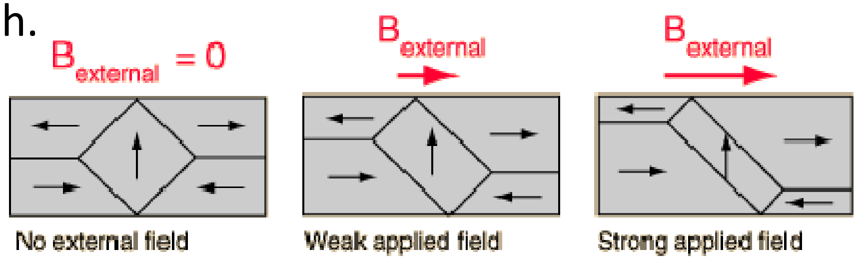


(a)



$\mathbf{B}_0$

These illustrations of domains are conceptual only and not meant to give an accurate scale of the size or shape of domains. The microscopic evidence about magnetization indicates that the net magnetization of ferromagnetic materials in response to an external magnetic field may actually occur more by the growth of the domains parallel to the applied field at the expense of other domains rather than the reorientation of the domains themselves as implied in the sketch.



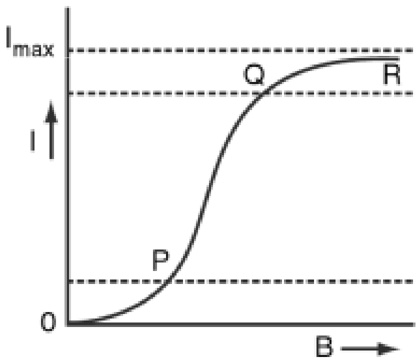


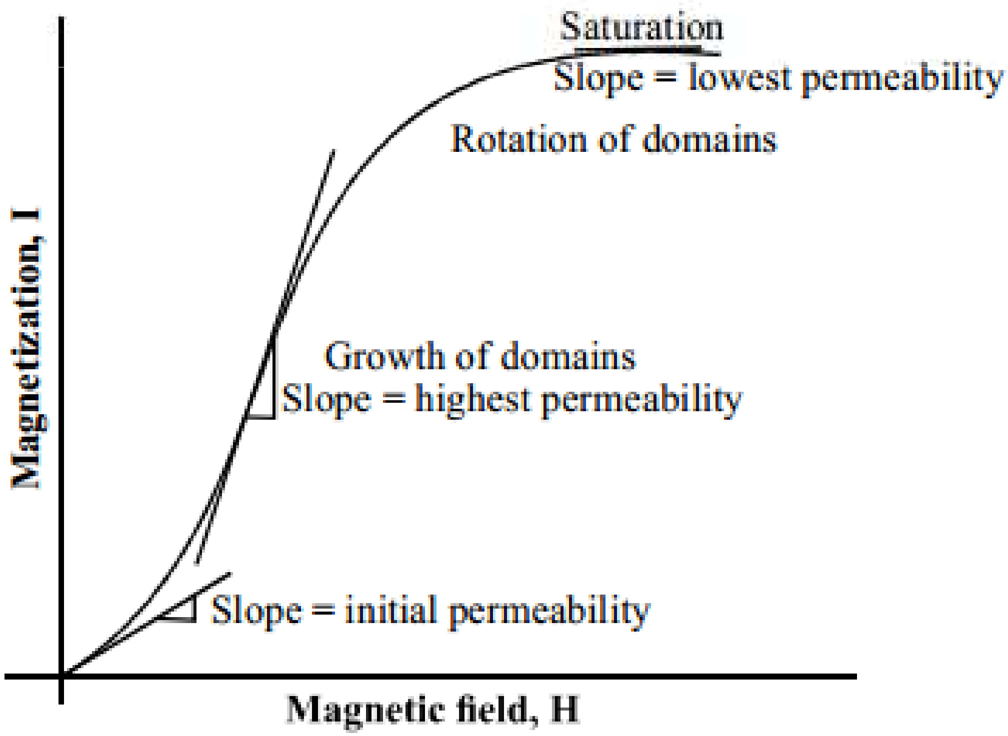
Fig. 18.6

When the substance is placed in a weak external magnetic field, the magnetisation produced is due to the displacement of boundaries of domains Fig. 18.5b and if the external magnetic field is strong, the magnetisation produced is mainly by the rotation of domains Fig. 18.5c. Fig. 18.6 represents the magnetisation curve for the ferromagnetic substance. In a very weak magnetic field, as represented in the part OP of the curve, the displacement of boundaries of domains is reversible and if we removed the external magnetic field, the boundaries of domains again come back to their original positions.

If we increase the external magnetic field, as represented in the part PQ of the curve, the displacement of boundaries of domains is irreversible and the material immediately becomes magnetised. If we again increase the magnetic field, as represented in the part QR of the curve, the magnetisation of the substance is because of rotation of domains in the direction of magnetising field.

Thus, the net effective magnetic field of the ferromagnetic substance is given by

$$H_{effective} = H + H_i$$



Saturation
Slope = lowest permeability
Rotation of domains
Growth of domains
Slope = highest permeability
Slope = initial permeability
Magnetization, I
Magnetic field, H

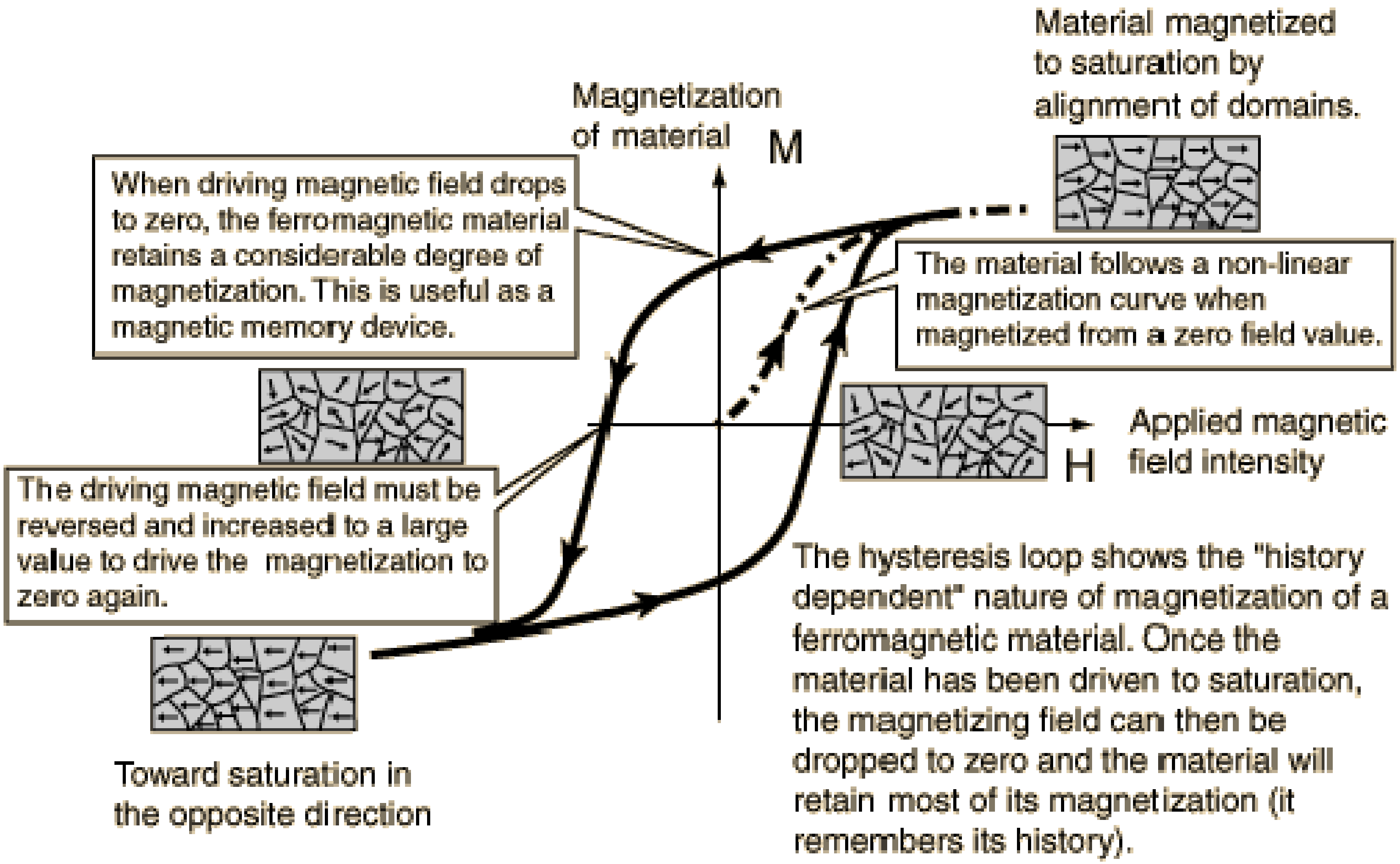
Explanation

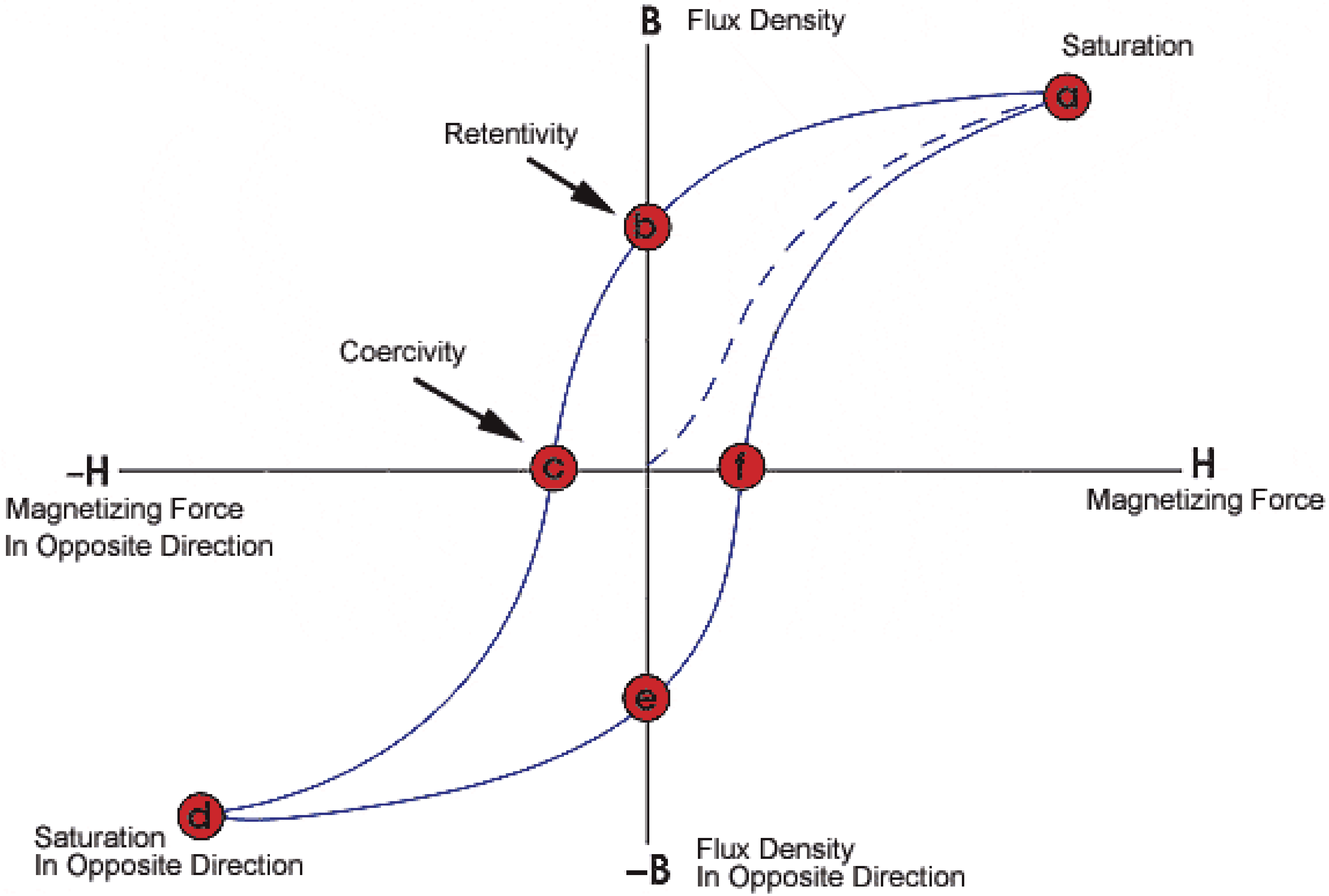
When the bulk material is un-magnetized, the net magnetization of these domains is zero, because adjacent domains may be orientated randomly in any number of directions, effectively canceling each other out figure 7.3(a). The average magnetic induction of a ferro-magnetic material is intimately related to the domain structure. When a magnetic field is imposed on the material, domains that are nearly lined up with the field grow at the expense of unaligned domains figure 7.3(b). This process continues until only the most favorably oriented domains remain. In order for the domains to grow, the Bloch walls must move, the external field provides the force required for this moment. When the domain growth is completed, a further increase in the magnetic field causes the domains to rotate and align parallel to the applied field figure 7.3(c). At this instant material reaches saturation magnetization and no further increase will take place on increasing the strength of the external field. Under these conditions the permeability of these materials becomes quite small. The variation of magnetization with applied magnetic field H is shown is figure 7.4.

HYSTERESIS

When a ferromagnetic material is magnetized in one direction, it will not relax back to zero magnetization when the imposed magnetizing field is removed. It must be driven back to zero by a field in the opposite direction.
If an alternating magnetic field is applied to the material, its magnetization will trace out a loop called a hysteresis loop.

- **The lack of retraceability of the magnetization curve is the property called hysteresis**
    - it is related to the existence of magnetic domains in the material.
    - Once the magnetic domains are reoriented, it takes some energy to turn them back again.
    - This property of ferrromagnetic materials is useful as a magnetic "memory". Some compositions of ferromagnetic materials will retain an imposed magnetization indefinitely and are useful as "permanent magnets". The magnetic memory aspects of iron and chromium oxides make them useful in audio tape recording and for the magnetic storage of data on computer disks.


Material magnetized to saturation by alignment of domains.
Magnetization of material
M
When driving magnetic field drops to zero, the ferromagnetic material retains a considerable degree of magnetization. This is useful as a magnetic memory device.
The material follows a non-linear magnetization curve when magnetized from a zero field value.
Applied magnetic field intensity
H
The driving magnetic field must be reversed and increased to a large value to drive the magnetization to zero again.
The hysteresis loop shows the "history dependent" nature of magnetization of a ferromagnetic material. Once the material has been driven to saturation, the magnetizing field can then be dropped to zero and the material will retain most of its magnetization (it remembers its history).
Toward saturation in the opposite direction


B Flux Density
Saturation
Retentivity
Coercivity
–H
Magnetizing Force
In Opposite Direction
H
Magnetizing Force
a
b
c
d
e
f
Saturation
In Opposite Direction
–B
Flux Density
In Opposite Direction

Retentivity - A measure of the residual flux density corresponding to the saturation induction of a magnetic material. In other words, it is a material's ability to retain a certain amount of residual magnetic field when the magnetizing force is removed after achieving saturation. (The value of **B** at point b on the hysteresis curve.)

Residual Magnetism or **Residual Flux** - the magnetic flux density that remains in a material when the magnetizing force is zero. Note that residual magnetism and retentivity are the same when the material has been magnetized to the saturation point. However, the level of residual magnetism may be lower than the retentivity value when the magnetizing force did not reach the saturation level.

Coercive Force - The amount of reverse magnetic field which must be applied to a magnetic material to make the magnetic flux return to zero. (The value of **H** at point c on the hysteresis curve.)

Permeability, m - A property of a material that describes the ease with which a magnetic flux is established in the component.

Reluctance - Is the opposition that a ferromagnetic material shows to the establishment of a magnetic field. Reluctance is analogous to the resistance in an electrical circuit.

ENERGY LOSS DUE TO HYSTERESIS

During the process of magnetisation, a loss of energy is always involved in aligning the domains in the direction of the applied magnetic field. When the direction of an external magnetic field is reversed, the absorbed energy is not completely recovered and rest energy in sample is lost in the form of heat. This loss of energy is called hysteresis loss.

It can be proved that the energy lost per unit volume of the substance in a complete cycle of magnetisation is equal to the area of the hysteresis loop. We consider a unit volume of the ferromagnetic substance, which has N magnetic domains. Let M be the magnetic moment of each magnetic domain which makes an angle θ with the direction of the magnetic field H.

So, the total magnetic moment per unit volume in the direction of magnetising field

$$= \sum_N M \cos\theta$$

$$= \text{Intensity of magnetisation (I)} \qquad \text{...(i)}$$

Since there is no magnetisation perpendicular to H, the total magnetic moment perpendicular to the magnetising field (H),

$$= \sum_N M \sin\theta = 0 \qquad \text{...(ii)}$$

$$\therefore \quad I = \sum_N M \cos\theta$$

$$\text{or} \qquad dI = -\sum_N M \sin\theta d\theta \qquad \text{...(iii)}$$

Here the negative sign shows that I decreases with increasing θ.

The work done in rotating the domain from this direction by an angle $d\theta$ towards H is

$$dW = -\mu_0 \sum_W MH \sin\theta \, d\theta$$

$$= \mu_0 H dI \quad \text{[by using Eq. (iii)]}$$

The work done per unit volume of substance in the complex cycle is

$$W = \oint \mu_0 H dI$$

$$= \mu_0 \oint H dI$$

$$= \mu_0 \times (\text{area of I–H loop})$$

Hence, the work done per unit volume of the substance per cycle of magnetisation is equal to μ_0 times the area of I–H curve and this energy is lost in the form of heat.

Hysteresis Loss due to B–H curve

The magnetic flux density (B) in substance is due to the magnetising field (H) and the intensity of magnetisation I. They are related as

$$B = \mu_0 (H + I) \qquad \text{...(vi)}$$

or $$dB = \mu_0 (dH + dI)$$

or $$dI = \frac{I}{\mu_0} dB - dH \qquad \text{...(vii)}$$

From Eqs. (v) and (vii) we get

$$W = \oint H dB - \mu_0 \oint H\, dH \qquad \text{...(viii)}$$

The value of $\oint H\, dH$ will be zero, because the curve between H and H is a straight line and will not enclose any area, i.e.,

$$\oint H\, dH = 0$$

Then Eq. (viii), takes the form,

$$W = \oint H\, dB = \text{ area of B–H curve.}$$

TEMPERATURE DEPENDENCE

Materials with ferro-magnetism (Examples: Fe, Co, Ni, Gd) possess magnetic susceptibilities approaching 10^6. Above the Curie temperature, ferro-magnetic materials behave as para-magnetic materials and their susceptibility is given by the Curie-Weiss law, defined as

$$\chi_m = \frac{C}{T - T_c}$$

where C – Curie constant, T – temperature, T_c – Curie temperature.

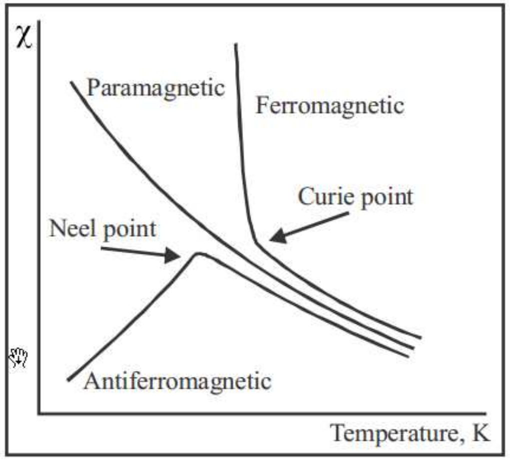


Figure 7.5 : Variation of susceptibility with temperature for ferromagnetic, antiferromagnetic and paramagnetic materials

Paramagnetic substances obey Curie Law which

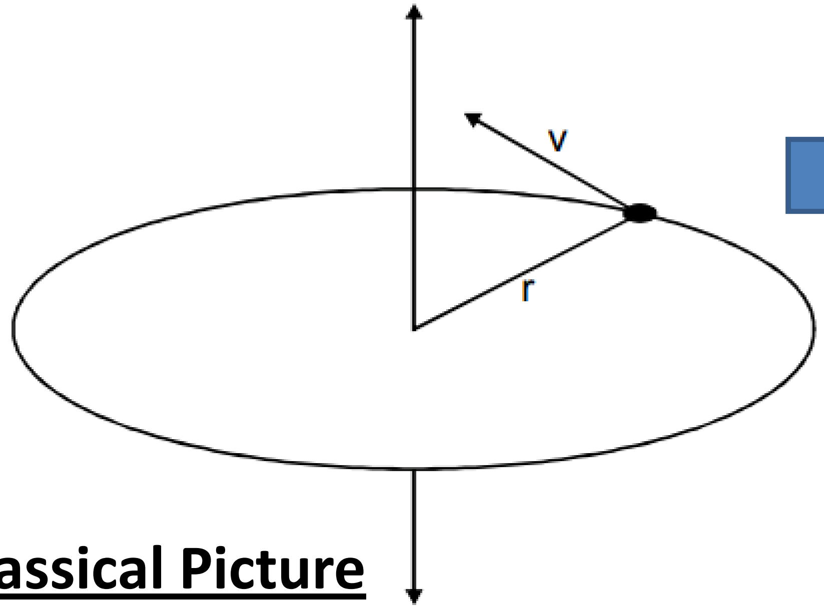
$$\chi_m = \frac{C}{T}$$

where C : Curie constant

T : Temperature

This law shows that with increasing temperature the alignment is more difficult and χ_m decreases.

Orbital Magnetic Moment



<u>Classical Picture</u>

$$i = -\frac{e}{T} = -\frac{ev}{2\pi r}$$

$$\mu = i \times A$$

$$= \frac{ev}{2\pi r}\pi r^2 = \frac{evr}{2}$$

$$L = mvr$$

$$\mu = \frac{-e}{2m}L$$

<u>Classical Picture</u>

$$\mu_l = -\frac{e}{2m}\frac{h}{2\pi}\sqrt{l(l+1)}$$

<u>Quantum Picture</u>

Orbital Magnetic Moment

$$\mu_l = -\frac{e}{2m}\frac{h}{2\pi}\sqrt{l(l+1)}$$

$$\mu_l = -g_l\sqrt{l(l+1)}\mu_B$$

where $g_l = 1$, $\mu_B = \frac{eh}{4\pi m}$ = Bohr magneton

Spin Magnetic Moment

$$\mu_s = -g_s\sqrt{s(s+1)}\frac{eh}{4\pi m}$$

Total Magnetic Moment

$$\mu = -g\mu_B\sqrt{J(J+1)}$$

where g is called g-factor or Lande's g-factor, given by

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

6. Calculate spectroscopic g-factor for

(i) Cr^{3+} (ii) Dy^{3+}

Solution : (i)

Electronic configuration for Cr^{3+} is $3d^3$

$$l = 2$$

The possible values of m_l for $l = 2$ are $-2, -1, 0, 1, 2$.

For $Cr3^+, L = \sum m_l = 3$

and $S = 3/2$ (for less than half filled orbitals $J = L - S$)

Therefore, $J = L - S = 3/2$

Spectroscopic notation : $^4F_{3/2}$

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} = 0.4$$

$$peff = g\sqrt{J(J+1)} = 3.87$$

(ii) Dy^{3+} has outer electronic configuration of $4f^9 6s^0$. Number of unpaired electrons in $Dy^{3+} = 5$,

So, $S = 5(1/2) = 5/2$

$L = 3 + 2 + 1 + 0 - 1 = 5$

(for half filled and more than half filled $J = L + S$)

$J = L + S = 5/2 + 5 = 15/2$

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} = 1.33$$

Find the percentage increase in magnetic induction when the space within a current-carrying toroid is filled with magnesium. Given χ for magnesium as 1.2×10^{-5}.

Solution Given $\chi = 1.2 \times 10^{-5}$.

Magnetic flux density

$$B = \mu_0 H$$

When the free space is filled with magnesium, then

$$B' = \mu_r \mu_0 H$$

$$\text{and } \mu_r = 1 + \chi$$

From Eqs. (ii) and (iii)

$$B' = (1 + \chi)B$$

Hence, the percentage increase in magnetic induction

$$= \frac{B' - B}{B} \times 100$$

By using Eqs. (i) and (iv), Eq. (v) becomes

$$= \frac{(1 + \chi)B - B}{B} \times 100 = \chi \times 100$$

$$= 1.2 \times 10^{-5} \times 100$$
$$= 1.2 \times 10^{-3}\%$$
$$= \mathbf{0.0012\%}$$

Example 10 An iron rod of 1.0 m length and cross-section 4 sq cm is in the form of a closed ring. If the permeability of iron is 50×10^{-4} Hm^{-1}. Show that the number of ampere turns required to produce a magnetic flux of 4×10^{-4} Wb through the closed ring is 200.

Solution Given $L = 1.0$ m, $A = 4 \times 10^{-4} m^2$, $\mu = 50 \times 10^{-4}$ H/m and $\phi = 4 \times 10^{-4}$ Wb.

Magnetic flux density $B = \frac{\phi}{A} =$

$$= \frac{4 \times 10^{-4}}{4 \times 10^{-4}} = 1.0 Wb/m^2$$

Also $B = \mu NI$

$\therefore$ Ampere turn $NI = \frac{B}{\mu} =$

$$= \frac{1.0}{50 \times 10^{-4}} = \mathbf{200\,A/m}$$

Example 11 The mean length of an iron ring having 200 turns of wire upon it is 0.5m and its cross-section is 4×10^{-4} m^2. What current through the winding should be sent to produce a flux of 4×10^{-4} Wb in the ring? Permeability of iron is 65×10^{-4} Wb/Am.

Solution Given $\mu = 6.5 \times 10^{-4}$ Wb/Am, $\phi = 4 \times 10^{-4}$ Wb and $A = 4 \times 10^{-4}$ m^2.

The formula used is $B = \frac{\phi}{A} = \frac{4\times10^{-4}}{4\times10^{-4}} = 1.0 \text{ Wb/m}^2$

Also $B = \mu NI$

or $I = \frac{B}{\mu N}$

where N is the number of turns per metre, i.e.

$$N = \frac{200}{0.5} = 400 \text{ turns/m}$$

Then, current $I = \frac{B}{\mu N} = \frac{1.0}{6.5\times10^{-4}\times400}$

$\mathbf{I = 3.85\ A}$

Example 1 In hydrogen atom, an electron revolves around a nucleus in an orbit of 0.53 Å radius. If the frequency of revolution of an electron is 6.6×10^{15} Hz, find the magnetic moment of the orbiting electron and calculate numerical value of Bohr magneton.

Solution Given $r = 0.53 \times 10^{-10}$ m and $n = 6.6 \times 10^{15}$ Hz.

Magnetic Moment $M = iA$

$$i = \frac{e}{T} = \frac{e}{1/n} = en = 1.6 \times 10^{-19} \times 6.6 \times 10^{15} \text{ A}$$

$$\text{Area} = \pi r^2 = 3.14 \times (0.53 \times 10^{-10})^2$$

$$\therefore M = iA = 1.6 \times 10^{-19} \times 6.6 \times 10^{15} \times 3.14 \times (0.53 \times 10^{-10})^2$$

$$= 9.314 \times 10^{-24} \text{ Am}^2$$

Bohr magneton is the smallest value of the orbital magnetic moment of the electron. For n = 1, Bohr magneton

$$\mu_B = \frac{eh}{4\pi m}$$

$$= \frac{1.6 \times 10^{-19} \times 6.6 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31}}$$

$$= 9.239 \times 10^{-24}$$

$$= \mathbf{9.24 \times 10^{-24}} \text{ J/T}$$

Example 2 Determine the magnetisation and flux density in silicon, if its magnetic susceptibility is -4.2×10^{-6} and the magnetic field in it is $1.19 \times 10^{5}\text{Am}^{-1}$. What would be the value of the relative permeability of the material?

Solution Given $\chi = -4.2 \times 10^{-6}$ and $H = 1.2 \times 10^{5}\ \text{Am}^{-1}$.

The formulae used are

$$\begin{aligned}
\text{Magnetisation } I &= \chi H \\
&= -4.2 \times 10^{-6} \times 1.19 \times 10^{5}\ \text{Am}^{-1} \\
&= -0.4998\ \text{Am}^{-1} \\
&= \mathbf{-0.50\ Am^{-1}}
\end{aligned}$$

$$\begin{aligned}
\text{Flux density } B &= \mu_0 (H + I) \\
&= 4\pi \times 10^{-7} \times (1.19 \times 10^{5} - 0.50) \\
&= 0.1495\ \text{T} \\
&= \mathbf{0.150\ T}
\end{aligned}$$

Relative permeability

$$B = \mu H = \mu_0 H \left(1 + \frac{I}{H}\right)$$

$$\begin{aligned}
\text{or} \quad \mu_r &= \frac{\mu}{\mu_0} = 1 + \frac{I}{H} \\
&= 1 + \frac{-0.50}{1.19 \times 10^{5}} \\
&= 1 - 0.42 \times 10^{-5} \\
&= \mathbf{0.999}
\end{aligned}$$