Assignment Stats - 1

Problem Statement - 1

Solu

x_i = 4, 4, 5, 5, 6, 6, 6, 7, 7, 7, 7, 7, 8, 8, 8, 9, 9, 10

N = 20

$$\text{Mean } (\mu) = \frac{1}{N} \sum_{i=1}^{N} x_i$$

$$\boxed{\mu = 6.85}$$

$\boxed{\text{Median} = 7}$

$\boxed{\text{Mode} = 7}$ (7 appeared 5 times)

$$\text{Std } \sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2}$$

$$\sigma^2 = \frac{\sum (x_i - \mu)^2}{N}$$

$$= \frac{50.55}{20}$$

$$= 2.5275$$

$$\sigma = \sqrt{2.5275}$$

$$\boxed{\sigma = 1.5898}$$

Problem statement-2

Solun

x_i = 28, 40, 68, 70, 75, 75, 75, 75, 80, 86, 89, 90, 90

97, 97, 100, 100, 100, 104, 104, 109, 113, 120, 120; 112, 120

123, 123, 130, 140, 145, 170, 174, 194, 217

N = 35

$$\mu = \frac{1}{N} \sum_{i=1}^{N} x_i$$

$\boxed{\mu = 107.51}$

$\boxed{\text{Median} = 100}$

$\boxed{\text{mode} = 75}$ (75, appeared 4 times)

$$\text{Std}(\sigma) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2}$$

$$\sigma^2 = \frac{\sum (x_i - \mu)^2}{N}$$

$$= \frac{52616.7}{35}$$

$$= 1503.33$$

$$\sigma = \sqrt{1503.33}$$

$$\boxed{\sigma = 38.77}$$

Problem statement - 3

Solun

x	$f(x)$
0	0.09
1	0.15
2	0.40
3	0.25
4	0.10
5	0.01

$$E(x) = \mu_x = (0 \times 0.09) + (1 \times 0.15) + 2(0.40) + (3 \times 0.25) + 4(4 \times 0.10) + (5 \times 0.01)$$

$$= 0 + 0.15 + 0.8 + 0.75 + 0.4 + 0.05$$

$$\boxed{\mu_x = 2.15}$$

$$Var(x) = (0-2.15)^2(0.09) + (1-2.15)^2(0.15) + (2-2.15)^2(0.40) + (3-2.15)^2(0.25) + (4-2.15)^2(0.10) + (5-2.15)^2(0.01)$$

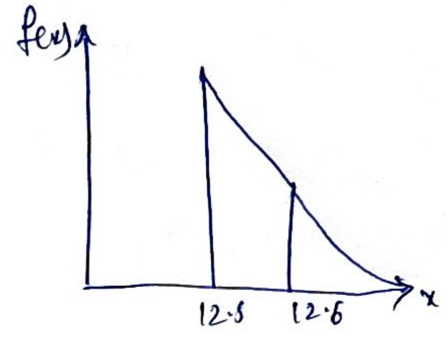
$$\boxed{Var(x) = 1.340}$$

$$\sigma_x = \sqrt{1.340}$$

$$\boxed{\sigma_x = 1.15} \simeq \underline{1.05}$$

Problem Statement - 4 Sec 9

Soln



$$P(D > 12.60) = \int_{12.6}^{\infty} f(d)\,dx = \int_{12.6}^{x} 20e^{-20(d-12.5)}\,dx$$

$$= -e^{20(d-12.5)} \Big|_{12.6}^{\infty}$$

$$= 0.135$$

Proportion of parts b/w 12.5 and 12.6 mm?

$$P(12.5 < X < 12.6) = 1 - P(X > 12.6)$$

$$= 1 - 0.135$$

$$= \underline{0.865}$$

CDF when diameter 11 mm

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(u)\,du$$

for $-\infty < x < \infty$.

as per continuous prob Distribution point is 0

$\therefore$ CDF is $\underline{0}$

Problem Statement-5

Soln

$n = 6$

$P = 0.3$

$q = 0.7$

$x = 2$

$$P(x) = \frac{n!}{(n-x)!\ (x!)} p^x (1-p)^{n-x}$$

$$= \frac{6!}{(6-2)!\ (2!)} (0.3)^2 (1-0.3)^{6-2}$$

$$\boxed{P(x) = 0.324}$$

$$\text{Mean}(\mu) = n \cdot p$$

$$= (6)(0.3)$$

$$= 1.8$$

$$\text{Variance} = n \cdot p(1-p)$$

$$= 6(0.3)(1-0.3)$$

$$= 1.26.$$

Problem statement 6

Soln

Prob that Gaurav will solve 5 que correctly $= \binom{8}{5}(0.75)^5 * (1-0.75)^3 = 0.2076$

Prob that Barakha will solves que correctly $= \binom{12}{5}(0.45)^5 * (1-0.45)^7 = 0.225$

Prob that Gaurav will solve 4 que correctly $= \binom{8}{4}(0.75)^4 * (1-0.75)^4 = 0.08652$

Prob that Barakha will solve 4 que correctly $= \binom{12}{4}(0.45)^4 * (1-0.75)^8 = 0.16996$

Prob that Gaurav will solve 6 que correctly $= \binom{8}{6}(0.75)^6 * (1-0.75)^2 = 0.31146$

Prob that Barkha will solve 6 que correctly $= \binom{12}{6}(0.45)^6 * (1-0.45)^6 = 0.21238$

Problem Statement 7

Soln

(*) Arrival rate of customers is 72 per hour

Given time of arrival - 4 min

Mean (λ) = $72/60 \times 4 = 72/15 = 4.8$

$\boxed{\lambda = 4.8}$

(*) Prob of arriving 5 customers in 4 min

$P(X=k) = \frac{e^{-\lambda} \lambda^k}{k!}$

$P(X=5) = \frac{e^{-4.8} (4.8)^5}{5!}$

$P(X=5) = 0.1748$

(*) Prob of arriving not more than 3 customers

$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= \frac{e^{-4.8}(4.8)^0}{0!} + \frac{e^{-4.8}(4.8)^1}{1!} + \frac{e^{-4.8}(4.8)^2}{2!} + \frac{e^{-4.8}(4.8)^3}{3!}$

$= \frac{e^{-4.8}(1)}{1} + \frac{e^{-4.8}(4.8)}{1} + \frac{e^{-4.8}(4.8)^2}{2} + \frac{e^{-4.8}(4.8)^3}{6}$

$= 0.0082 + 0.0395 + 0.0948 + 0.1517$

$\boxed{P(X \leq 3) = 0.2942}$

(*) Prob of arriving more than 3 customer's

$P(X>3) = 1 - P(X \leq 3)$

$= 1 - 0.2942$

$\boxed{P(X>3) = 0.7058}$

Problem Statement - 8

Soln

The time taken to write 455 words is

$= \frac{455}{7} = 5.909$ min

Excepted no of errors in 5.909 min is

$= 5.909/10 = 0.591$

So the rate parameter $\lambda = 0.591$

The pmf of a poisson's distribution is

$$f(x) = \frac{\lambda^x \cdot e^{-\lambda}}{x!}$$

The prob that 2 errors will be committed in a 455 word report is

$$P = \frac{0.591^2 \cdot e^{-0.591}}{2!}$$

$= 0.097$

If the number of words decreases, the time taken to write them will decrease. If the time decrease's, the Expected no of error's in that time period will also decreases.

Hence λ will decrease.

If the no of words is 255, then

$\lambda = \frac{255}{77} * 6/60 = 0.331$

Hence, λ has decreased

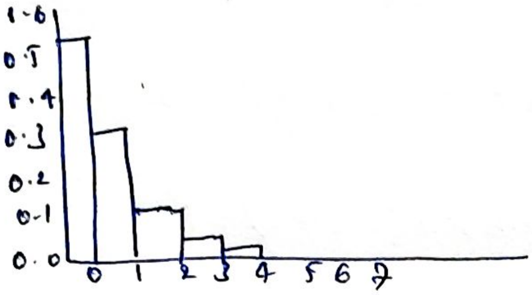
In the case when we want to know the prob of 2 error's, the more closer λ is 2. higher will be the prob.

So if the no of words increases to 1000, λ will increase to 1.299 which is closer to 2 then the case the no of words decreases to 255 in which λ decreases to 0.331 and gets farther from 2.

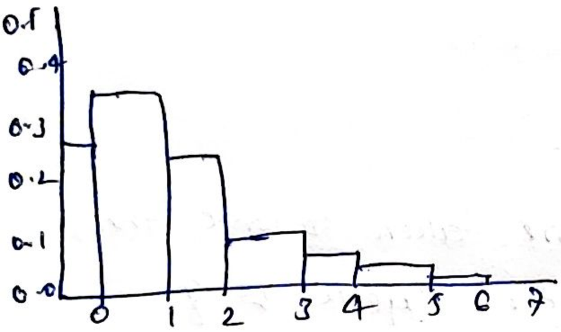
Cont..

Hence, the prob of making 2 error will increases if the no of words increased, and the prob of making 2 errors will be decrease if the no of words is decreased

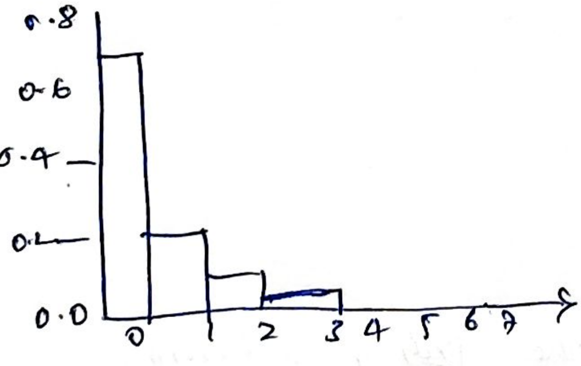
(x) the graph when $\lambda = 0.591$



the graph when no of words increases to 1000 and $\lambda = 1.299$ is



The graph when the no of words decreases to 255 the $\lambda = 0.33$ is



As we can see, for higher no of words, the prob of making 2 errors $(x=2)$ is the highest

Problem Statement - 10

Soln

(a) $P(Z>1.26)$, $P(Z<-0.86)$, $P(Z>-1.37)$, $P(-1.25<Z<0.37)$

$P(Z\leq -4.6)$

$P(Z<-0.86)$
$= 1-P(Z\geq 0.86)$
$= 1-0.8051$
$= 0.1949$

$\Rightarrow P(Z>1.26)$
$= 1-P(Z\leq 1.26)$
$= 1-0.8962$
$= 0.1038$

$P(Z>-1.37)$
$= 1-P(Z\leq -1.37)$
$= 1-0.0853$
$= 0.9147$

$P(-1.25<Z<0.37)$

$P(Z<0.37)$
$= 0.6443$

$P(Z>-1.25)$
$= 0.1056.$

$0.6443+0.1056$
$= 0.5387.$

$P(Z\leq -4.6)$

max value of Z -3.49 is 0.0002

$\therefore P(Z\leq -4.6)$ is $\simeq 0$

(b) $P(Z>z)=0.05$

$P(Z>z)=0.05$
$= 0.95$

(c) $P(-z<Z<z)=0.99$

$P(Z<z)=0.99$
$= 2.33$

$P(-z<Z)=0.01$
$= -2.33$

Problem statement - 11

Soln

$\mu = 10mA$ $\quad$ $P(x > 13)$

$\sigma^2 = 4$

$\sigma = \sqrt{4}$ $\quad$ $Z = \frac{13-10}{2}$

$= 2$

$= 3/2 = 1.5$

$= \underline{0.9332}$

$Z = 1 - 0.9332$

$= \underline{0.0668}$

$P(9 < x < 11)$

$P(x < 11) = \frac{11-10}{2} = 1/2$

$= 0.5$

$= \underline{0.6915}$

$P(x > 9) = \frac{9-10}{2} = -1/2$

$= 0.5$

$= 0.3085$

$P(9 < x < 11) = 0.6915 - 0.3085$

$= 0.383$

$P(Z < 2.06)$ is 0.98

$= 2.06$

$\boxed{Z = 2.06}$

$x = 2(2.06) + 10$

$= \underline{14.12}$

Problem Statement 12

Soln: $\mu = 0.2508$

$\sigma = 0.0005$

$0.2508 < x < 0.2515$

$$Z = \frac{0.2510 - 0.2508}{0.0005}$$

$= 1.4$

$0.2506 > x > 0.2485$

$$z = \frac{0.2485 - 0.2506}{0.0005}$$

$= -4.6$

$P(z) = P(1.4)$

$= 0.9192$

$\mu = 0.02500$

$$Z = \frac{0.2515 - 2500}{0.0005}$$

$= 3$

$$Z = \frac{0.2485 - 2500}{0.0005}$$

$= -3$

$P(Z = 3) = 0.9987$

$P(Z = -3) = 0.0013$

$\therefore = 0.9987 - 0.0013$

$\boxed{= 0.9974}$