Lab 3

Problem 1:

1. $10000000000n^2$ $\quad$ n^3

$O(n^2) < O(n^3)$ ✓

2. $n^2 \log n$ ✓ $\quad$ $n(\log n^{10})$

$\log(n^2 \log n)$ $\quad$ $\log(n \log n^{10})$

$\log n^2 + \log\log n$ $\quad$ $\log n + \log \log n^{10}$

$2\log n + \log\log n > \log n + 10 \log\log n$

3. $n^{\log n}$ ✓ $\quad$ $2^{\sqrt{n}}$

$\log n^{\log n}$ $\quad$ $\log 2^{\sqrt{n}}$

$\log n \log n$ $\quad$ $\sqrt{n} \log 2$

$\log^2 n$ $\quad$ $\sqrt{n}$

$2\log n > \frac{1}{2}\log n$

4. 2^n $\quad$ 2^{2n} ✓

$\log 2^n$ $\quad$ $\log 2^{2n}$

$n \log 2$ $\quad$ $2n \log 2$

$n < 2n$

However, in terms of asymptotic notation, both would be $O(2^n)$.

Problem 2:

1. The best case would be when the input n would be less than 4 in which case it would take $\Theta(1)$ time.
2. The worst case would be when the input is a prime number, in which case it would take $\Theta(\sqrt{n})$ time.
3. The average case is not very straightforward to calculate, but in the majority of cases, the function will need to check until the square root of a number to ascertain if it is prime. Hence, the average case time complexity will stay at $\Theta(\sqrt{n})$.

The time complexity would be $\Theta(\sqrt{n})$.