

# Programming Assignment 3

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How to run: Store all the files in a single folder and run main to run ODE45 of MATLAB. For Gazebo simulation run rrbot\_control.

## Cubic polynomials trajectory generator

The trajectory\_generator function takes in initial and final state of one the variable of the system as a single matrix and start and end time of the trajectory. The equation of motion for states in the “step a” are show below:

$$th1\_des = (\pi * t^3)/500 - (3 * \pi * t^2)/100 + \pi;$$

$$th1dot\_des = (3 * \pi * t^2)/500 - (3 * \pi * t)/50;$$

$$th2\_des = (\pi * t^3)/1000 - (3 * \pi * t^2)/200 + \pi/2;$$

$$th2dot\_des = (3 * \pi * t^2)/1000 - (3 * \pi * t)/100;$$

## Standard manipulator form

Below mentioned is the standard manipulator form the dynamic equation of the 2-link robot:

$$T = m*[theta\_ddot1; theta\_ddot2] + c*[theta\_dot1; theta\_dot2] + G$$

$$T = [theta\_ddot1 * (m1 * r1^2 + I1 + I2 + (m2 * (2 * l1^2 + 4 * \cos(theta2) * l1 * r2 + 2 * r2^2))/2) + theta\_ddot2 * (I2 + (m2 * (2 * r2^2 + 2 * l1 * \cos(theta2) * r2))/2) - g * m2 * (r2 * \sin(theta1 + theta2) + l1 * \sin(theta1)) - g * m1 * r1 * \sin(theta1) + l1 * r2 * theta\_dot1 * \sin(theta2) - l1 * m2 * r2 * theta\_dot2 * \sin(theta2) * (theta\_dot1 + theta\_dot2) ;$$

$$theta\_ddot2 * (m2 * r2^2 + I2) + theta\_ddot1 * (I2 + (m2 * (2 * r2^2 + 2 * l1 * \cos(theta2) * r2))/2) - g * m2 * r2 * \sin(theta1 + theta2) + l1 * m2 * r2 * theta\_dot1 * \sin(theta2) * (theta\_dot1 + theta\_dot2) - l1 * m2 * r2 * theta\_dot1 * theta\_dot2 * \sin(theta2)]$$

## Feedback Linearization

Virtual inputs are calculated as shown below:

$$v_1 = -k_{p1}*(\theta_1 - q_{1\_t(1)}) - k_{d1}*(\dot{\theta}_1 - q_{1\_t(2)}) + \dot{v}_d(1)$$

$$v_2 = -k_{p2}*(\theta_2 - q_{2\_t(1)}) - k_{d2}*(\dot{\theta}_2 - q_{2\_t(2)}) + \dot{v}_d(2)$$

Then  $v_1$  and  $v_2$  are substituted into the original equation of motion calculated in assignment1 to get  $T_1$  and  $T_2$ :

$$T_1 = \text{subs}(u_1, \{\theta_{\ddot{1}}, \theta_{\ddot{2}}\}, \{v_1, v_2\})$$

$$T_2 = \text{subs}(u_2, \{\theta_{\ddot{1}}, \theta_{\ddot{2}}\}, \{v_1, v_2\})$$

After obtaining  $T_1$  and  $T_2$  we form new equation as shown below, by subtracting  $T_1$  and  $T_2$  from  $u_1$  and  $u_2$  respectively. And solve those equation for  $\theta$  double dot, which are used in the ODE function.

$$eq_1 = u_1 - T_1;$$

$$eq_2 = u_2 - T_2;$$

$$\text{sol} = \text{solve}([eq_1 == 0, eq_2 == 0], [\theta_{\ddot{1}}, \theta_{\ddot{2}}]);$$

$$\theta_{\ddot{1}} = \text{sol}.\theta_{\ddot{1}} ;$$

$$\theta_{\ddot{2}} = \text{sol}.\theta_{\ddot{2}} ;$$

## ODE function

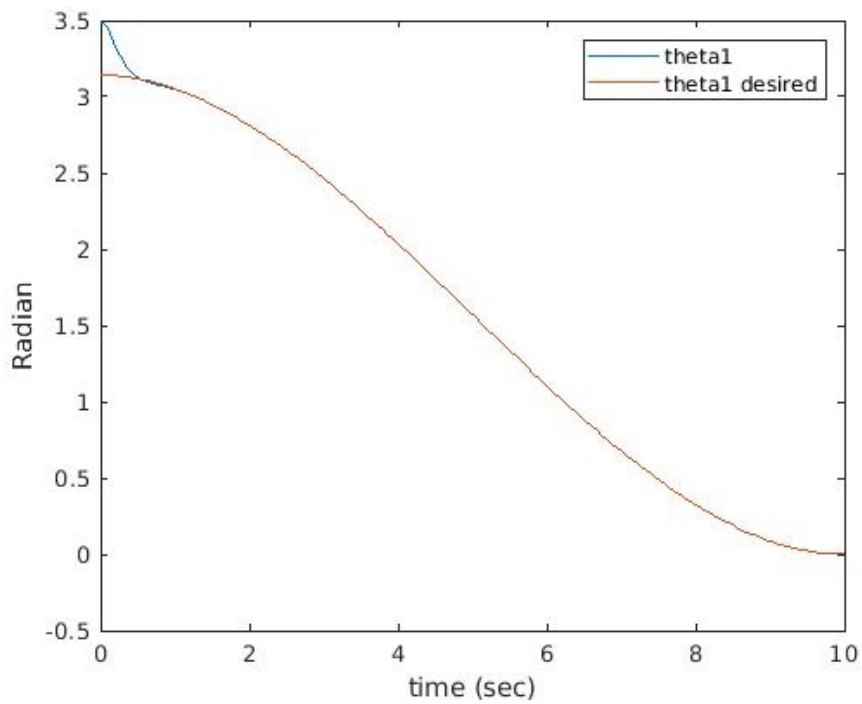
The only change in ODE function is the replacement of the  $\theta_{1\_dd}$  and  $\theta_{2\_dd}$  by the values that we calculated above:

$$\begin{aligned} \theta_{\ddot{1}} = & (10*\pi*k_{p1})/9 - \pi/15 + (\pi*t)/75 - k_{d1}*\dot{\theta}_1 - k_{p1}*\theta_1 - (\pi*k_{d1}*t)/15 + \\ & (\pi*k_{d1}*t^2)/150 - (\pi*k_{p1}*t^2)/30 + (\pi*k_{p1}*t^3)/450; \end{aligned}$$

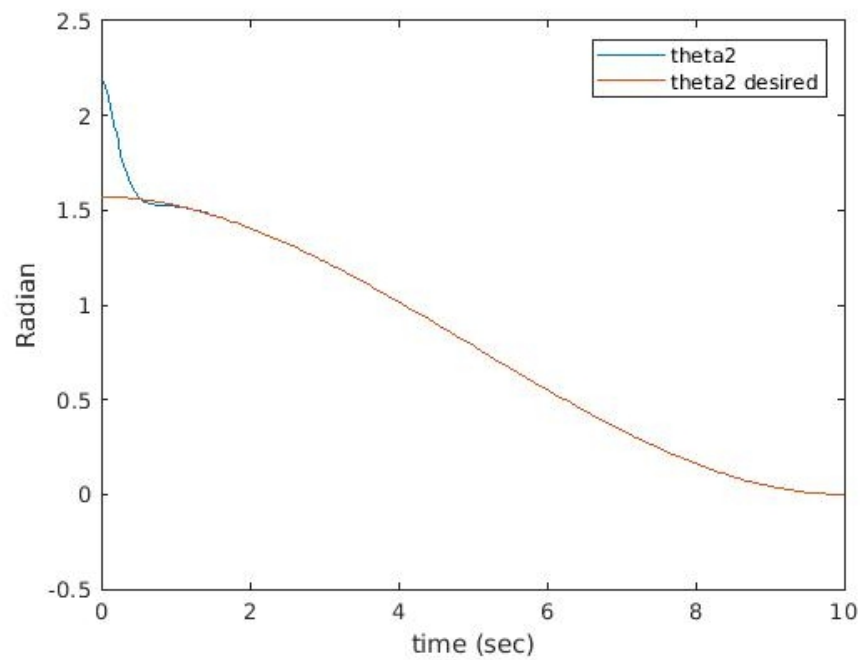
$$\begin{aligned} \theta_{\ddot{2}} = & (25*\pi*k_{p2})/36 - \pi/24 + (\pi*t)/120 - k_{d2}*\dot{\theta}_2 - k_{p2}*\theta_2 - (\pi*k_{d2}*t)/24 \\ & + (\pi*k_{d2}*t^2)/240 - (\pi*k_{p2}*t^2)/48 + (\pi*k_{p2}*t^3)/720; \end{aligned}$$

## ODE function output

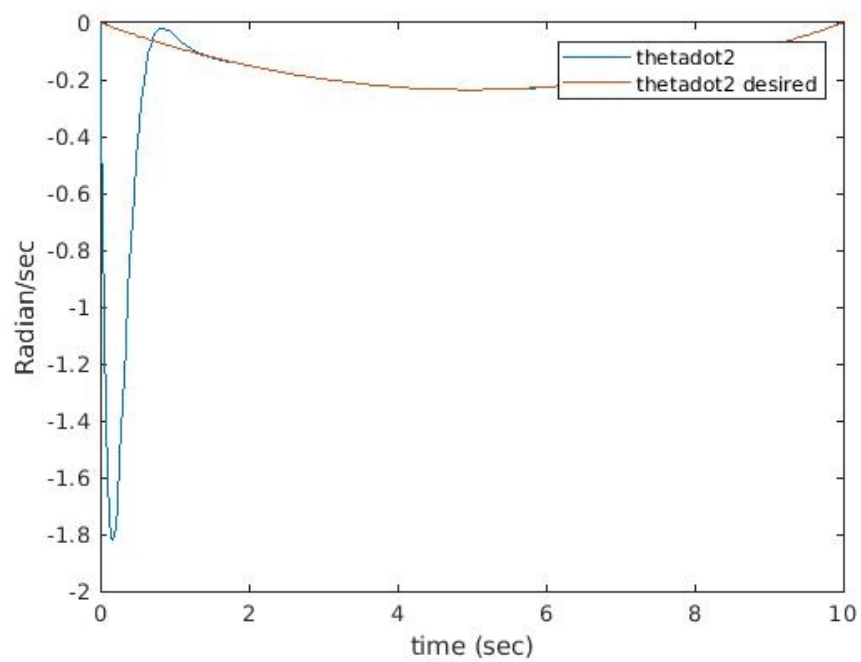
Theta1:



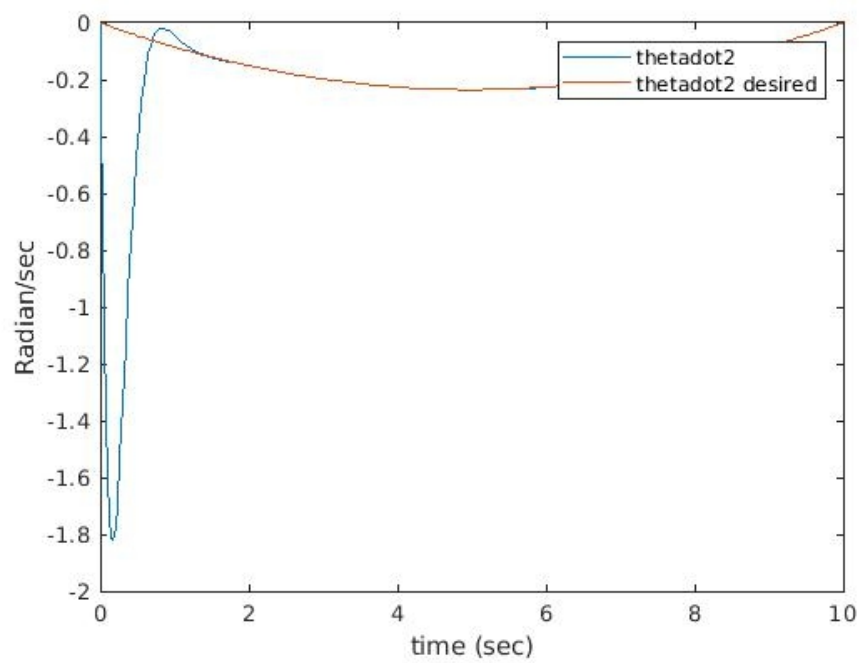
Theta2:



Theta1\_dot:

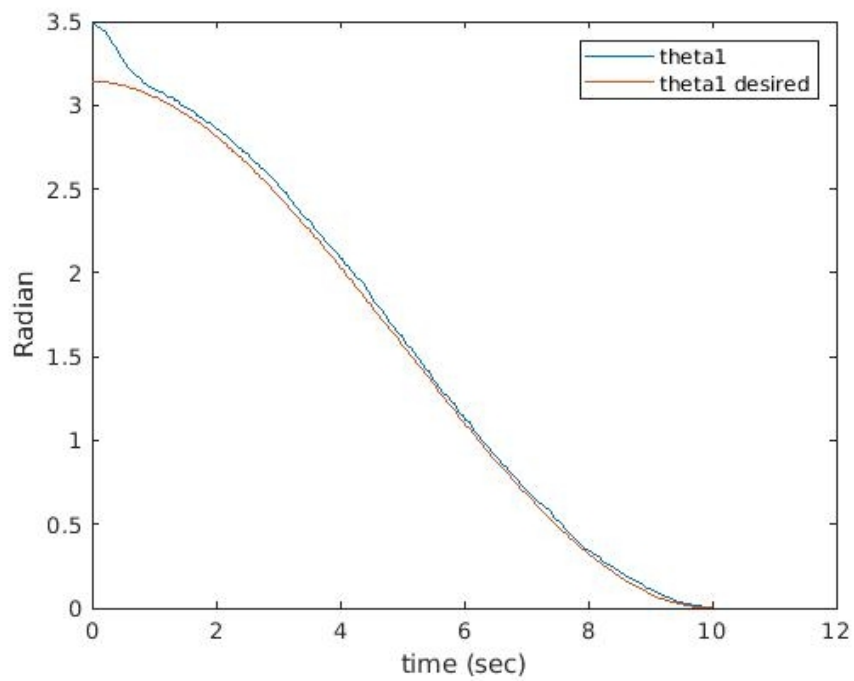


Theta2\_dot:

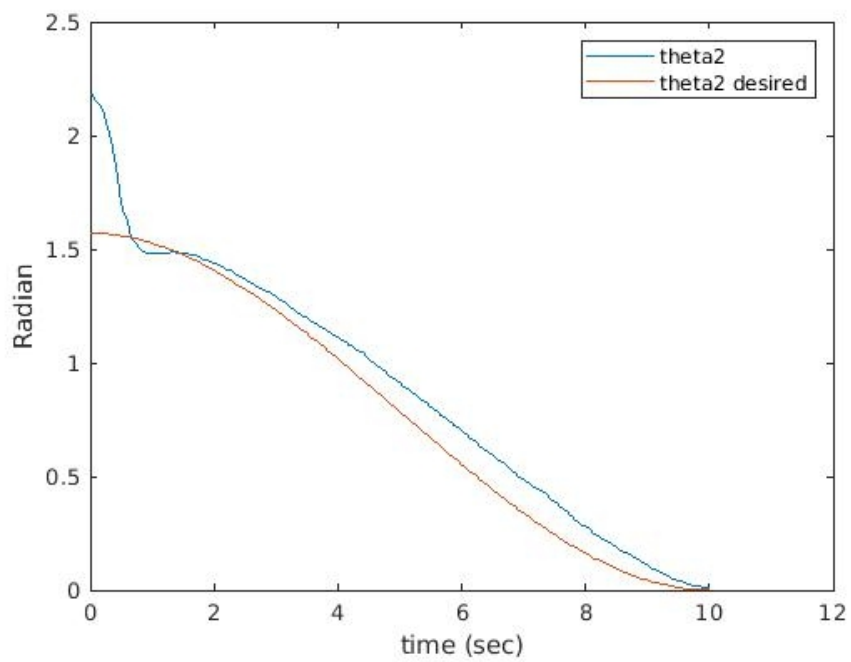


## Gazebo Simulation

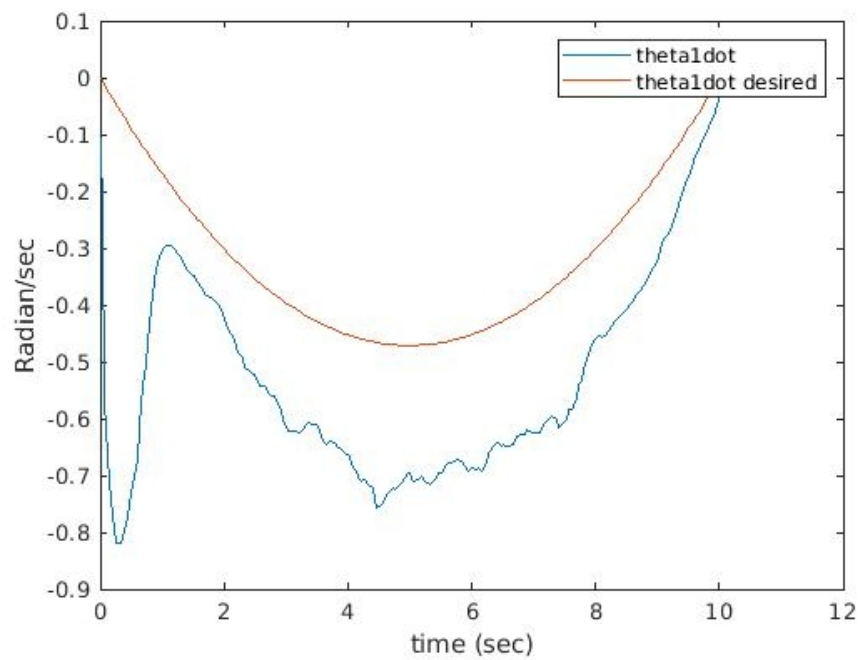
Theta1:



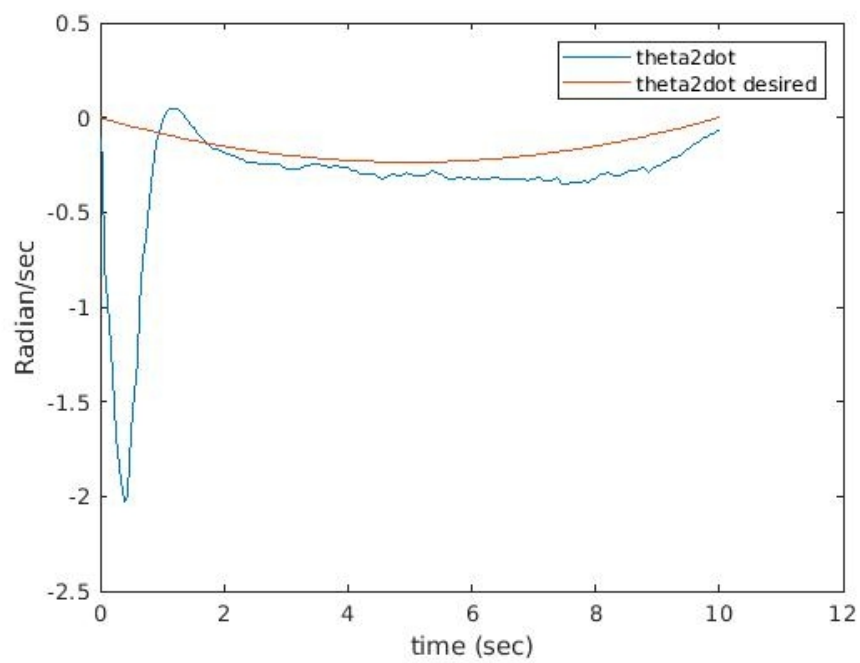
Theta2:



Theta1\_dot:



Theta2\_dot:



Torque input:

