RBE 521 Homework 2

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Solution 1

(a) Leg Length for ZYZ Euler angle: [128.3063, 141.8588, 156.4394, 157.3413, 143.6902, 129.2379]

(b) Leg Length for XYZ Euler angle: [132.0922 ,151.6577 ,161.0500 ,151.0929 ,133.3069 ,123.8025]

(c) Leg Length for both cases are not same because the sequence of rotation axis is not the same.

Rotation Matrix for ZYZ is obtained as follows:

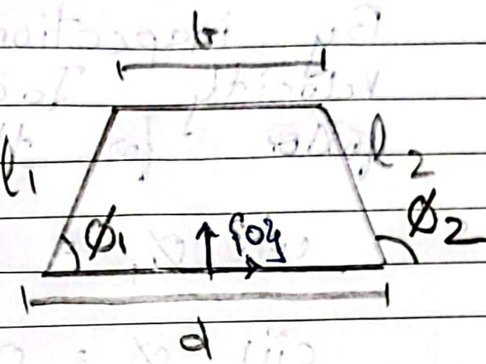
$R_{zyz} = R_z * R_{y*} R_z$

While Rotation Matrix for XYZ is obtained as follows:

$R_{xyz} = R_x * R_{y*} R_z$

Clearly, $R_{zyz} \neq R_{xyz}$. Due to this the translational position of the mobile platform remains same but the orientation of the mobile platform changes.

* Solution 2



$l_{min} = 100$

$l_{max} = 200$

$b = 300$

$d = 500$

For $\theta_1 = \theta_3 = 0$, we know that the mobile and fixed platform are horizontal

Velocity Jacobian, $J_{vi} = [\, n_i^T \;\; [R s_i \times n_i]^T \,]$

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$, R is identity matrix because mobile platform is always horizontal

$$\therefore s_1 = \begin{bmatrix} -b/2 \\ 0 \end{bmatrix} = \begin{bmatrix} -150 \\ 0 \end{bmatrix}, \quad s_2 = \begin{bmatrix} b/2 \\ 0 \end{bmatrix} = \begin{bmatrix} 150 \\ 0 \end{bmatrix}$$

$$\vec{n}_1 = \begin{bmatrix} \cos\phi_1 \\ \sin\phi_1 \end{bmatrix}, \quad \vec{n}_2 = \begin{bmatrix} \cos\phi_2 \\ \sin\phi_2 \end{bmatrix}$$

As the mobile platform is always horizontal and workspace is a COW, we can ignore the angular velocity part of J_{vi}

$$\therefore J_{vi} = [\, n_i^T \,]$$

$$J_v = \begin{bmatrix} \cos\phi_1 & \sin\phi_1 \\ \cos\phi_2 & \sin\phi_2 \end{bmatrix}$$

By inspection we notice that the velocity Jacobian loses rank ~~for the~~ ~~for~~ for the following condition:

(i) $\phi_1 = \phi_2$

(ii) $\phi_1 = 0 \qquad \phi_2 = \pi$

Due to geometry of the robot and maximum and minimum leg length condition cases for $\phi_1 = \phi_2$ lie outside the work space of the robot.

$$l_i \vec{n_i} = \vec{O} + R\vec{s_i} - \vec{u_i}$$

$$l_1 \begin{bmatrix} \cos\phi_1 \\ \sin\phi_1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -150 \\ 0 \end{bmatrix} - \begin{bmatrix} -250 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} l_1 \\ 0 \end{bmatrix} = \begin{bmatrix} x+100 \\ y \end{bmatrix}, \text{ for } \phi_1 = 0$$

$$l_2 \begin{bmatrix} \cos\phi_2 \\ \sin\phi_2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 150 \\ 0 \end{bmatrix} - \begin{bmatrix} 250 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -l_2 \\ 0 \end{bmatrix} = \begin{bmatrix} x-100 \\ y \end{bmatrix}, \text{ for } \phi_2 = \pi$$

From inspection we get,

$x = 0$, $y = 0$, $l_1 = l_2 = 100$

Thus, when end effect is at $(0, 0)$ and both leg length are 100 we get singularity configuration.

* Solution 3

(i) XYZ euler angles

For rotation about x-axis

$$\vec{\omega_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \dot{a}$$

After 1st rotation about x-axis new y-axis can be represented as

$$y = \begin{bmatrix} 0 \\ \cos(a) \\ \sin(a) \end{bmatrix}$$

Therefore, $\vec{\omega_2} = \begin{bmatrix} 0 \\ \cos(a) \\ \sin(a) \end{bmatrix} \dot{b}$

After rotation about x-axis and then about y-axis, new z-axis can be represented as:

$$z = \begin{bmatrix} \sin(b) \\ -\cos(b) \times \sin(a) \\ \cos(b) \times \cos(a) \end{bmatrix}$$

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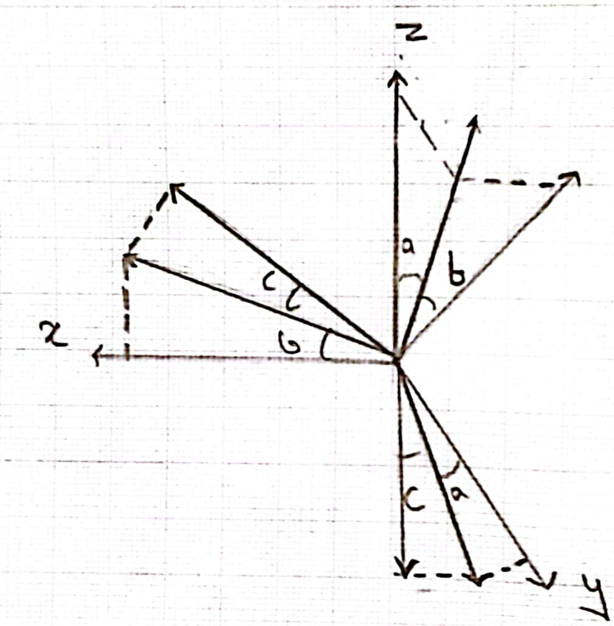
Therefore, $\vec{\omega}_3 = \begin{bmatrix} \sin b \\ -\cos(b) \times \sin(a) \\ \cos(b) \times \cos(a) \end{bmatrix} \dot{c}$

$$\vec{\omega} = \vec{\omega}_1 + \vec{\omega}_2 + \vec{\omega}_3 = \begin{bmatrix} \dot{a} + \sin(b)\dot{c} \\ \cos(a)\dot{b} + (-\cos(b)\sin(a))\dot{c} \\ \sin(a)\dot{b} + (\cos(b)\cos(a))\dot{c} \end{bmatrix}$$

$$\vec{\omega} = \begin{bmatrix} 1 & 0 & \sin(b) \\ 0 & \cos(a) & -\cos(b)\sin(a) \\ 0 & \sin(a) & \cos(b)\cos(a) \end{bmatrix} \begin{bmatrix} \dot{a} \\ \dot{b} \\ \dot{c} \end{bmatrix}$$

$$\therefore \quad B(\alpha) = \begin{bmatrix} 1 & 0 & \sin(b) \\ 0 & \cos(a) & -\cos(b)\sin(a) \\ 0 & \sin(a) & \cos(b)\cos(a) \end{bmatrix}$$



XYZ Euler angles

(ii) Z Y X Euler angles

For rotation in z-axis

$$\vec{\omega}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \dot{a}$$

After rotation in z-axis new y axis can be represented as

$$y = \begin{bmatrix} -\sin a \\ \cos a \\ 0 \end{bmatrix}$$

$$\vec{\omega}_2 = \begin{bmatrix} -\sin a \\ \cos a \\ 0 \end{bmatrix} \dot{b}$$

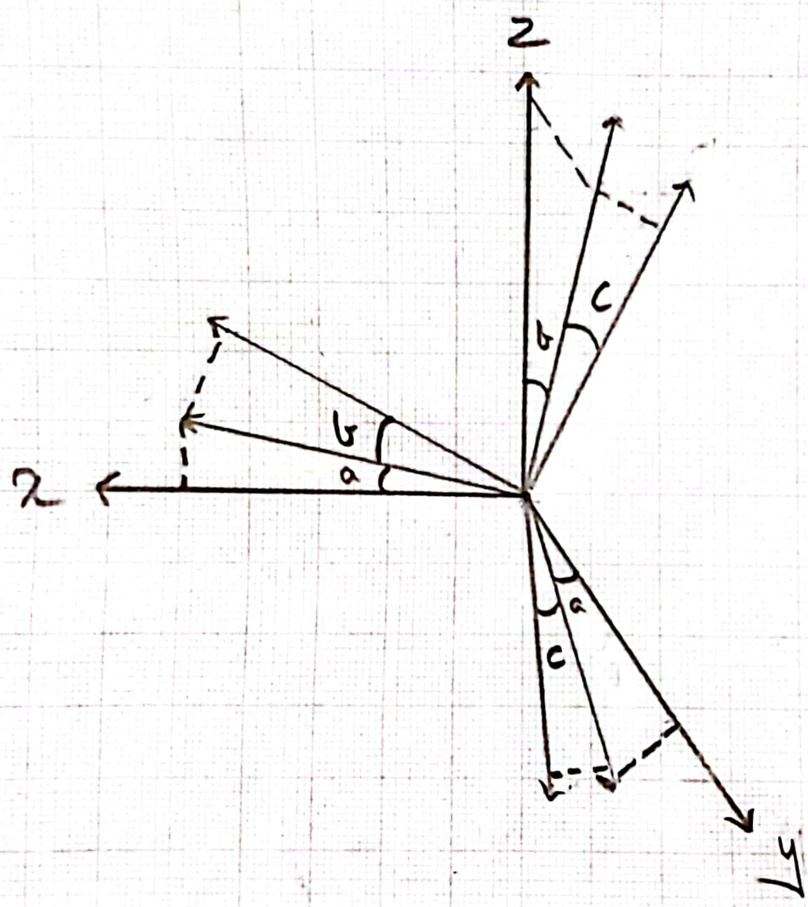
After rotation in z-axis and y-axis new x-axis can be represented as:

$$x = \begin{bmatrix} \cos(a)\cos(b) \\ \cos(b)\sin(a) \\ -\sin(b) \end{bmatrix}$$

$$\vec{\omega}_3 = \begin{bmatrix} \cos(a)\cos(b) \\ \cos(b)\sin(a) \\ -\sin(b) \end{bmatrix} \dot{c}$$

$$\vec{\omega} = \begin{bmatrix} 0 & -\sin a & \cos(a)\cos(b) \\ 0 & \cos a & \cos(b)\sin(a) \\ 1 & 0 & -\sin b \end{bmatrix} \begin{bmatrix} \dot{a} \\ \dot{b} \\ \dot{c} \end{bmatrix}$$

$$\therefore B(\alpha) = \begin{bmatrix} 0 & -\sin a & \cos(a)\cos(b) \\ 0 & \cos a & \cos(b)\sin(a) \\ 1 & 0 & -\sin b \end{bmatrix}$$



ZYX Euler angles