

AI1110 Assignment 2

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Abstract—This document contains the solution for ICSE 2019 class 12 maths Q.15(a)

Problem 15(a): If \mathbf{a} and \mathbf{b} are perpendicular vectors, $\|\mathbf{a} + \mathbf{b}\| = 13$ and $\|\mathbf{a}\| = 5$, find the value of $\|\mathbf{b}\|$.

Solution: We know that

$$\|\mathbf{a} + \mathbf{b}\|^2 = \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 + 2\mathbf{a} \cdot \mathbf{b} \quad (1)$$

Given, \mathbf{a} and \mathbf{b} are perpendicular, hence $\mathbf{a} \cdot \mathbf{b} = 0$, therefore substituting in (1),

$$\|\mathbf{a} + \mathbf{b}\|^2 = \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 \quad (2)$$

Given,

$$\|\mathbf{a} + \mathbf{b}\| = 13 \quad (3)$$

$$\|\mathbf{a}\| = 5 \quad (4)$$

Substituting (3) and (4) in (2),

$$13^2 = 5^2 + \|\mathbf{b}\|^2 \quad (5)$$

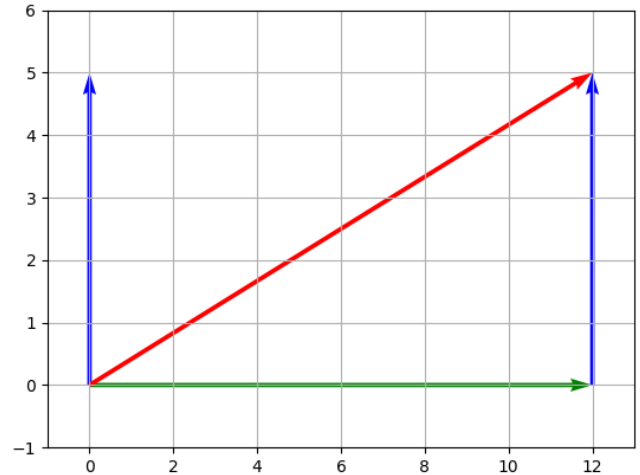
$$\|\mathbf{b}\|^2 = 13^2 - 5^2 \quad (6)$$

$$\|\mathbf{b}\|^2 = 169 - 25 \quad (7)$$

$$\|\mathbf{b}\|^2 = 144 \quad (8)$$

$$\|\mathbf{b}\| = \sqrt{144} \quad (9)$$

$$\therefore \|\mathbf{b}\| = 12 \quad (10)$$



In the figure, blue arrow with its tail at origin represents \mathbf{a} , which can also be displaced to have its tail at the coordinate (12,0) to complete the triangle.

The red arrow represents $\mathbf{a} + \mathbf{b}$ (by Triangle Law of Vector Addition) and green arrow represents \mathbf{b} . Vectors \mathbf{a} and \mathbf{b} are perpendicular, $\|\mathbf{a}\| = 5$ and $\|\mathbf{a} + \mathbf{b}\| = 13$, hence by the diagram, $\|\mathbf{b}\|$ should be 12 since by Baudhāyana Sulbasūtra, 5, 12, and 13 form a triplet which is quite famous ($5^2 + 12^2 = 13^2$).

The output of the python code used for verification of the answer: