AI1110 Assignment 2

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***Abstract*—This document contains the solution for ICSE 2019 class 12 maths Q.15(a)**

Problem 15(a): If $\mathbf{a}$ and $\mathbf{b}$ are perpendicular vectors, $|\mathbf{a}+\mathbf{b}| = 13$ and $|\mathbf{a}| = 5$, find the value of $|\mathbf{b}|$.

Solution: We know that

$$|\mathbf{a}+\mathbf{b}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2 + 2\mathbf{a}.\mathbf{b} \tag{1}$$

Given, $\mathbf{a}$ and $\mathbf{b}$ are perpendicular, hence $\mathbf{a}.\mathbf{b} = 0$, therefore substituting in (1),

$$|\mathbf{a}+\mathbf{b}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2 \tag{2}$$

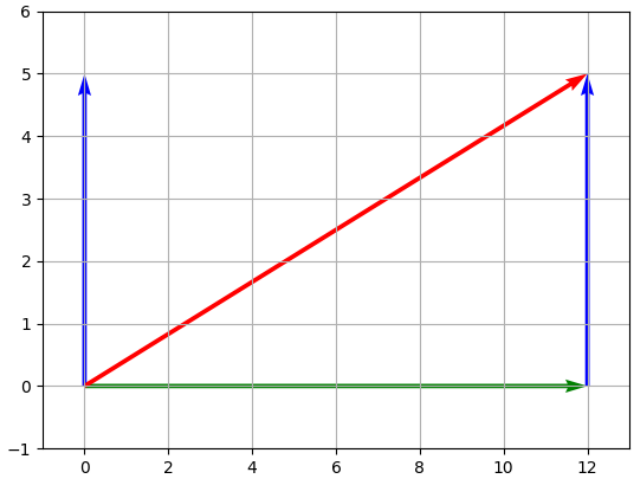
Given,

$$|\mathbf{a}+\mathbf{b}| = 13 \tag{3}$$
$$|\mathbf{a}| = 5 \tag{4}$$

Substituting (3) and (4) in (2),

$$13^2 = 5^2 + |\mathbf{b}|^2 \tag{5}$$
$$|\mathbf{b}|^2 = 13^2 - 5^2 \tag{6}$$
$$|\mathbf{b}|^2 = 169 - 25 \tag{7}$$
$$|\mathbf{b}|^2 = 144 \tag{8}$$
$$|\mathbf{b}| = \sqrt{144} \tag{9}$$
$$\therefore |\mathbf{b}| = 12 \tag{10}$$

The output of the python code used for verification of the answer:



In the figure, blue arrow with its tail at origin represents $\mathbf{a}$, which can also be displaced to have its tail at the coordinate (12,0) to complete the triangle.
The red arrow represents $\mathbf{a}+\mathbf{b}$ (by Triangle Law of Vector Addition) and green arrow represents $\mathbf{b}$. Vectors $\mathbf{a}$ and $\mathbf{b}$ are perpendicular, $|\mathbf{a}| = 5$ and $|\mathbf{a}+\mathbf{b}| = 13$, hence by the diagram, $|\mathbf{b}|$ should be 12 since by Baudhāyana Sulbasūtra, 5, 12, and 13 form a triplet which is quite famous ($5^2 + 12^2 = 13^2$).