AI1110 Assignment 2

Prasham Walvekar (CS21BTECH11047)

***Abstract*—This document contains the solution for ICSE 2019 class 12 maths Q.15(a)**

Problem 15(a): If $\mathbf{a}$ and $\mathbf{b}$ are perpendicular vectors, $\|\mathbf{a}+\mathbf{b}\| = 13$ and $\|\mathbf{a}\| = 5$, find the value of $\|\mathbf{b}\|$.

Solution: We know that

$$\|\mathbf{a}+\mathbf{b}\|^2 = \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 + 2\mathbf{a}^\top\mathbf{b} \quad (1)$$

Given, $\mathbf{a}$ and $\mathbf{b}$ are perpendicular, hence $\mathbf{a}^\top\mathbf{b} = 0$, therefore substituting in (1),

$$\|\mathbf{a}+\mathbf{b}\|^2 = \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 \quad (2)$$

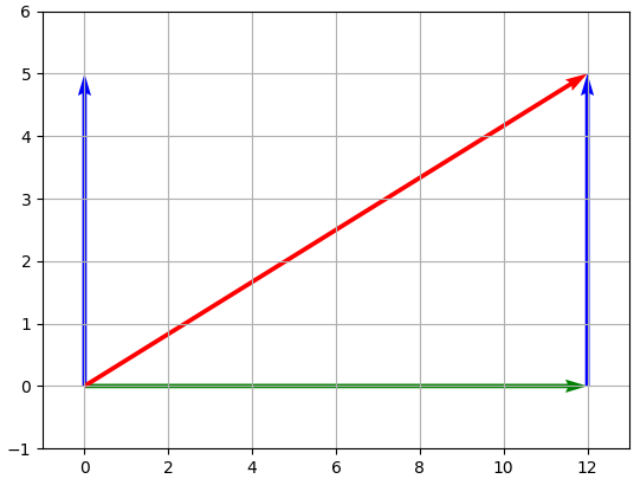
Given,

$$\|\mathbf{a}+\mathbf{b}\| = 13 \quad (3)$$
$$\|\mathbf{a}\| = 5 \quad (4)$$

Substituting (3) and (4) in (2),

$$13^2 = 5^2 + \|\mathbf{b}\|^2 \quad (5)$$
$$\|\mathbf{b}\|^2 = 13^2 - 5^2 \quad (6)$$
$$\|\mathbf{b}\|^2 = 169 - 25 \quad (7)$$
$$\|\mathbf{b}\|^2 = 144 \quad (8)$$
$$\|\mathbf{b}\| = \sqrt{144} \quad (9)$$
$$\therefore \|\mathbf{b}\| = 12 \quad (10)$$

The output of the python code used for verification of the answer:



In the figure, blue arrow with its tail at origin represents $\mathbf{a}$, which can also be displaced to have its tail at the coordinate (12,0) to complete the triangle.
The red arrow represents $\mathbf{a}+\mathbf{b}$ (by Triangle Law of Vector Addition) and green arrow represents $\mathbf{b}$. Vectors $\mathbf{a}$ and $\mathbf{b}$ are perpendicular, $\|\mathbf{a}\| = 5$ and $\|\mathbf{a}+\mathbf{b}\| = 13$, hence by the diagram, $\|\mathbf{b}\|$ should be 12 since by Baudhāyana Sulbasūtra, 5, 12, and 13 form a triplet which is quite famous ($5^2 + 12^2 = 13^2$).