Assignment 7

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CS21BTECH11047

July 3, 2022

Question

Question - 15.5

Consider a population in which the number of offsprings for any individual is at most two. Show that if the probability of occurrence of two offsprings/individuals is less than the probability of occurrence of zero offspring/individual, then the entire population is bound to extinct with probability one.

Solution - I

Let us assume that the various offspring of different individuals are independent, identically distributed, random variables with common distribution given by (over all generations)

$$p_k = \Pr(y = k) = \Pr(\text{an individual has k offspring}) \geq 0 \tag{1}$$

and common moment generating function

$$\Pr(z) = E\{z^y\} = \sum_{k=0}^{\infty} p_k z^k \tag{2}$$

Since, $k \leq 2$, we can write

$$\Pr(z) = p_0 + p_1 z + p_2 z^2 \tag{3}$$

where

$$p_0 + p_1 + p_2 = 1 \tag{4}$$

Solution II

Notice that

$$\Pr(z) - z = p_0 - (1 - p_1)z + p_2 z^2 \tag{5}$$
$$= p_0 - (p_0 + p_2)z + p_2 z^2 \tag{6}$$
$$= (z - 1)(p_2 z - p_0) \tag{7}$$

and hence, the two roots of the equation $\Pr(z) = z$ are given by

$$z_1 = 1 \tag{8}$$
$$z_2 = \frac{p_0}{p_2} \tag{9}$$

Thus, According to the question, if $p_2 < p_0$, then $z_2 > 1$ and hence the smallest positive root of $\Pr(z) = z$ is 1 which represents the probability of extinction. It implies that a tribe that does not produce offspring in abundance is bound to extinct.