AI1110 Assignment Q-1.5

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1.5: Verify your result theoretically given that $E[U^k] = \int_{-\infty}^{\infty} x^k dF_U(x)$
Solution:

$$F_U(x) = \begin{cases} x, & x \in (0,1) \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

$$dF_U(x) = \begin{cases} dx, & x \in (0,1) \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

$$E[U^k] = \int_{-\infty}^{\infty} x^k dF_U(x) \tag{3}$$

$$E[U^k] = \int_0^1 x^k dx \tag{4}$$

$$E[U^k] = \frac{1}{k+1} \tag{5}$$

To verify the result obtained for mean and variance from the C code and by the integral:
The result given by the C code was,
Mean = 0.50007 and Variance = 0.083301.
Now, using the formula

$$E[U^k] = \frac{1}{k+1} \tag{6}$$

$$E[U] = \frac{1}{1+1} = \frac{1}{2} \tag{7}$$

$$E[U^2] = \frac{1}{2+1} = \frac{1}{3} \tag{8}$$

$$\tag{9}$$

Therefore, Mean = $E[U] = 0.5$
Variance =

$$E[U^2] - (E[U])^2 = \frac{1}{3} - \frac{1}{4} \tag{10}$$

$$= \frac{1}{12} = 0.0833 \tag{11}$$

Hence the values of mean and variance are almost similar, and is hence theoretically verified.