AI1110 Assignment 2

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ICSE 2019 12th Board Paper Question 15(a):

If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, $\left|\vec{a}+\vec{b}\right| = 13$ and $|\vec{a}| = 5$, find the value of $\left|\vec{b}\right|$.

Solution:

We know that

$$\left|\vec{a}+\vec{b}\right|^2 = |\vec{a}|^2 + \left|\vec{b}\right|^2 + 2\vec{a}.\vec{b} \quad (1)$$

Given, $\vec{a}$ and $\vec{b}$ are perpendicular, hence $\vec{a}.\vec{b} = 0$, therefore substituting in (1),

$$\left|\vec{a}+\vec{b}\right|^2 = |\vec{a}|^2 + \left|\vec{b}\right|^2 \quad (2)$$

Given,

$$\left|\vec{a}+\vec{b}\right| = 13 \quad (3)$$

$$|\vec{a}| = 5 \quad (4)$$

Substituting (3) and (4) in (2),

$$13^2 = 5^2 + \left|\vec{b}\right|^2 \quad (5)$$

$$\left|\vec{b}\right|^2 = 13^2 - 5^2 \quad (6)$$

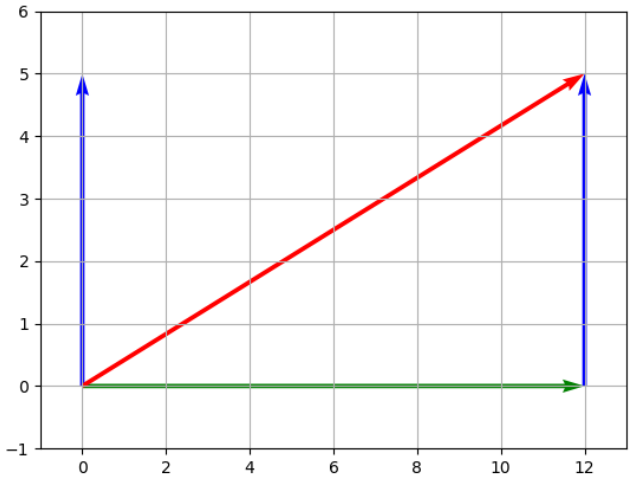
$$\left|\vec{b}\right|^2 = 169 - 25 \quad (7)$$

$$\left|\vec{b}\right|^2 = 144 \quad (8)$$

$$\left|\vec{b}\right| = \sqrt{144} \quad (9)$$

$$\therefore \left|\vec{b}\right| = 12 \quad (10)$$

The output of the python code used for verification of the answer:



In the figure, blue arrow with its tail at origin represents $\vec{a}$, which can also be displaced to have its tail at the coordinate (12,0) to complete the triangle.
The red arrow represents $\vec{a} + \vec{b}$ (by Triangle Law of Vector Addition) and green arrow represents $\vec{b}$. Vectors $\vec{a}$ and $\vec{b}$ are perpendicular, $|\vec{a}| = 5$ and $\left|\vec{a} + \vec{b}\right| = 13$, hence by the diagram, $\left|\vec{b}\right|$ should be 12 since by Baudhāyana Sulbasūtra, 5, 12, and 13 form a triplet which is quite famous ($5^2 + 12^2 = 13^2$)