

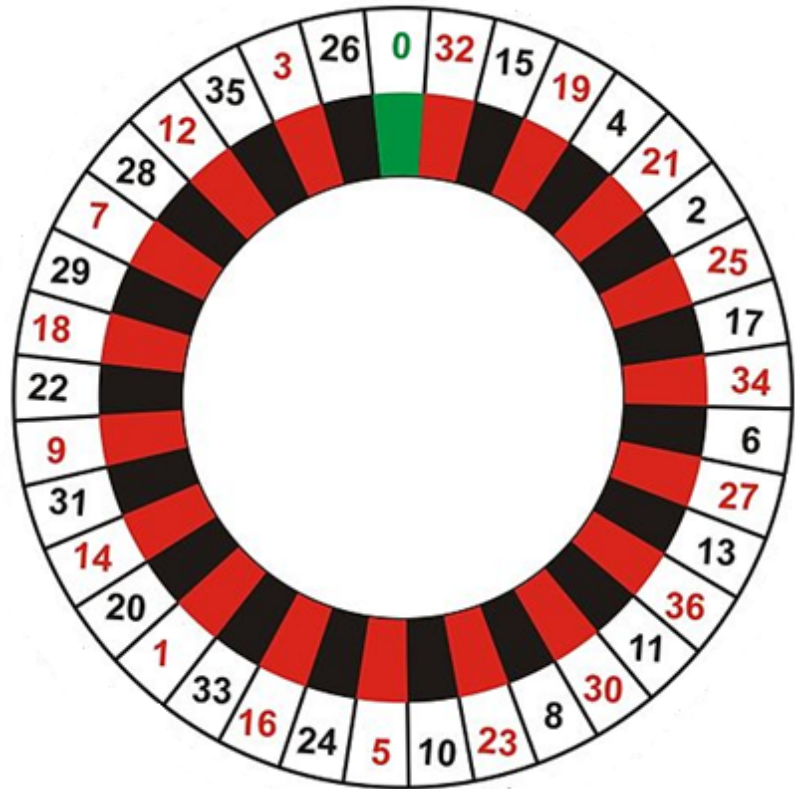
# Inferential Statistics and Probability, Segment 2

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# Casinos Not in the Business of Being Fair



American  
Roulette



CC-BY Darsie (modified)

European  
Roulette

# Comparing Different Games

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**Simulate betting a pocket for 20 trials of 100 spins each**

**Exp. return for Fair Roulette = 22.4%**

**Exp. return for European Roulette = -8.2%**

**Exp. return for American Roulette = 4.4%**

**Simulate betting a pocket for 20 trials of 1000 spins each**

**Exp. return for Fair Roulette = 3.68%**

**Exp. return for European Roulette = -5.5%**

**Exp. return for American Roulette = -4.24%**

**Simulate betting a pocket for 20 trials of 10000 spins each**

**Exp. return for Fair Roulette = 0.998%**

**Exp. return for European Roulette = -6.364%**

**Exp. return for American Roulette = -4.582%**

**Simulate betting a pocket for 20 trials of 100000 spins each**

**Exp. return for Fair Roulette = 0.125%**

**Exp. return for European Roulette = -3.313%**

**Exp. return for American Roulette = -5.5936%**

# Sampling Space of Possible Outcomes

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- Never possible to guarantee perfect accuracy through sampling
- Not to say that an estimate is not precisely correct
- How many samples do we need to look at before we can have justified confidence on our answer?
  - Depends upon variability in underlying distribution

# Quantifying Variation in Data

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$$\text{variance}(X) = \frac{\sum_{x \in X} (x - \mu)^2}{|X|}$$

$$\sigma(X) = \sqrt{\frac{1}{|X|} \sum_{x \in X} (x - \mu)^2}$$

- Standard deviation simply the square root of the variance
- Outliers can have a big effect
- Standard deviation should always be considered relative to mean

# For Those Who Prefer Code

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```
def getMeanAndStd(X):  
    mean = sum(X)/float(len(X))  
    tot = 0.0  
    for x in X:  
        tot += (x - mean)**2  
    std = (tot/len(X))**0.5  
    return mean, std
```

# Confidence Levels and Intervals

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- Instead of estimating an unknown parameter by a single value (e.g., the mean of a set of trials), a confidence interval provides a range that is likely to contain the unknown value and a confidence that the unknown value lays within that range
- “The return on betting on 2 twenty times in European roulette is -3.3%. The margin of error is +/- 1 percentage point with a 95% level of confidence.”

# Empirical Rule

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- Under some assumptions discussed later
  - ~68% of data within one standard deviation of mean
  - ~95% of data within 2 standard deviations of mean
  - ~99.7% of data within 3 standard deviations of mean



# Applying Empirical Rule

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```
numTrials = 20
resultDict = {}
games = (FairRoulette, EuRoulette, AmRoulette)
for G in games:
    resultDict[G().__str__()] = []
for numSpins in (100, 1000, 10000):
    print('\nSimulate betting a pocket for', numTrials,
          'trials of', numSpins, 'spins each')
    for G in games:
        pocketReturns = findPocketReturn(G(), 20, numSpins, False)
        mean, std = getMeanAndStd(pocketReturns)
        resultDict[G().__str__()].append((numSpins,
                                             100*mean, 100*std))
    print('Exp. return for', G(), '=', str(round(100*mean, 3))
          + '%,', ' +/- ' + str(round(100*1.96*std, 3))
          + '% with 95% confidence')
```

# Results

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**Simulate betting a pocket for 20 trials of 1000 spins each**

**Exp. return for Fair Roulette = 3.68%, +/- 27.189% with 95% confidence**

**Exp. return for European Roulette = -5.5%, +/- 35.042% with 95% confidence**

**Exp. return for American Roulette = -4.24%, +/- 26.494% with 95% confidence**

**Simulate betting a pocket for 20 trials of 100000 spins each**

**Exp. return for Fair Roulette = 0.125%, +/- 3.999% with 95% confidence**

**Exp. return for European Roulette = -3.313%, +/- 3.515% with 95% confidence**

**Exp. return for American Roulette = -5.594%, +/- 4.287% with 95% confidence**

**Simulate betting a pocket for 20 trials of 1000000 spins each**

**Exp. return for Fair Roulette = 0.012%, +/- 0.846% with 95% confidence**

**Exp. return for European Roulette = -2.679%, +/- 0.948% with 95% confidence**

**Exp. return for American Roulette = -5.176%, +/- 1.214% with 95% confidence**