

1 Knapsack Problem

Algorithm 1: Knapsack

Input: W (weight capacity), $wt[]$ (weights), $val[]$ (values), n (number of items)

Output: Maximum value that can be put in a knapsack of capacity W

Initialize $K[n+1][W+1]$ with 0;

for $i = 0$ *to* n **do**

for $w = 0$ *to* W **do**

if $i = 0$ *or* $w = 0$ **then**

$K[i][w] \leftarrow 0$;

else if $wt[i-1] \leq w$ **then**

$K[i][w] \leftarrow \max(val[i-1] + K[i-1][w-wt[i-1]], K[i-1][w])$;

else

$K[i][w] \leftarrow K[i-1][w]$;

end

end

end

return $K[n][W]$;

2 Coin Change Problem

Algorithm 2: Coin Change

Input: coins[] (coin denominations), amount (target amount)
Output: Minimum number of coins needed to make up the amount

Initialize dp[amount+1] with amount+1;
dp[0] \leftarrow 0;
for $i = 1$ to amount **do**
 for each coin in coins **do**
 if coin $\leq i$ **then**
 dp[i] \leftarrow min(dp[i], dp[i - coin] + 1);
 end
 end
end
if dp[amount] \neq amount **then**
 return -1;
else
 return dp[amount];
end

3 Longest Common Subsequence (LCS)

Algorithm 3: Longest Common Subsequence

Input: S1, S2 (two sequences)

Output: Length of LCS and the LCS itself

$m \leftarrow \text{length}(S1)$, $n \leftarrow \text{length}(S2)$;

Initialize $\text{dp}[m+1][n+1]$ with 0;

for $i = 1$ *to* m **do**

for $j = 1$ *to* n **do**

if $S1[i-1] = S2[j-1]$ **then**

$\text{dp}[i][j] \leftarrow \text{dp}[i-1][j-1] + 1$;

else

$\text{dp}[i][j] \leftarrow \max(\text{dp}[i-1][j], \text{dp}[i][j-1])$;

end

end

end

$\text{lcs} \leftarrow$ empty string;

$i \leftarrow m$, $j \leftarrow n$;

while $i \neq 0$ *and* $j \neq 0$ **do**

if $S1[i-1] = S2[j-1]$ **then**

 Append $S1[i-1]$ to front of lcs;

$i \leftarrow i - 1$;

$j \leftarrow j - 1$;

else if $\text{dp}[i-1][j] \neq \text{dp}[i][j-1]$ **then**

$i \leftarrow i - 1$;

else

$j \leftarrow j - 1$;

end

end

return ($\text{dp}[m][n]$, lcs);

4 Karatsuba Integer Multiplication

Algorithm 4: Karatsuba Integer Multiplication

Input: x, y (two integers)

Output: Product of x and y

if $x \leq 10$ or $y \leq 10$ **then**

return $x * y$;

$n \leftarrow \max(\text{number of digits in } x, \text{number of digits in } y)$;

$m \leftarrow n / 2$;

$a \leftarrow$ left half of x ;

$b \leftarrow$ right half of x ;

$c \leftarrow$ left half of y ;

$d \leftarrow$ right half of y ;

$ac \leftarrow \text{Karatsuba}(a, c)$;

$bd \leftarrow \text{Karatsuba}(b, d)$;

$ad_bc \leftarrow \text{Karatsuba}(a + b, c + d) - ac - bd$;

return $ac * 10^{2m} + ad_bc * 10^m + bd$;

5 Matrix Multiplication

Algorithm 5: Matrix Multiplication

Input: $A[n][n]$, $B[n][n]$ (two $n \times n$ matrices)

Output: $C[n][n]$ (product of A and B)

Initialize $C[n][n]$ with 0;

for $i = 1$ *to* n **do**

for $j = 1$ *to* n **do**

for $k = 1$ *to* n **do**

$C[i][j] \leftarrow C[i][j] + A[i][k] * B[k][j];$

end

end

end

return C ;

6 Closest Pair Problem

Algorithm 6: Closest Pair

Input: P (set of points)

Output: Smallest distance between any two points in P

Sort P by x-coordinate;

return ClosestUtil(P , $|P|$);

Function *ClosestUtil*(P , n):

if $n \leq 3$ **then**

return BruteForce(P , n);

$\text{mid} \leftarrow n / 2$;

$\text{midPoint} \leftarrow P[\text{mid}]$;

$\text{dl} \leftarrow \text{ClosestUtil}(P[1..\text{mid}], \text{mid})$;

$\text{dr} \leftarrow \text{ClosestUtil}(P[\text{mid}+1..n], n - \text{mid})$;

$d \leftarrow \min(\text{dl}, \text{dr})$;

$\text{strip}[] \leftarrow$ points in P whose x-distance from midPoint $\leq d$;

return $\min(d, \text{StripClosest}(\text{strip}, |\text{strip}|, d))$;

Function *StripClosest*($\text{strip}[]$, size , d):

 Sort strip by y-coordinate;

for $i = 1$ **to** size **do**

$j \leftarrow i + 1$;

while $j \leq \text{size}$ **and** $(\text{strip}[j].y - \text{strip}[i].y) \leq d$ **do**

$d \leftarrow \min(d, \text{dist}(\text{strip}[i], \text{strip}[j]))$;

$j \leftarrow j + 1$;

end

end

return d ;

7 Maxima Set Problem

Algorithm 7: Maxima Set

Input: S (set of points)

Output: Maxima set of S

if $|S| \leq 1$ **then**

return S ;

Sort S by x-coordinate;

$mid \leftarrow |S| / 2$;

$p \leftarrow S[mid]$;

$L \leftarrow \{\text{point} \in S \mid \text{point} < p\}$;

$G \leftarrow \{\text{point} \in S \mid \text{point} \geq p\}$;

$M1 \leftarrow \text{FindMaximaSet}(L)$;

$M2 \leftarrow \text{FindMaximaSet}(G)$;

$q \leftarrow \text{point in } M2 \text{ with minimum x-coordinate}$;

$M1 \leftarrow M1 - \{r \in M1 \mid r.x \leq q.x \text{ and } r.y \leq q.y\}$;

return $M1 \cup M2$;
