**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans:** Probability that the services manager will not meet his demand will be100-73.4 = 26.6 or 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**Ans:** Mean = 38, SD = 6, Z score = (Value - Mean)/SD

Z score for 44 = (44 - 38)/6 = 1 = 84.13 %

People above 44 age = 100 - 84.13 = 15.87% ≈ 63 out of 400

Z score for 38 = (38 - 38)/6 = 0 = 50%

Hence People between 38 & 44 age = 84.13 - 50 = 34.13 % ≈ 137 out of 400

Hence More employees at the processing center are older than 44 than between 38 and 44. is **FALSE**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans:** So, number of employees with probability 0.912 of them being under age 30=0.0912\*400=36.48 (36) out of 400.

Therefore, the statement B of the question is also **TRUE**

1. If *X1* ~ *N* (μ, σ2) and *X*2 ~ *N* (μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans:** As we know that if X ~ N (μ, σ2) and Y ~ N (μ2, σ22) are two independent Random variables than X + Y ~ N (μ1 + μ2, σ12+ σ22) and X - Y ~ N (μ1 - μ2, σ12+ σ22).

Similarly, if Z = ax +by, Where X and Y are as defined above i.e. z is linear combination of X and Y then Z ~ N (aμ1 + bμ2 a² σ1² + b² σ2²)

therefore, in the question

2X1~ N (2µ,4 σ2) and X1 +X2 ~ N (µ + µ, σ2+ σ2) ~ N (2µ, 2σ2)2X1 – (X1 +X2) =

N (4µ ,6 σ2)

1. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans**: Z\*σ + µ = X

Z (-0.005) \*20+100 = (2.57) \*20+100 = 151.4

Z (0.005) \*20+100 = (-2.57) \*20+100 = 48.6

**ANS: - D. 48.5 ,151.5**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Ans:** Mean profit is RS 540 million

Std deviation is RS 225 million

Range is RS

State norms interval (0.95, 540, 225)

**Range is RS 99.0081034, 980.991896**

1. Specify the 5th percentile of profit (in Rupees) for the company

**Ans:** formula X= X=μ + Zσ; wherein from z table, 5 percentile = -1.645

X = 540(-1.645) \*225

**X = 169.875**

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans:** Probability od division 1 making a loss p(X<0)

Stats.norm.cdf (0, 5, 3) = 0.0477903

Probability od division 2 making a loss p(X<0)

Stats.norm.cdf (0, 7, 4) = 0.0400591