# Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with $\mu$ = 45 minutes and $\sigma$ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

    A. 0.3875
    B. 0.2676
    C. 0.5
    D. 0.6987

Answer-

Ans- We have a normal distribution with mu= 45 and sigma= 8.0.

Let X be the amount of time it takes to complete the repair on a customer's car.

To finish in one hour you must have X ≤ 50

to find P(X > 50).

P(X > 50) = 1 - Pr(X ≤ 50).

Z = (X - 45)/8.0

P(X ≤ 50) = P(Z ≤ (50 - 45)/8.0)

= P(Z ≤ 0.625)=73.4%

Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or 0.2676

So Answer is B

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean $\mu$ = 38 and Standard deviation $\sigma$=6. For each statement below, please specify True/False. If false, briefly explain why.
    A. More employees at the processing center are older than 44 than between 38 and 44.
    B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.



Answer - We have a normal distribution with mu= 38 and sigma= 6.

Let X be the number of employees.

a)Probability of employees greater than age of 44

$P(X > 44) = 1 - P(X \leq 44)$

$Z = (X - 38)/6$

$P(X \leq 44) = Pr(Z \leq (44 - 38)/6)$

$= P(Z \leq 1)=84.1345\%$

Probability that the employee will be greater than age of 44is 100-84.1345=15.86

So the probability of number of employees between 38-44 years of age = P(X<44)-0.5=84.1345-0.5= 34.1345%

Therefore the statement that More employees at the processing center are older than 44 than between 38 and 44 is TRUE.

b) Probability of employees less than age of 30 = Pr(X<30).

$Z = (30 - 38)/6$

$P(X \leq 30) = P(Z \leq (30 - 38)/6)$

$= Pr(Z \leq -1.333)=9.12\%$

So the number of employees with probability 0.912 of them being under age 30 is 0.0912*400=36.48. Approx. 36 Employees

Therefore the statement B of the question is also TRUE.

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 $X_1$ and $X_1 + X_2$? Discuss both their distributions and parameters.

Answer- As we know that if X ∼ N(μ1, σ1^2 ) and Y ∼ N(μ2, σ2^2 ) are two independent random variables then X + Y ∼ N(μ1 + μ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(μ1 − μ2, σ1^2 + σ2^2 ) .

Similarly if Z = aX + bY , where X and Y are as defined above,

i.e Z is linear combination of X and Y , then Z ∼ N(aμ1 + bμ2, a^2σ1^2 + b^2σ2^2 ).

Therefore

2X1~ N(2 u,4 σ^2) and

X1+X2 ~ N(μ + μ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )

2X1-(X1+X2) = N( 4μ,6 σ^2)

4. Let X ~ N(100, $20^2$). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

Answer

The Probability of getting value between a and b should be 0.99.

the Probability outside the a and b area is 0.01 (i.e. 1-0.99).

The Probability towards left from a = -0.005 (i.e. 0.01/2).

The Probability towards right from b = +0.005 (i.e. 0.01/2).

So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X

values. $Z=(X- \mu) / \sigma$

For Probability 0.005 the Z Value is -2.57

$Z * \sigma + \mu = X$

$Z(-0.005)*20+100 = -(-2.57)*20+100$

$= 151.4$

$Z(+0.005)*20+100 = (-2.57)*20+100$

$= 48.6$

So, option D is correct.

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
    A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

    Answer : (-77.38865513011706, 1157.388655130117) Rs in Millions

    B. Specify the $5^{th}$ percentile of profit (in Rupees) for the company

    Answer: 5th percentile of profit (in Million Rupees) is 23.4

    C. Which of the two divisions has a larger probability of making a loss in a given year?

Answer: Probability of Division 1 making a loss in a given year is more than Division 2