

Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with $\mu = 45$ minutes and $\sigma = 8$ minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

A. 0.3875
B. 0.2676
C. 0.5
D. 0.6987

Ans:

Since work begins 10 mins after the car is dropped, the time left to complete work is 50 mins.
Prob. that service manager can't meet his commitment = $P(X > 50) = 1 - P(X \leq 50)$ (X is the time taken to complete work). Convert 50 to z-score

Standard normal variable $Z = (X - \mu) / \sigma = (X - 45) / 8$

$P(X \leq 50) = P(Z \leq (50 - 45) / 8) = P(Z \leq 0.625) = 0.73237 = 73.237\%$ (the number in z-table)

Prob. That service manager will not meet his commitment is: $100 - 73.237 = 26.763\% = 0.2676$

So, the answer is **B**

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean $\mu = 38$ and Standard deviation $\sigma = 6$. For each statement below, please specify True/False. If false, briefly explain why.
A. More employees at the processing center are older than 44 than between 38 and 44.
B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:

$\mu = 38$ & $\sigma = 6$

- a) Prob. Of employees $> 44 = P(x > 44) = 1 - P(x \leq 44)$

$$Z = (X - \mu) / \sigma = (X - 38) / 6$$

$$P(x \leq 44) = P(z \leq (44 - 38) / 6) = P(z \leq 1) = 0.84134 = 84.134\%$$

$$\text{Prob. That employees will be greater than 44} = 100 - 84.134 = 15.866$$

$$\text{Prob. Of employees between 38 \& 44} = P(x \leq 44) - P(x \leq 38)$$

$$\text{Here, } P(x \leq 44) = 0.84134$$

$$P(x \geq 38) = P(z \geq (38 - 38) / 6) = P(z \geq 0) = 0.5$$

$$\text{Therefore, } P(x \leq 44) - P(x \geq 38) = 0.84134 - 0.5 = 0.34134 = 34.134\%$$

So, the statement "More employees at the processing center are older than 44 than between 38 and 44" is **TRUE**

b) Prob. Of employees less than 30 = $\Pr(x < 30)$

$$Z = (X - \mu) / \sigma = (30 - 38) / 6$$

$$\Pr(x < 30) = \Pr(z < (30 - 38) / 6) = \Pr(z < -1.3333) = 0.09176 = 9.17\%$$

So, the number of employees with prob. 0.0917 of them being under 30 = $400 * 0.0917 = 36.68 = 36$

The statement of "training program for employees under the age of 30 at the center would be expected to attract about 36 employees" is **TRUE**.

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are iid normal random variables, then what is the difference between $2X_1$ and $X_1 + X_2$? Discuss both their distributions and parameters.

Ans: NOT SURE....

4. Let $X \sim N(100, 20^2)$. Find two values, a and b , symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

- A. 90.5, 105.9
- B. 80.2, 119.8
- C. 22, 78
- D. 48.5, 151.5
- E. 90.1, 109.9

Ans:

The Prob. Of getting value between a & b is 0.99

So, the prob. Of getting value outside a & b is $1 - 0.99 = 0.01$

The prob. Towards left of $a = -0.01/2 = -0.05$

The prob. Towards right of $b = 0.01/2 = 0.05$

Since we have prob. Of a & b , we need to calculate the prob. Of X - the random variable at a & b which has these probabilities.

By finding std normal variable (z), need to calculate X :

$$Z = (X - \mu) / \sigma$$

For a prob. Of 0.005, z values is -2.57

$$Z * \sigma + \mu = x$$

$$-(-2.57) * 20 + 100 = \mathbf{151.4}$$

$$(-2.57) \cdot 20 + 100 = 48.6$$

Option D is the Correct answer

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $\text{Profit}_1 \sim N(5, 3^2)$ and $\text{Profit}_2 \sim N(7, 4^2)$ respectively. Both the profits are in \$ Million. Answer the following questions about the total profit of the company in Rupees. Assume that \$1 = Rs. 45
- A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
 - B. Specify the 5th percentile of profit (in Rupees) for the company
 - C. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: NOT SURE....