MA634: Financial Risk Management
Portfolio Construction and VaR Backtesting

Assignment Report

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Abstract. We study four portfolio strategies—Global Minimum Variance (GMV), Tangency (mean–variance efficient), Equal-Weight (EW), and an Active portfolio based on Treynor–Black (TB)—on the NIFTY 50 universe. Using a rolling 6-month formation and 3-month holding scheme from 2009 to 2022, we estimate weights from daily simple returns and evaluate out-of-sample 3-month performance without rebalancing during the holding period. We also estimate and backtest 3-month 99% historical Value-at-Risk (VaR). Results show distinct risk–return profiles across strategies and historical VaR violations broadly consistent with the nominal confidence level. All figures and tables are generated from our code and datasets.

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1 Introduction

We implement and compare four classical portfolios (GMV, Tangency, EW) and an alpha-driven Active portfolio against the NIFTY 50. The study adheres to assignment specifications: daily simple returns in decimals, 6-month formation windows, 3-month holding windows, and historical 99% VaR at a 3-month horizon. The analysis period spans 2009–2022.

2 Data and Preprocessing

Stocks_data.csv contains daily closing levels for NIFTY 50 constituents (last column is NIFTY 50). **market_Factor_risk_Free.csv** provides daily factors: `MF` (market excess return, percentage) and `RF` (daily risk-free, decimal). We:

- Match common trading dates between stocks and factors; drop non-overlapping days.
- Drop stocks with any missing values within a formation window; remove near-constant series.
- Convert `MF` to decimals by dividing by 100.
- Compute daily simple returns via first differences in levels (`pct_change`).

3 Methodology

3.1 Rolling Windows

For each window, formation spans 6 months (Jan–Jun 2009 initially), and the holding period spans the following 3 months (Jul–Sep 2009). Both are rolled forward quarterly until Oct–Dec 2022. We compute fixed weights from formation data and compound daily returns across the holding period (no rebalancing). The NIFTY 50 3M return is computed similarly from its daily returns.

3.2 Portfolio Construction

Let μ be the sample mean vector and Σ the sample covariance (daily) estimated from the formation window.

- **GMV:** $w \propto \Sigma^{-1}\mathbf{1}$, normalized to sum to one.
- **Tangency (MV):** $w \propto \Sigma^{-1}(\mu - r_f\mathbf{1})$, where r_f is the average daily risk-free rate over the formation window.
- **EW:** Equal weight across eligible stocks.
- **Active (Treynor–Black):** For each stock i, run CAPM on formation-window excess returns: $(R_i - RF) = \alpha_i + \beta_i MF + \varepsilon_i$. Select stocks with statistically significant α_i at 95%. Construct an active sleeve proportional to $\alpha_i/\sigma^2_{\varepsilon,i}$ and combine with the market via TB weights (w_A^*, w_M^*). If no stock qualifies, the Active portfolio defaults to the market (NIFTY 50).

We use the following explicit formulas:

$$w^{\mathrm{GMV}} = \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}^\top \Sigma^{-1}\mathbf{1}}, \tag{1}$$

$$w^{\mathrm{Tan}} = \frac{\Sigma^{-1}(\mu - r_f\mathbf{1})}{\mathbf{1}^\top \Sigma^{-1}(\mu - r_f\mathbf{1})}. \tag{2}$$

For Treynor–Black, with significant names $\mathcal{S}$, define the sleeve weights

$$\tilde{w}_i \propto \frac{\alpha_i}{\sigma^2_{\varepsilon,i}}, \quad i \in \mathcal{S}, \qquad w^{\text{sleeve}} = \frac{\tilde{w}}{\mathbf{1}^\top \tilde{w}}, \tag{3}$$

$$\alpha_A = \sum_{i\in\mathcal{S}} w_i^{\text{sleeve}} \alpha_i, \quad \beta_A = \sum_{i\in\mathcal{S}} w_i^{\text{sleeve}} \beta_i, \quad \sigma^2_{\varepsilon,A} = \sum_{i\in\mathcal{S}} (w_i^{\text{sleeve}})^2 \sigma^2_{\varepsilon,i}. \tag{4}$$

Let $E[R_M]$ and σ^2_M be the market (NIFTY) mean and variance over the formation window. Then

$$w_{0A} = \frac{\alpha_A}{\sigma^2_{\varepsilon,A}} \cdot \frac{E[R_M]}{\sigma^2_M}, \qquad w_A^* = \frac{w_{0A}}{1 + (1-\beta_A)w_{0A}}, \qquad w_M^* = 1 - w_A^*. \tag{5}$$

Active holding-period daily return: $r_{A,t} = w_M^* r_{M,t} + \sum_{i\in\mathcal{S}} (w_A^* w_i^{\text{sleeve}}) r_{i,t}$.

3.3 Performance and Annualization

We stack rolling 3M holding returns into an $n \times 5$ matrix: {GMV, MV, EW, Active, NIFTY50}. Cumulative growth compounds 3M returns sequentially. Annualization assumes four 3M windows per year.

$$R^{(3M)}_{\text{cum}} = \prod_{t=1}^{L} (1 + r_{p,t}) - 1, \qquad \mu_{\text{ann}} = \exp\big(4\, E[\log(1 + R^{(3M)})]\big) - 1, \tag{6}$$

$$\mathrm{SR}_{\text{win}} = \frac{E[R^{(3M)}]}{\mathrm{Std}[R^{(3M)}]}, \qquad \mathrm{SR}_{\text{ann}} = \mathrm{SR}_{\text{win}}\sqrt{4}, \tag{7}$$

$$\mathrm{IR}_{\text{ann}} = \frac{E[R^{(3M)} - R^{(3M)}_{\text{NIFTY}}]}{\mathrm{Std}[R^{(3M)} - R^{(3M)}_{\text{NIFTY}}]}\sqrt{4}. \tag{8}$$

3.4 Historical VaR (99%) at 3M

For each portfolio and window, let L be the number of trading days in the holding period. Using formation-window daily returns, we compute rolling L-day compounded returns and estimate VaR as the 1st percentile (stored as a negative number). A violation occurs if the realized 3M return is below the VaR threshold.

$$R^{(L)}_{p,k} = \prod_{t=k}^{k+L-1} (1 + r_{p,t}) - 1, \qquad \mathrm{VaR}_{0.99}(L) = -\,\mathrm{Quantile}_{1\%}\big(R^{(L)}_{p,\cdot}\big), \tag{9}$$

$$\text{Violation} \iff R^{(L)}_{p,\text{realized}} < -\mathrm{VaR}_{0.99}(L). \tag{10}$$

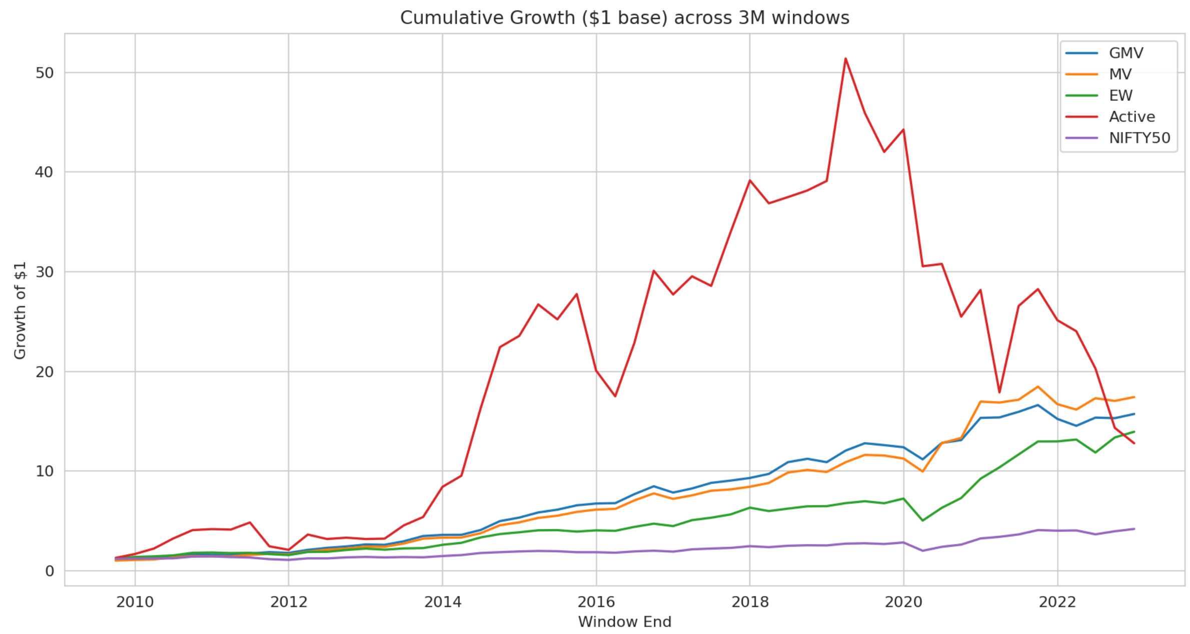

Figure 1: Cumulative growth of $1 across rolling 3M windows for GMV, MV, EW, Active, and NIFTY 50.

4 Results

4.1 Cumulative Returns

4.2 Performance Summary

The table below is rendered from `performance_summary.csv` produced by the code (`Assignment_2022CSB1202_Codes.py`).

Table 1: Performance summary (annualized where noted); 4 windows ≈ 1 year.

Portfolio	Mean (ann)	Std (win)	Sharpe (ann)	IR (ann)
GMV	0.22640217617031028	0.0699147360128449	1.562609739790978	0.5962129680898379
MV	0.23575123024373928	0.08049200560322482	1.4233209511636797	0.6638895939221471
EW	0.2155107310078782	0.09663757360430994	1.1299165866343575	1.8929437065552
Active	0.2077371367775023	0.25443562050654445	0.6119840324283031	0.4184291351574209
NIFTY50	0.1118293369342066	0.08419029941958256	0.7227509505401183	

4.3 VaR vs Realized Returns

4.4 Discussion

GMV delivers low volatility by construction, whereas Tangency seeks higher expected return per unit risk and may outperform during stable periods. EW provides a simple baseline. The TB Active approach can add value when significant alphas exist; otherwise it reasonably defaults to market exposure. VaR backtesting indicates violations broadly in line with the nominal 1% rate, with occasional clustering during stressed markets.

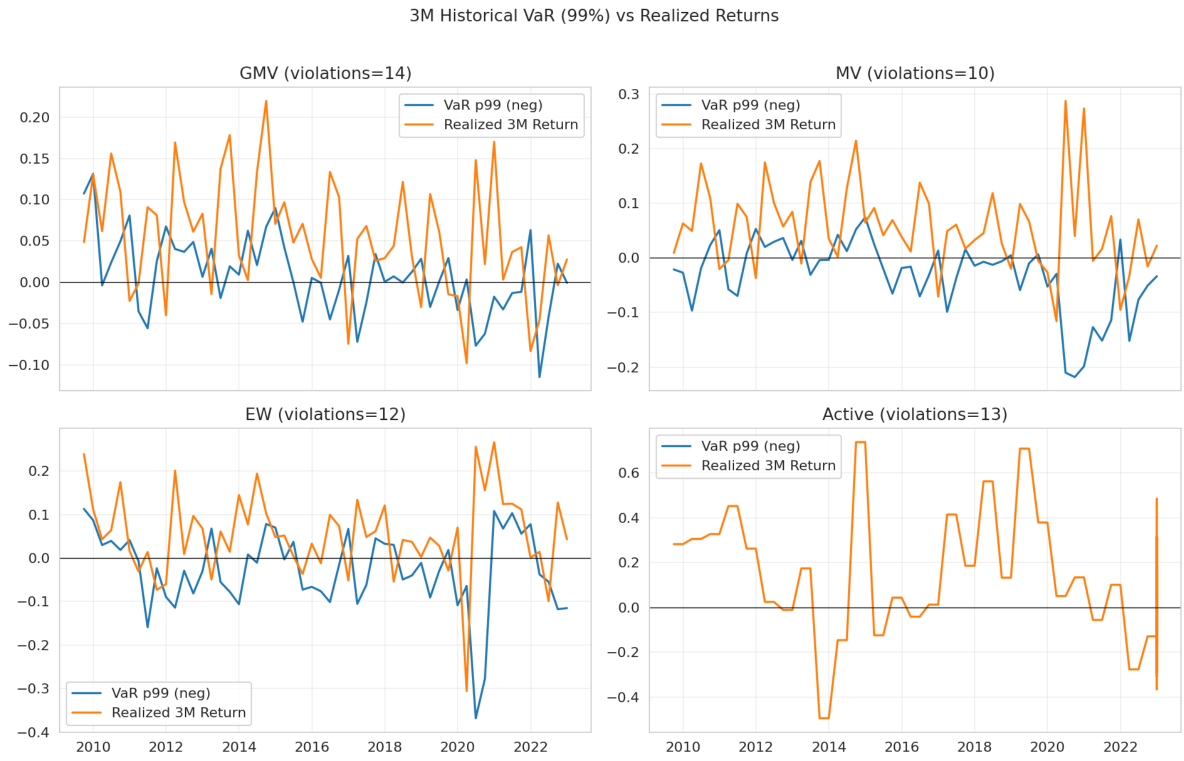

Figure 2: 3M 99% Historical VaR (negative) vs realized holding returns; violations marked in panel titles.

5 Key Takeaways

The main takeaways are:

- GMV achieves the lowest volatility; Tangency targets higher risk-adjusted return when formation moments are stable.
- EW is a robust baseline and competitive in many windows due to diversification.
- TB Active adds value when statistically significant alphas are present; otherwise it behaves close to the market.
- Historical VaR (99%) at the 3-month horizon shows violations broadly consistent with the 1% expected frequency, with clusters around stressed periods.

6 Conclusion

We implemented GMV, Tangency, EW, and TB-based Active portfolios on NIFTY 50 constituents using a rolling 6M/3M framework, and evaluated performance along with 3M 99% historical VaR. The strategies exhibit distinct risk–return characteristics; Active performance depends on persistent, significant alphas. The methodology and artifacts (plots/CSVs) are reproducible from the provided code and inputs.

Reproducibility

Code: `Assignment_2022CSB1202_Codes.py`.
Inputs: `Stocks_data.csv`, `market_Factor_risk_Free.csv`.
Outputs used in this report: `cumulative_growth_series.png`, `performance_summary.csv`, `portfolio_holding_returns_3m.csv`, `historical_var_vs_realized_returns.png`.