Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

Answer-:

The average time = 45min + 10 min as work start after 10 min=55min

from normal distribution-:

z = (X-μ)/б

= (60-55)/8

= 5/8

=0.625

The probability that the service manager cannot meet his commitment=1-pnorm (0.625)

The probability that the service manager cannot meet his commitment = 0.2676

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.
   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer-:

Mean=38, Standard Deviation=6

A) Z_score=(Value-mean)/SD

= (44-38)/6

=1

P value from Z-table is 0.84134 i.e. 84.13%

For the people above 44 age = 100 - 84.13 = 15.87%

B) Z_score=(Value-mean)/SD
= (38-38)/6
=0

P value from Z-table is 0.50 i.e. 50.00%

People between 38 & 44 age = 84.13 - 50 = 34.13 %

Hence,

More employees at the processing center are older than 44 than between 38 and 44 is **FALSE.**

Z score for 30 = (30 - 38)/6 = -1.33

P value from Z-table is 0.0915 i.e. 9.15 %, almost 36 people out of 400

Hence training program for employees under the age of 30 at the center would be expected to attract about 36 employees - **TRUE**

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

Answer-:

According to the Central Limit Theorem, any large sum of independent, identically distributed(iid) random variables is approximately Normal.

The Normal distribution is defined by two parameters, the mean, μ, and the variance, σ^2 and written as $X \sim N(\mu, \sigma^2)$

Given $X1 \sim N(\mu, \sigma^2)$ & $X2 \sim N(\mu, \sigma^2)$ are two independent identically distributed random variables.

From the properties of normal random variables,

if $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$ are two independent identically distributed random variables then

- the sum of normal random variables is given by

$$X + Y \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2),$$

- and the difference of normal random variables is given by

$$X - Y \sim N(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$$

When $Z = aX$, the product of X is given by

$$Z \sim N(a\mu_1, a^2\sigma_1^2)$$

When $Z = aX + bY$, the linear combination of X and Y is given by

$$Z \sim N(a\mu_1 + b\mu_2, a^2\sigma_1^2 + b^2\sigma_2^2)$$

Given to find, $2X_1$

Thus, following the property of multiplication, we get

$2X_1 \sim N(2\mu, 2^2\sigma^2) \implies 2X_1 \sim N(2\mu, 4\sigma^2)$

and following the property of addition,

$X_1 + X_2 \sim N(\mu + \mu, \sigma^2 + \sigma^2) \sim N(2\mu, 2\sigma^2)$

And the difference between the two is given by

$2X_1 - (X_1 + X_2) \sim N(2\mu - 2\mu, 2\sigma_1^2 + 4\sigma_2^2) \sim N(0, 6\sigma^2)$

The mean of $2X_1$ and $X_1 + X_2$ is same but the var(σ^2) of $2X_1$ is 2 times more than the variance of $X_1 + X_2$.

The difference between the two says that the two given variables
are identically and independently distributed.

4. Let X ~ N(100, 20^2). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

Answer-:

Given: $p(a<x<b) = 0.99$, mean =100, SD= 20

To Find:

Identify symmetric values for the standard normal distribution such that the area enclosed is .99

From the above details, we have to excluded area of (1-0.99=0.01)/2=0.005 in each of the left and right tails.

Hence, we want to find the 0.5th and the 99.5th percentiles Z score values→

Using Python→

Z value is given as stats.norm.ppf(pvalue)

Z value at 0.5th percentile is given as,

$$Z(0.5) = \text{stats.norm.ppf}(0.005) = -2.576$$

Z value at 99.5 percentile is given as,

$$Z(99.5) = \text{stats.norm.ppf}(0.995) = 2.576$$

Z = (x-mean)/SD

= (x - 100)/20

x = 20z+100

a = -(20*2.576) + 100= 48.5

b = (20*2.576) +100= 151.5

Two values symmetric about mean for the given standard normal distribution are= [48.5,151.5].

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
   B. Specify the 5^{th} percentile of profit (in Rupees) for the company
   C. Which of the two divisions has a larger probability of making a loss in a given year?

Answer-:

Please check the notebook.