**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer-:

The average time = 45min + 10 min as work start after 10 min=55min

from normal distribution-:

z = (X-μ)/б

= (60-55)/8

= 5/8

=0.625

The probability that the service manager cannot meet his commitment=1-pnorm (0.625)

The probability that the service manager cannot meet his commitment = 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer-:

Mean=38, Standard Deviation=6

1. Z\_score=(Value-mean)/SD

= (44-38)/6

=1

P value from Z-table is 0.84134 i.e. 84.13%

For the people above 44 age = 100 - 84.13 = 15.87%

1. Z\_score=(Value-mean)/SD

= (38-38)/6

=0

P value from Z-table is 0.50 i.e. 50.00%

People between 38 & 44 age = 84.13 - 50 = 34.13 %

Hence,

More employees at the processing center are older than 44 than between 38 and 44 is **FALSE.**

Z score for 30 = (30 - 38)/6 = -1.33

P value from Z-table is 0.0915 i.e. 9.15 %, almost 36 people out of 400

Hence training program for employees under the age of 30 at the center would be expected to attract about 36 employees - **TRUE**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Answer-:

According to the Central Limit Theorem, any large sum of independent, identically distributed(iid) random variables is approximately Normal.

# The Normal distribution is defined by two parameters, the mean, https://tex.z-dn.net/?f=%5Cmu, and the variance, σ² and written as X~N(µ, σ² )

GivenX1~N(µ, σ² ) & X2~N(µ, σ²)  are two independent identically distributed random variables.

From the properties of normal random variables,

if https://tex.z-dn.net/?f=X%20%5Csim%20N(%5Cmu_1%2C%20%5Csigma_1%5E2) and https://tex.z-dn.net/?f=Y%20%5Csim%20N(%5Cmu_2%2C%20%5Csigma_2%5E2) are two independent identically distributed random variables then

* the sum of normal random variables is given by

https://tex.z-dn.net/?f=X%20%2B%20Y%20%5Csim%20N(%5Cmu_1%20%2B%20%5Cmu_2%2C%20%5Csigma_1%5E2%20%2B%20%5Csigma_2%5E2%20),

* and the difference of normal random variables is given by

https://tex.z-dn.net/?f=X%20-%20Y%20%5Csim%20N(%5Cmu_1%20-%20%5Cmu_2%2C%20%5Csigma_1%5E2%20%2B%20%5Csigma_2%5E2%20)

When  https://tex.z-dn.net/?f=Z%20%3D%20aX, the product of X is given by

https://tex.z-dn.net/?f=Z%20%5Csim%20N(a%5Cmu_1%20%2C%20a%5E2%5Csigma_1%5E2%20)

When  https://tex.z-dn.net/?f=Z%20%3D%20aX%20%2B%20bY, the linear combination of X and Y is given by

https://tex.z-dn.net/?f=Z%20%5Csim%20N(a%5Cmu_1%20%2B%20b%5Cmu_2%2C%20a%5E2%5Csigma_1%5E2%20%2B%20b%5E2%5Csigma_2%5E2%20)

Given to find, https://tex.z-dn.net/?f=2X_1

Thus, following the property of multiplication, we get

https://tex.z-dn.net/?f=2X_1%20%5Csim%20N(2%5Cmu%2C%202%5E2%5Csigma%5E2)%5Cimplies2X_1%20%5Csim%20N(2%5Cmu%2C%204%5Csigma%5E2)

and following the property of addition,

https://tex.z-dn.net/?f=X_1%2BX_2%5Csim%20N(%5Cmu%20%2B%20%5Cmu%2C%20%5Csigma%5E2%20%2B%20%5Csigma%5E2%20)%20%5Csim%20N(2%5Cmu%2C%202%5Csigma%5E2%20)

And the difference between the two is given by

https://tex.z-dn.net/?f=2X_1-(X_1%2BX_2)%20%5Csim%20N(2%5Cmu%20-%202%5Cmu%2C%202%5Csigma_1%5E2%20%2B%204%5Csigma_2%5E2%20)%5Csim%20N(%200%2C6%20%5Csigma%5E2)

The mean of https://tex.z-dn.net/?f=2X_1 and https://tex.z-dn.net/?f=X_1%2BX_2 is same but the var(https://tex.z-dn.net/?f=%5Csigma%5E2) of  https://tex.z-dn.net/?f=2X_1 is 2 times more than the variance of https://tex.z-dn.net/?f=X_1%2BX_2.

The difference between the two says that the two given variables are identically and independently distributed.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Answer-:

Given:  p(a<x<b) = 0.99, mean =100, SD= 20

To Find:

Identify symmetric values for the standard normal distribution such that the area enclosed is .99

From the above details, we have to excluded area of (1-0.99=0.01)/2=0.005 in each of the left and right tails.

Hence, we want to find the 0.5th and the 99.5th percentiles Z score values🡪

Using Python🡪

Z value is given as stats.norm.ppf(pvalue)

Z value at 0.5th percentile is given as,

                                         Z(0.5) = stats.norm.ppf(0.005)= -2.576

Z value at 99.5 percentile is given as,

                          Z(99.5) = stats.norm.ppf(0.995) = 2.576

Z = (x-mean)/SD

= (x - 100)/20

x = 20z+100

      a = -(20\*2.576) + 100= 48.5

      b = (20\*2.576) +100= 151.5

Two values symmetric about mean for the given standard normal distribution are= [48.5,151.5].

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Answer-:

**Please check the notebook**.