Sampling and Reconstruction of Bandlimited Signals with Time Encoding – Karen Adam, Adam Schofield & Martin Vetterli

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Abstract/Aim of your project

- This project provides an alternative way of sampling a signal using time encoding technique.
- It is more efficient than classical sampling as it only stores spike intervals instead of amplitude, time pairs at a given sampling rate, both in space and power consumption.
- It is also not affected by unknown shifts in devices and offers higher accuracy when using multiple channels compared to classical sampling methods.

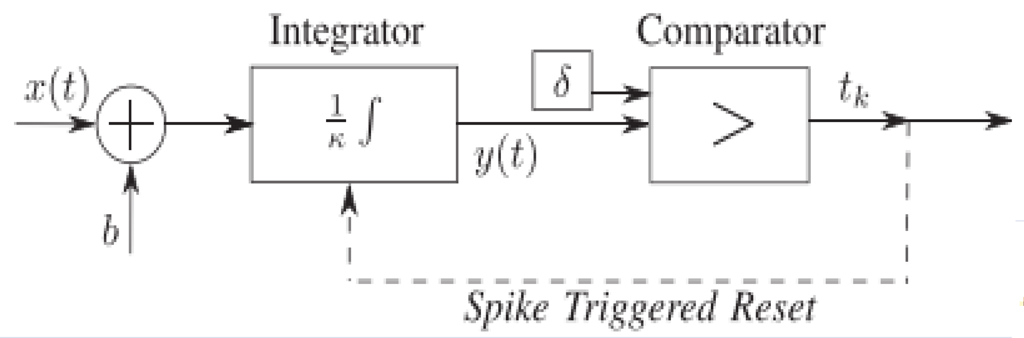
Applications or importance of the task

- The application of this methods is in all possible domains of digital signal processing wherever classical sampling method is used like in communication systems.
- In fact, time encoding can help us in designing higher-precision sampling hardware as high-precision clocks are more readily available than high-precision quantizers.
- It can be used in situations where the signal is zero for a long time, by reducing power consumption.
- Biological neural networks are often constructed using spiking neurons, which could be better understood using this method.

Challenges or motivation of work

Almost all current sampling theories represent a signal using (time, amplitude) pairs. However, this is quite different from the way encoding is done in nature, where a neuron takes an input, it outputs a series of action potentials—the timings of which encode the original input. Time encoding machines can be made to resemble biological neurons to different degrees using LIF neurons.

Method /Algorithms/ important concepts



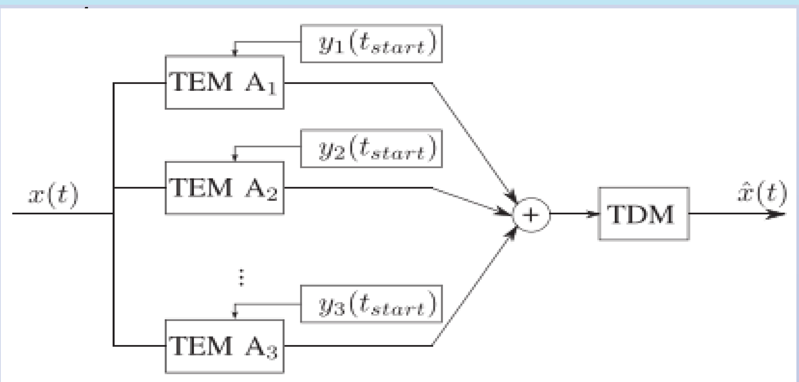
$$\int_{t_k}^{t_{k+1}} x(u)\,du = 2\kappa\delta - b\left(t_{k+1}-t_k\right)$$

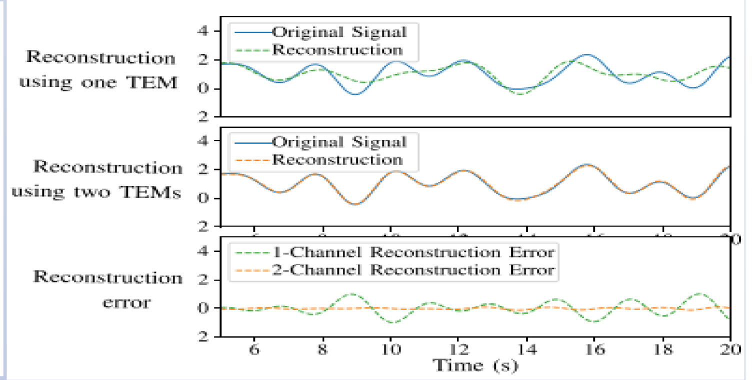
$$\mathcal{R}\left(y(t)\right) = \sum_{k\in\mathbb{Z}} \int_{t_k}^{t_{k+1}} y(u)\,du\; g(t-s_k)$$

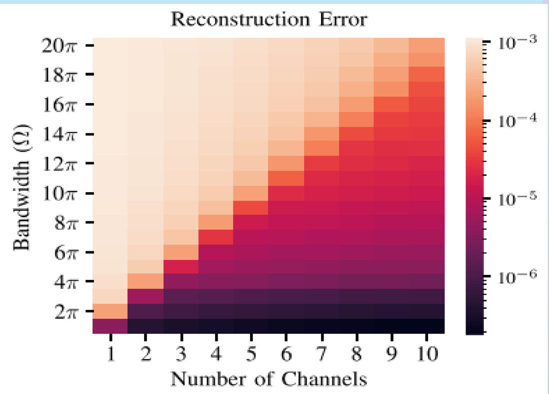
$$\Omega < \frac{\pi(b-c)}{2\kappa\delta} \qquad x_0 = \mathcal{R}(x),\quad x_{l+1} = x_l + \mathcal{R}(x-x_l) \qquad t_{k+1}-t_k \le \frac{2\kappa\delta}{b-c}$$

The above block diagram shows TEM, and the right algorithms is the reconstruction operator using sinc interpolation in the bandwidth.

Extension to Multi Channel Time







It is possible to modify the concept to allow multiple channels to improve accuracy for higher bandwidth reconstruction. This could work without any loss in accuracy due to unknown shifts in the integrators, and the reconstruction is the mean of all channels.

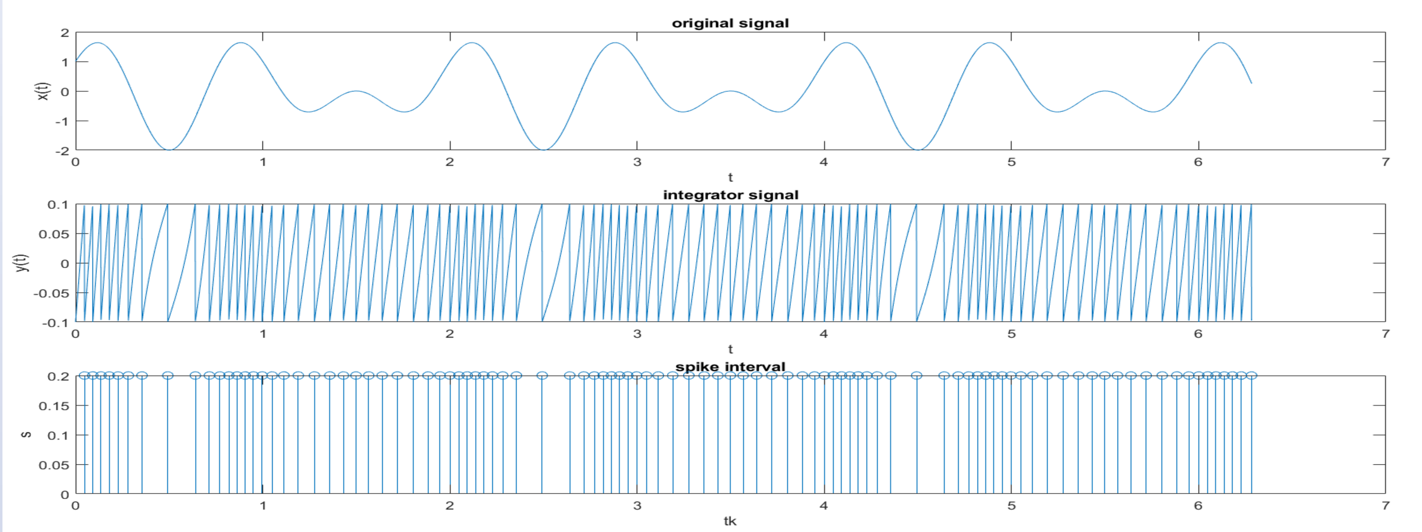
$$\mathcal{R}_{1\cdots M} = \frac{1}{M}\sum_{i=1}^{M}\mathcal{R}_i$$

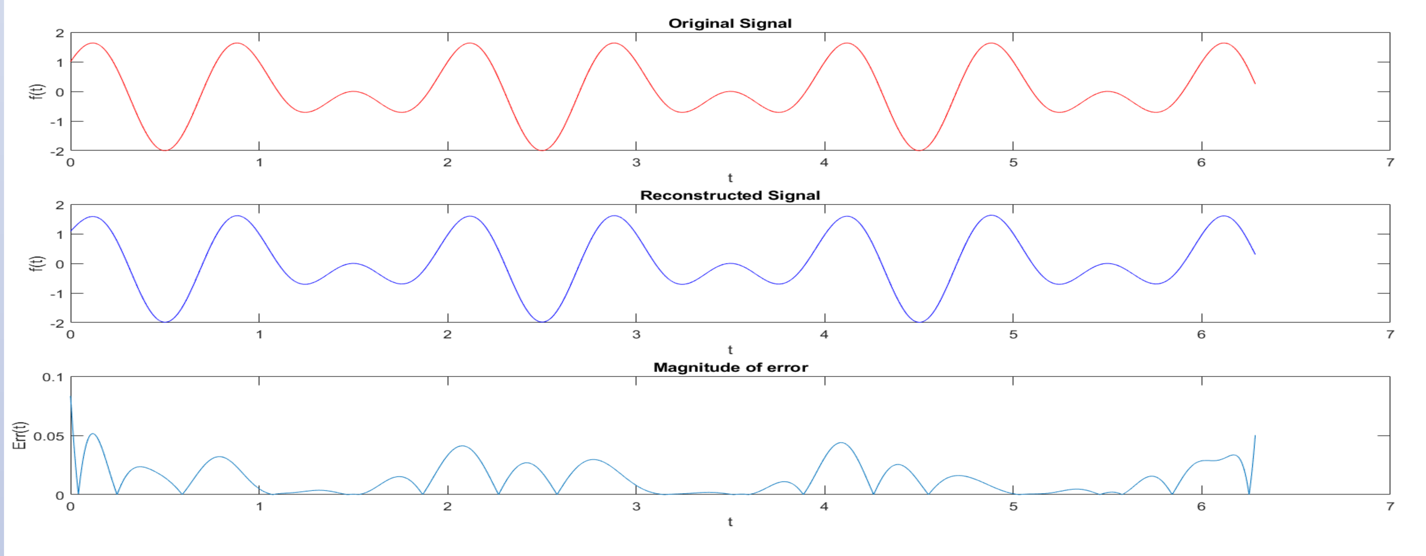
$$x_0 = \mathcal{R}_{1\cdots M}(x),\qquad x_{\ell+1} = x_\ell + \mathcal{R}_{1\cdots M}(x-x_\ell)$$

Conclusion

We have studied time encoding of 2Ωbandlimited signals, proposed an algorithm for reconstructing an input signal from its samples, and provided sufficient conditions on Ω for the algorithm to converge to the correct solution. Our setting has focused on reconstructing signals using TEMs with the same parameters κ, δ and b, where b > 0. This algorithm can be extended to scenarios where these parameters vary.

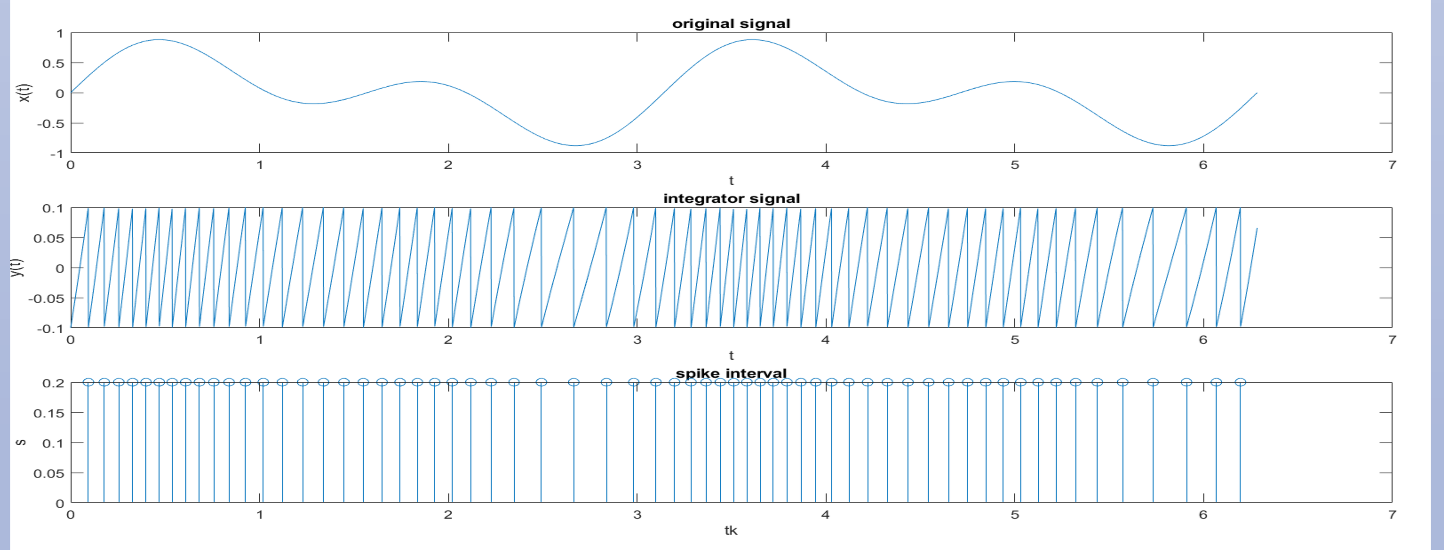
Results/ MATLAB plots

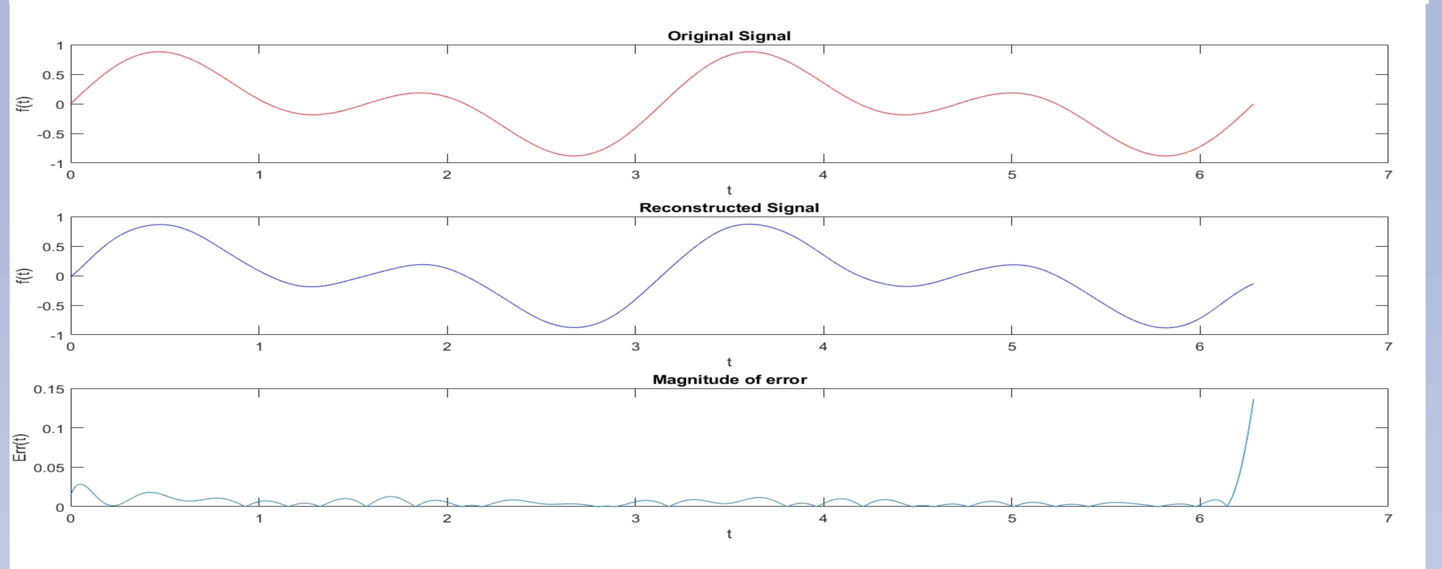




TEM and TED output graphs of the functions :-

- x(t) = cos(2.pi.t)+sin(3.pi.t);
- x(t) = cos(t).sin(3.t);





References

[1] A. A. Lazar and L. T. Toth, “Perfect recovery and sensitivity analysis of time encoded bandlimited signals,” IEEE Trans. Circuits Syst. I: Regular Papers, vol. 51, no. 10, pp. 2060–2073, Oct. 2004.
[2] A. A. Lazar, “Multichannel time encoding with integrate-and-fire neurons,” Neurocomputing, vol. 65, pp. 401–407, 2005.
[3] H. G. Feichtinger, J. C. Príncipe, J. L. Romero, A. S. Alvarado, and G. A. Velasco, “Approximate reconstruction of bandlimited functions for the integrate and fire sampler,” Adv. Comput. Math., vol. 36, no. 1, pp. 67–78, 2012.