Final Project Report

Sampling and Reconstruction of Bandlimited Signal with Multi-Channel Time Encoding

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Abstract :

This project provides an alternative way of sampling a signal using time encoding technique.

It is more efficient than classical sampling as it only stores spike intervals instead of amplitude, time pairs at a given sampling rate, both in space and power consumption.

It is also not affected by unknown shifts in devices and offers higher accuracy when using multiple channels compared to classical sampling methods.

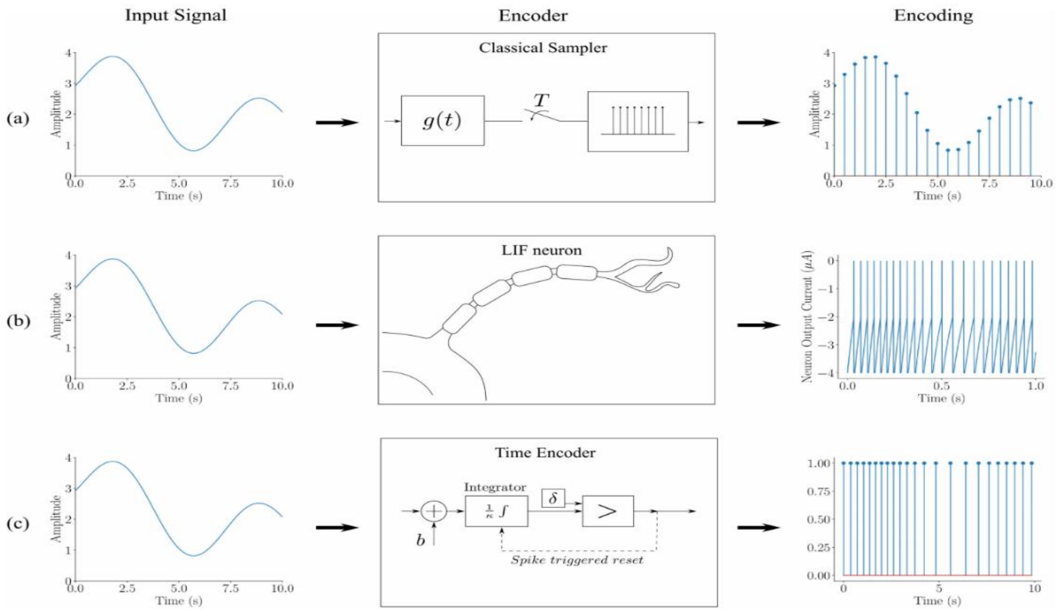
Objectives :

1. Sampling a bandlimited signal using a Time Encoding Machine instead of the Classical Sampling Theorem.
2. Reconstruction of the time-encoded sequence using Time Decoding Machine.
3. Understanding the accuracy of reconstruction on using multiple channel Time Encoding Machines.

Important Concepts and short discussion about existing method :

Almost all current sampling theories represent a signal using (time, amplitude) pairs. However, this is quite different from the way encoding is done in nature, where a neuron takes an input, it outputs a series of action potentials—the timings of which encode the original input. Time encoding machines can be made to resemble biological neurons to different degrees using LIF neurons.

The current existing method used for sampling a signal is by the Sampling Theorem where the output we obtain is an (amplitude, time) pair but when we use a Time-Encoding Machine to sample the input signal we get the output as a sequence of time sample t_k.



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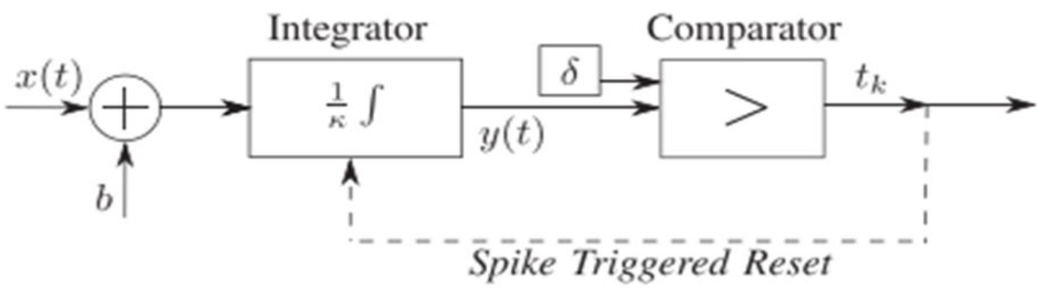
Concept 1 – Time Encoding Machine (TEM):-



From the above diagram we can observe that a Time-Encoding Machine works in a way similar to a perfect Integrate and Fire Neuron with parameters b(input bias), k(integrator constant), δ(threshold). Here we can observe that as the when the amplitude of the signal is high then the time samples t_k are closely spaced as compared to when the amplitude is low. Hence, we can intuitively say that the time samples (or) output y(t) indirectly also contains the information of amplitude as well.

Concept 2 – Time Decoding Machine (TDM):-

The Time-Decoding Machine reconstructs the signal using the t_k array generated by Time-Encoding Machine and also requires the parameters k, b, δ.

Mathematically it is known that x(t) satisfies the below equation.

$$\int_{t_k}^{t_{k+1}} x(u)\,du = 2\kappa\delta - b\left(t_{k+1} - t_k\right)$$

Now we use a recursive algorithm to reconstruct the signal and its formula is given below

$$x_0 = \mathcal{R}(x),$$
$$x_{l+1} = x_l + \mathcal{R}\left(x - x_l\right)$$

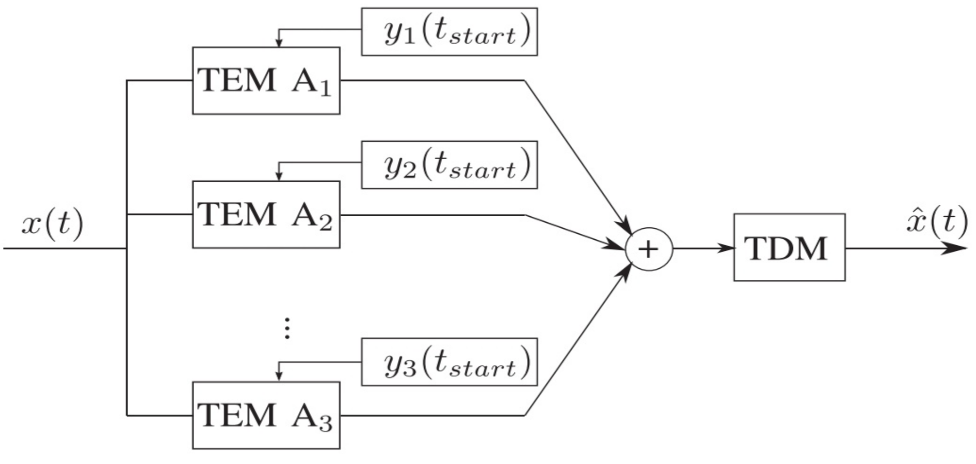
Here $R(x)$ is a reconstruction operator which is given by

$$\mathcal{R}\left(y(t)\right) = \sum_{k\in\mathbb{Z}} \int_{t_k}^{t_{k+1}} y(u)\,du \;\; g(t - s_k)$$

Now as we perform the Recursion infinite number of times the value $R(x - x_l)$ tends to 0 and hence $x_{l+1} \to x_l$ and this becomes the reconstructed Signal.

Concept 3 – Multi Channel Implementation:-

Now instead of using a Single Channel Time-Encoding Machine, if we use Multiple Channel Time-Encoding Machines (M > 1) and it has some over a Single Channel one, like M times that bandwidth without any trade-offs due to out of sync, shifted integrator devices.

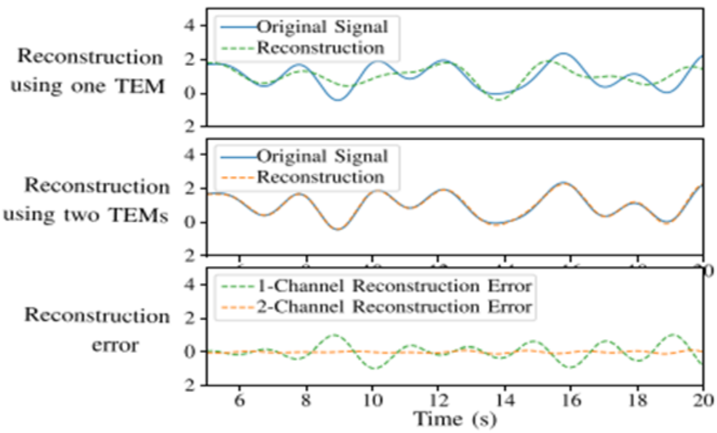


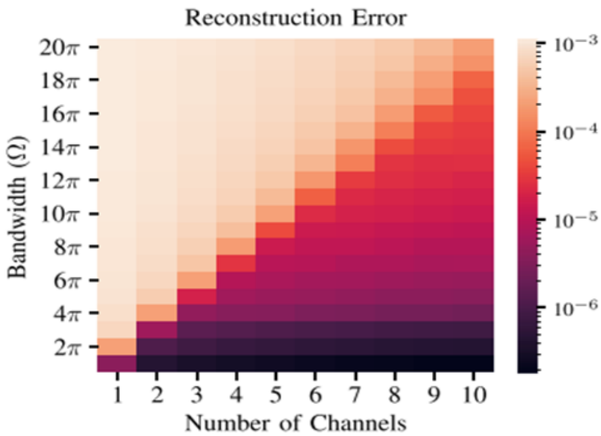
We define a Multi-Channel Time Encoding Machine as follows. Assuming $y_1(t)$, $y_2(t),\ldots,y_m(t)$ be the outputs of M Single Channel TEM having parameters b,k,δ for the same input x(t) then $y_{i+1}(t)=(y_i(t) + \alpha_i) \bmod 2\delta$ and $y_1(t)=(y_M(t) + \alpha_M) \bmod 2\delta$ where i = 1,2,…M and α_i are the integrator shifts in the devices.

The reconstruction using Multi-Channel is given by $R_{1\ldots\ldots M}(x)$ and is given by the formula as shown below, where R_i denotes the reconstruction by the i^{th} decoder.

$$\mathcal{R}_{1\cdots M} = \frac{1}{M}\sum_{i=1}^{M}\mathcal{R}_i \qquad \begin{aligned} x_0 &= \mathcal{R}_{1\cdots M}(x), \\ x_{\ell+1} &= x_\ell + \mathcal{R}_{1\cdots M}\left(x - x_\ell\right) \end{aligned}$$

And as we could derive that as the number of channels increases or the bandwidth of the signal decreases, the reconstruction Error decreases. From the below 2 figures we can observe the same





Proposed Method

After understanding the above discussed concepts of TEM, TDM and its extension to Multi-Channel we use them for the implementation.

Firstly the given signal is given to a Time-Encoding Machine with parameters k, b, δ which samples the signals and provides a sequence of time array t_k. Then we provide these t_k samples along with the parameters k, b, δ to the Time-Decoding machine which reconstructs the Signal.

Now as explained above that if we desire to increase the accuracy of reconstruction then we use the Multi-Channel Reconstruction which gives better results.

There are certain criteria which the Signal has to fulfil so as to be properly constructed. They are :

1. The Signal should be bounded in magnitude i.e., $|x(t)| \leq c$
2. The Signal should be bandlimited and the condition can be mathematically expressed as

$$\Omega < \frac{\pi(b-c)}{2k\delta} \quad \text{and} \quad t_{k+1} - t_k \leq \frac{2k\delta}{(b-c)}$$

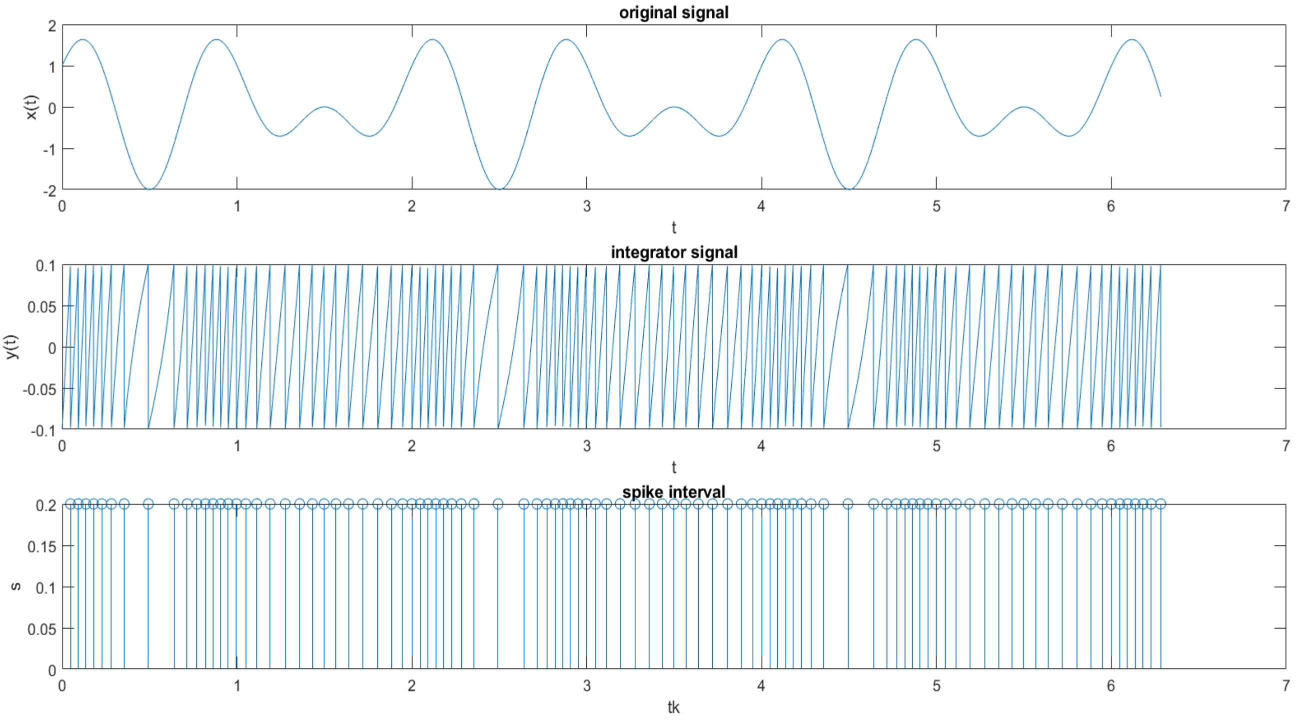
Here Ω is the bandwidth of the Signal.

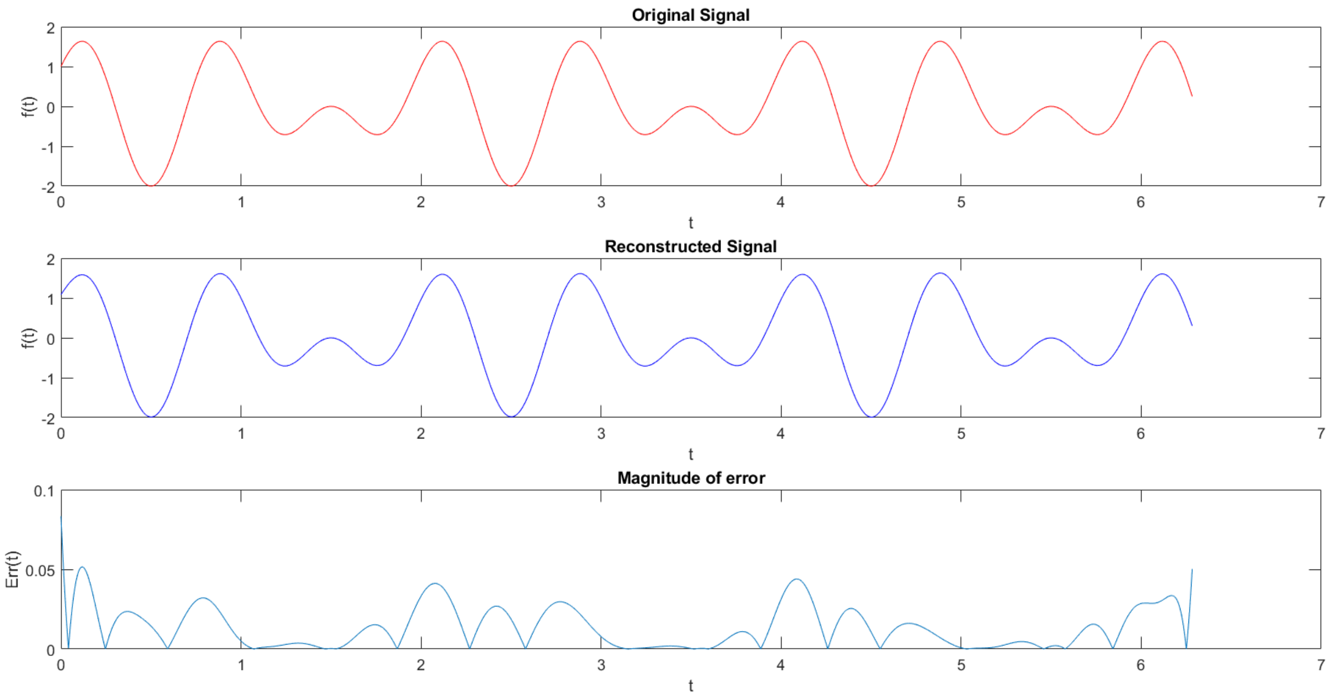
The above discussed Method has been implemented for Single Channel in MATLAB.

Simulations and Results :

Function 1 :

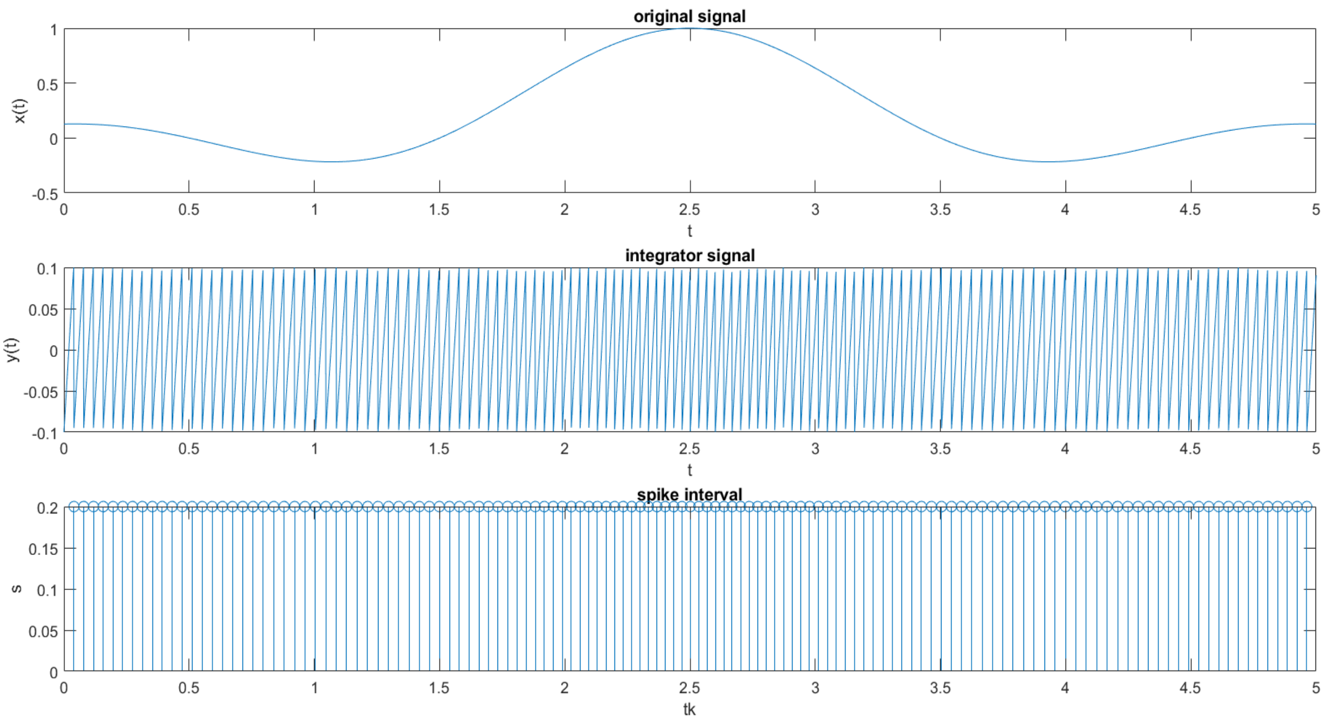
$$\boldsymbol{x(t) = cos(2\pi t) + sin(3\pi t)}$$

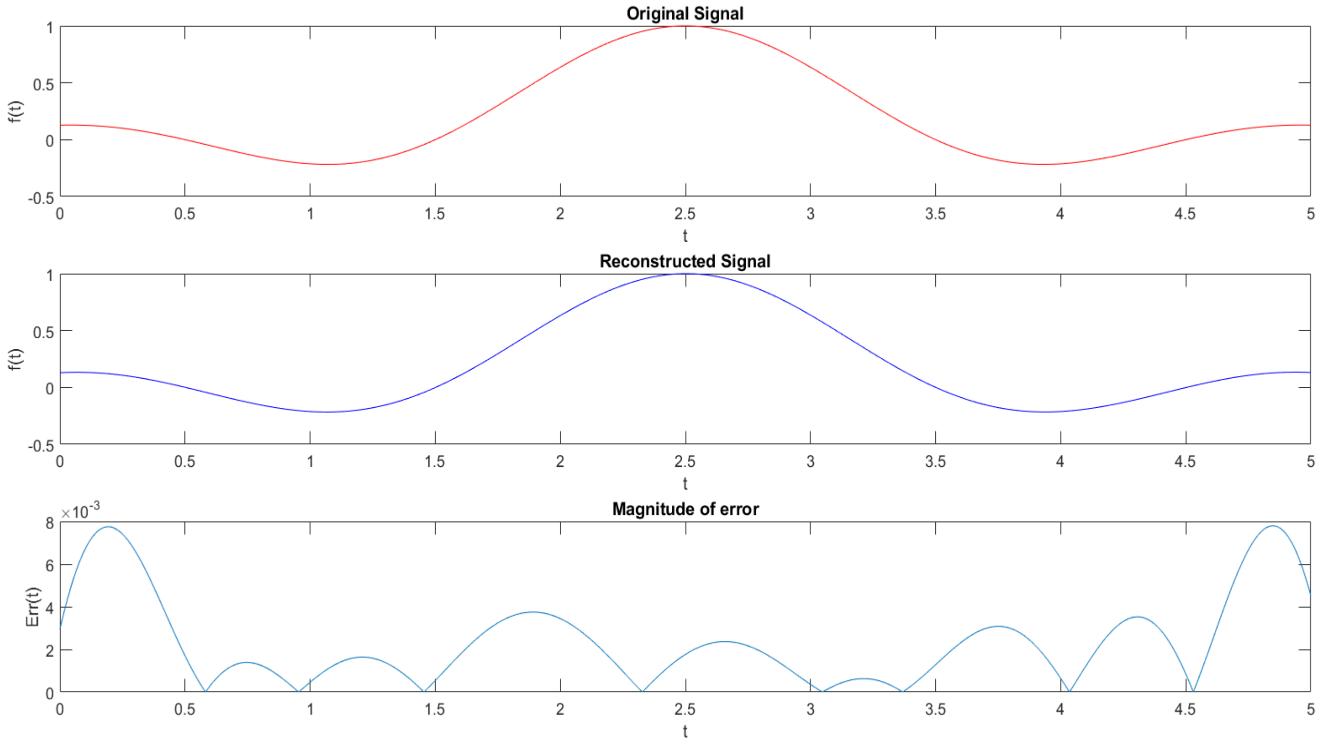




Function 2:

$$x(t) = sinc(t - 2.5)$$





Applications:

The application of this methods is in all possible domains of digital signal processing wherever classical sampling method is used like in communication systems.

In fact, time encoding can help us in designing higher-precision sampling hardware as high-precision clocks are more readily available than high precision quantizers.

It can be used in situations where the signal is zero for a long time, by reducing power consumption.

Biological neural networks are often constructed using spiking neurons, which could be better understood using this method.

Project Summary:

The motive behind this project is to develop a better sampling and reconstruction model for signals, which offers theoretical and practical advantages over the classical model. This model is also closer to what is found in nature. The implementation of the model will be done using MATLAB to simulate, and can be used with high accuracy in embedded systems as well

Conclusion :

We have studied time encoding of 2Ω bandlimited signals, proposed an algorithm for reconstructing an input signal from its samples, and provided sufficient conditions on Ω for the algorithm to converge to the correct solution. Our setting has focused on reconstructing signals using TEMs with the same parameters κ, δ and b, where $b > 0$. This algorithm can be extended to scenarios where these parameters vary.

References :

1. Main Reference - https://ieeexplore.ieee.org/document/8962206
2. POCS - https://en.m.wikipedia.org/wiki/Projections_onto_convex_sets
3. Basic Concept - https://ieeexplore.ieee.org/document/1344228