A Very Brief Literature Review

The value of a time series asset class is dependent on and varies with a set of macro level time series variables. We will use the Error Correction Model (ECM) to explicate the relationship. This relationship determines both short term and long term behavior (Davidson et al., 1978). Instead of deriving relations directly from equilibrium-based economic theory, we will impose the error correction mechanism as an adjustment hypothesis a la Aldrich (1989).

If two time series variables y and x are both I(1) and cointegrated then they are generated by Error Correction Models (Engle & Granger, 1987) where characteristically the change in one variable is related to the change in another variable as well as the error term in the previous period. More generally we can write:

$$y_t = \alpha_0 + \alpha_1 x_t + \varepsilon$$

ε here is I(0).

Vector Autoregression models can be used to forecast economic time series and design and evaluate economic models (Sims, 1980a). A simple VAR model involving 2 lags on three variables y_1, y_2, y_3 is shown below:

$$\begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \end{bmatrix} = \begin{bmatrix} \mu_{1t} \\ \mu_{2t} \\ \mu_{3t} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} y_{1t-1} \\ y_{2t-1} \\ y_{3t-1} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{21} & b_{31} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} y_{1t-2} \\ y_{2t-2} \\ y_{3t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix}$$

This model is a vector of 3 variables y_1, y_2, y_3 and is autoregressive because in each equation, at least one of the regressors is a lagged dependent variable.

A typical 2 variable Error Correction Model can be operationalized using Vector Autoregression (VAR) getting us the Vector Error Correction Model (VECM). A VECM is a restricted VAR with nonstationary variables that are cointegrated.

$$\Delta y_t = \beta_{y0} + \beta_{yy1}\Delta y_{t-1} + \beta_{yx1}\Delta x_{t-1} + \lambda_y(y_{t-1} - \alpha_0 - \alpha_1 x_{t-1}) + v_t{}^y$$

$$\Delta x_t = \beta_{x0} + \beta_{xy}\,\Delta y_{t-1} + \beta_{xx1}\Delta x_{t-1} + \lambda_x(y_{t-1} - \alpha_0 - \alpha_1 x_{t-1}) + v_t{}^x$$

The Model

We can obviously model a VECM with multiple explanatory variables. Also there seems to be merit in looking at nonlinear VARs. I will try to make sense of elasticities and the underlying reason for the nonlinearity in the model. I will estimate the parameters of such a model for individual asset classes in a commodity index. Up-to-date data comes from QuantConnect.com. To generate arbitrage there is an apparent need to pick an asset class from the index. I will go a step further and optimize the returns for the index investing in an asset class mix.

References

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