MANUFACTURING MODELING AND ANALYSIS

Multi-Stage Single Product/Multi Product Factory

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PROBLEM STATEMENT

Consider a factory that consists entirely of single-server workstations with service time data for each workstation given by Table 1.

Workstation i	E(T(i))	C(i)*C(i)
1	7.8 min	1.0355
2	7.8 min	1.7751
3	9.6 min	0.3906
4	3.84 min	2.4414

Table 1. Workstation characteristics

Arrivals from an external source enter into the factory at the first workstation, and the arrivals are exponentially distributed with a mean rate of 5 jobs per hour. After initial processing, 2/3 of the jobs are sent to Workstation 2 and 1/3 are sent to Workstation 3. After the second step of processing, jobs are tested at Workstation 4, and only 40% of the jobs are found to be acceptable. Ten percent of the completed jobs fail the testing completely and are scrapped, at which time a new job is started to replace the scrapped jobs. Fifty percent of the jobs partially fail the testing and can be reworked. Sixty percent of the partial failures are sent to Workstation 3 and the others are sent to Workstation 2. After reworking, the jobs are sent again for testing at Workstation 4 with the same percentage of passing, partially failing, and completely failing the testing. (Figure 1. illustrates these job flows and switching probabilities.) Management is interested in the mean cycle time for jobs, factory inventory levels, and workloads at each workstation.

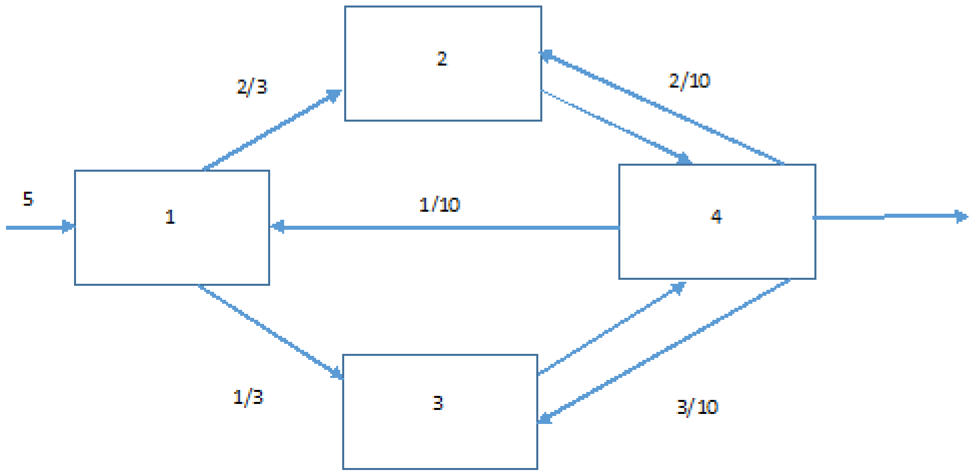

Figure 1. Factory Topology

SOLUTION

1) Workstation arrival rates:

The inflow rate into each workstation is a function of external inflows into the system and internal routing characteristics. From Figure 1, the equations that define these rates are:

$$\lambda_1 = 5 + \frac{1}{10}\lambda_5$$

$$\lambda_2 = 0 + \frac{2}{3}\lambda_1 + \frac{2}{10}\lambda_4$$

$$\lambda_3 = 0 + \frac{1}{3}\lambda_1 + \frac{3}{10}\lambda_4$$

$$\lambda_4 = 0 + \lambda_2 + \lambda_3$$

The solution to this system of equations is $(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = (6.25, 6.667, 5.833, 12.5)$

2) Workstation utilizations:

The workload at each of the workstations is this arrival rate multiplied by the given expected processing time from Table 1.

Workstation i	**λ(i)**	**E(T(i))**	**Utilization = u(i)**	**u(i)*u(i)**	**1-u(i)*u(i)**
1	6.250/hr	0.130 hr	0.8125	0.6602	0.3398
2	6.667/hr	0.130 hr	0.8667	0.7512	0.2488
3	5.833/hr	0.160 hr	0.9333	0.8710	0.1290
4	12.5/hr	0.064 hr	0.8000	0.6400	0.3600

Table 2: Workstation Utilization

The resulting utilization rates are in the range of 80-90% which allows us to compute steady state system characteristics.

3) Squared Coefficients of Variation:

Consider an arrival stream that is formed by merging of n individual arrival processes. The individual streams have mean arrival rates $\lambda_i = \frac{1}{E(T_i)}$ and squared coefficients of variation denoted by ${C_i}^2$ for i = 1,2,..,n.The mean arrival rate λ_a and the squared coefficient of variation, ${C_a}^2$, for a renewal process used to approximate the merged arrival process are given by,

$$\lambda_a = \sum_{i=1}^{n} \lambda_i$$

$${C_a}^2 = \sum_{i=1}^{n} \frac{\lambda_i}{\lambda_a} {C_i}^2 \qquad (1)$$

The number of departures, N, between routings to the target workstation is a random variable and is distributed according to the geometric distribution. Thus,

$$\Pr(N = n) = f(n) = p(1-p)^{(n-1)}$$

Where p is the probability that a given job is routed to the second workstation.

$$\mathrm{E}(N) = \frac{1}{p}$$

$$\mathrm{Var}(N) = \frac{1-p}{p^2}$$

To compute the time between visits to the second workstation for jobs departing from the first workstation, we define the random variable, T.

$$T = T_1 + T_2 + \cdots + T_N$$

Since this is a sum of N iid random variables,

$$\mathrm{E}(T) = \frac{E(T_1)}{p}$$

$$\mathrm{E}(T_a) = \frac{E(T_d)}{p}$$

and

$$\mathrm{Var}(T) = \frac{V(T_1)}{p} + \frac{(1-p)E(T_1)^2}{p^2}$$

$$C_a{}^2 = pC_d{}^2 + (1-p)$$

Now, the squared coefficient of variation of the inter-departure times can be approximated by,

$$C_d{}^2(G|G|c) = (1-u^2)C_a{}^2 + u^2\frac{(C_s{}^2 + \sqrt{c} - 1)}{\sqrt{c}}$$

Thus using these approximations along with equation (1) and the routing probabilities from Figure 1, we get

$$C_a{}^2(1) = 0.0072C_a{}^2(4) + 1.0112$$

$$C_a{}^2(2) = 0.1416C_a{}^2(1) + 0.0270C_a{}^2(4) + 0.9104$$

$$C_a{}^2(3) = 0.0405C_a{}^2(1) + 0.0694C_a{}^2(4) + 1.0708$$

$$C_a{}^2(4) = 0.1327C_a{}^2(2) + 0\ .0602C_a{}^2(3) + 0.8699$$

Solving this linear system of equations,

$$(C_a{}^2(1), C_a{}^2(2), C_a{}^2(3), C_a{}^2(4)) = (1.0190{,}1.0840{,}1.1874{,}1.0852)$$

4) Decomposition:

With the determination of arrival rates and coefficients of squared variation, each workstation is analyzed as if it were an isolated workstation. Here, the individual cycle time is given by,

$$CT = \frac{({C_a}^2 + {C_S}^2)}{2} * \left(\frac{u}{1-u}\right) * E(T_s) + E(T_s)$$

Thus from Table 1 & 2,

CT(1) = 0.709 hr

CT(2) = 1.338 hr

CT(3) = 1.927 hr

CT(4) = 0.515 hr

Using the Little's law,

WIP = CT*throughput

Thus from Table 2,

WIP (1) = 4.429

WIP (2) = 8.920

WIP (3) = 11.243

WIP (4) = 6.443

Therefore total Work-In-Progress is the sum of individual WIPs,

WIP = 31.03

Total Cycle Time,

CT = WIP/5 = 31.03/5 = 6.206 hr

Going forward, such a methodology of modeling and analysis can be readily extended to multi-stage multi-product settings. Also computer simulations can be used to get more accurate readings on the required factory level metrics. These high level metrics can in-turn be further improved by applying lean six sigma to the explanatory variables.

APPENDIX: EXTENSION TO MULTI PRODUCT FACTORY MODELS

A few key performance metrics for an m-product factory layout:

1) Workstation workloads: The workload at workstation k is computed as the sum of the product visits multiplied by their mean processing times.

$$WL_k = \sum_{i=1}^{m} \lambda_{ik} E[T_s(i,k)]$$

2) The mean service time is given by,

$$E[T_s(k)] = \sum_{i=1}^{m} \frac{\lambda_{ik} E[T_s(i,k)]}{\lambda_k}$$

$$E[T_s(k)] = \frac{WL_k}{\lambda_k}$$

3) The second moment is given by

$$E[T_s(k)^2] = \sum_{i=1}^{m} \frac{\lambda_{ik} E[T_s(i,k)^2]}{\lambda_k}$$

But,

$$E(X^2) = E(X)^2[1 + C(x)^2]$$

$$E[T_s(k)^2] = \sum_{i=1}^{m} \frac{\lambda_{ik} E[T_s(i,k)]^2 [1 + C_s(i,k)^2]}{\lambda_k}$$

Therefore,

$$C_s{}^2(k) = \frac{\sum_{i=1}^{m} \frac{\lambda_{ik} E[T_s(i,k)]^2 [1 + C_s(i,k)^2]}{\lambda_k}}{(\sum_{i=1}^{m} \frac{\lambda_{ik} E[T_s(i,k)]}{\lambda_k})^2} - 1$$

Now, the composite routing matrix is given by,

$$p_{jk} = \sum_{i=1}^{m} \frac{\lambda_{ik}}{\lambda_k} [p^i{}_{jk}]$$

And the cycle time is given by,

$$CT^i{}_s = \frac{\sum_{k=1}^{n} \lambda_{ik} (CT_q(k) + E[T_s(i,k)])}{\sum_{j=1}^{n} \gamma_{ij}}$$

Finally, we can compute the Work-In-Progress using the Little's Law

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