



Sunbeam Institute of Information Technology Pune and Karad

Module – Data Structures

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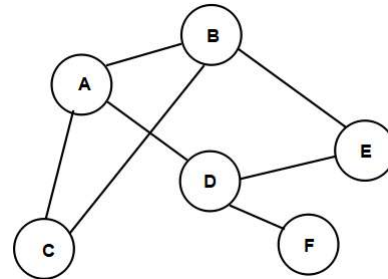


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Graph : Terminologies

- **Graph** is a non linear data structure having set of vertices (nodes) and set of edges (arcs).
 - $G = \{V, E\}$
Where V is a set of vertices and E is a set of edges
 - **Vertex (node)** is an element in the graph
 $V = \{A, B, C, D, E, F\}$
 - **Edge (arc)** is a line connecting two vertices
 $E = \{(A,B), (A,C), (B,C), (B,E), (D, E), (D,F), (A,D)\}$
- Vertex A is set be **adjacent** to B, if and only if there is an edge from A to B.
- **Degree of vertex** :- Number of vertices adjacent to given vertex
- **Path** :- Set of edges connecting any two vertices is called as path between those two vertices.
 - Path between A to D = $\{(A, B), (B, E), (E, D)\}$
- **Cycle** :- Set of edges connecting to a node itself is called as cycle.
 - $\{(A, B), (B, E), (E, D), (D, A)\}$
- **Loop** :- An edge connecting a node to itself is called as loop. Loop is smallest cycle.

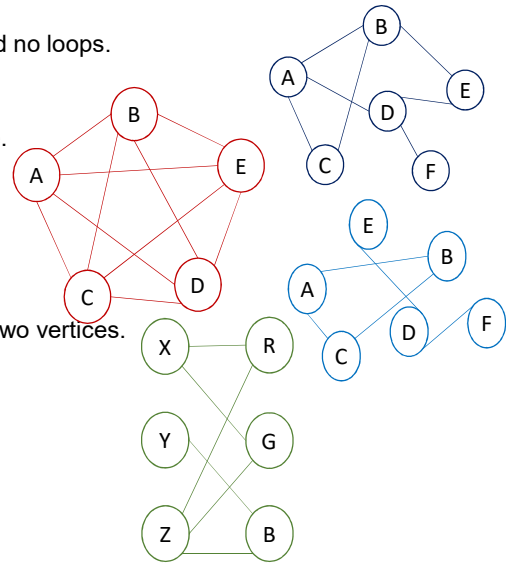


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Graph : Types

- **Simple Graph**
 - Graph not having multiple edges between adjacent nodes and no loops.
- **Complete Graph**
 - Simple graph in which node is adjacent with every other node.
 - Un-Directed graph: Number of Edges = $n(n-1)/2$
where, n – number of vertices
 - Directed graph: Number of edges = $n(n-1)$
- **Connected Graph**
 - Simple graph in which there is some path exist between any two vertices.
 - Can traverse the entire graph starting from any vertex.
- **Bi-partite graph**
 - Vertices can be divided in two disjoint sets.
 - Vertices in first set are connected to vertices in second set.
 - Vertices in a set are not directly connected to each other.

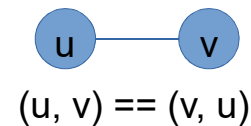


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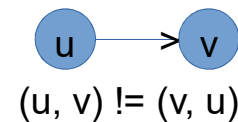
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Graph : Types

- **Undirected graph.**
 - If we can represent any edge either (u,v) OR (v,u) then it is referred as **unordered pair of vertices** i.e. **undirected edge**.
 - **graph which contains undirected edges referred as undirected graph.**



- **Directed Graph (Di-graph)**
 - If we cannot represent any edge either (u,v) OR (v,u) then it is referred as an **unordered pair of vertices** i.e. **directed edge**.
 - **graph which contains set of directed edges referred as directed graph (di-graph).**
 - graph in which each edge has some direction



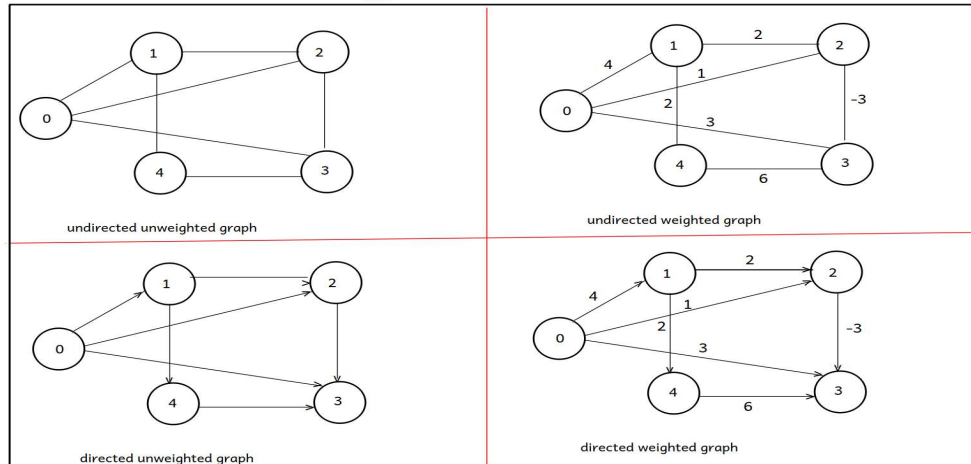
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Graph : Types

• Weighted Graph

- A graph in which edge is associated with a number (ie weight)

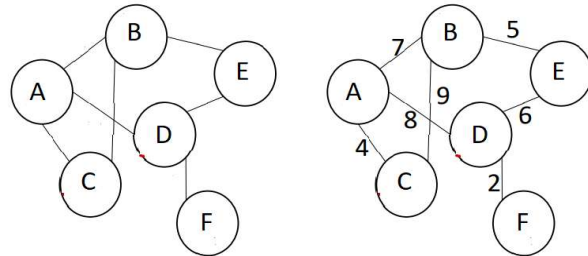


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Graph Implementation – Adjacency Matrix

- If graph have V vertices, a $V \times V$ matrix can be formed to store edges of the graph.
- Each matrix element represent presence or absence of the edge between vertices.
- For non-weighted graph, 1 indicate edge and 0 indicate no edge.
- For weighted graph, weight value indicate the edge and infinity sign ∞ represent no edge.
- For un-directed graph, adjacency matrix is always symmetric across the diagonal.
- Space complexity of this implementation is $O(V^2)$.



	A	B	C	D	E	F
A	0	1	1	1	0	0
B	1	0	1	0	1	0
C	1	1	0	0	0	0
D	1	0	0	0	1	1
E	0	1	0	1	0	0
F	0	0	0	1	0	0

	A	B	C	D	E	F
A	∞	7	4	8	∞	∞
B	7	∞	9	∞	5	∞
C	4	9	∞	∞	∞	∞
D	8	∞	∞	∞	6	2
E	∞	5	∞	6	∞	∞
F	∞	∞	∞	2	∞	∞

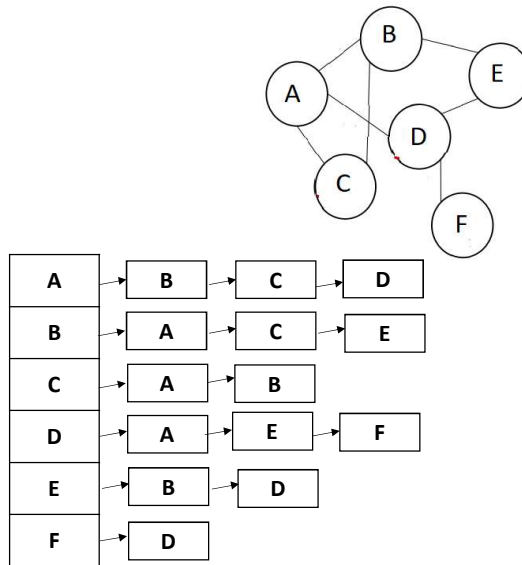


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Graph Implementation – Adjacency List

- Each vertex holds list of its adjacent vertices.
- For non-weighted graphs only, neighbor vertices are stored.
- For weighted graph, neighbor vertices and weights of connecting edges are stored.
- Space complexity of this implementation is $O(V+E)$.
- If graph is sparse graph (with fewer number of edges), this implementation is more efficient (as compared to adjacency matrix method).

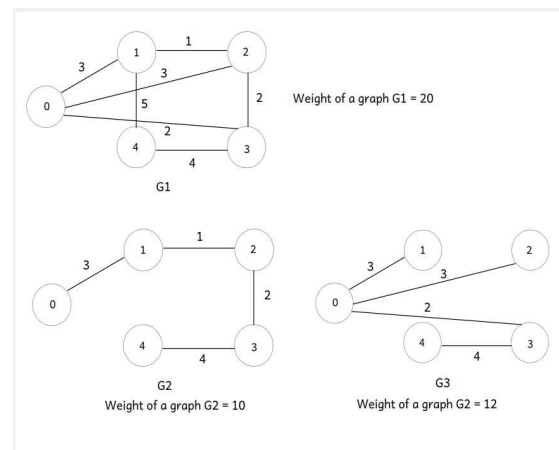


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Spanning Tree

- Tree is a graph without cycles. Includes all V vertices and $V-1$ edges.
- Spanning tree is connected sub-graph of the given graph that contains all the vertices and sub-set of edges.
- Spanning tree can be created by removing few edges from the graph which are causing cycles to form.
- One graph can have multiple different spanning trees.
- In weighted graph, spanning tree can be made who has minimum weight (sum of weights of edges). Such spanning tree is called as Minimum Spanning Tree.
- Spanning tree can be made by various algorithms.
 - BFS Spanning tree
 - DFS Spanning tree
 - Prim's MST
 - Kruskal's MST



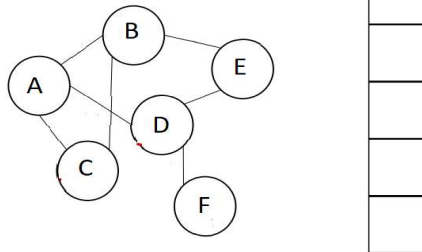
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Graph Traversal - BFS & DFS

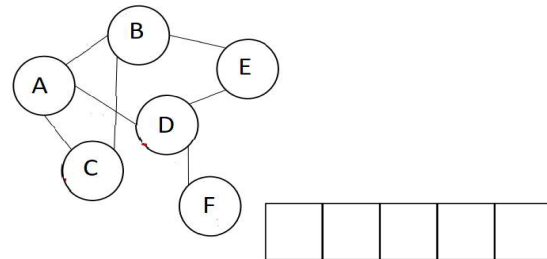
• DFS algorithm

1. Choose a vertex as start vertex.
2. Push start vertex on stack & mark it.
3. Pop vertex from stack.
4. Print the vertex.
5. Put all non-visited neighbours of the vertex on the stack and mark them.
6. Repeat 3-5 until stack is empty.



• BFS algorithm

1. Choose a vertex as start vertex.
2. Push start vertex on queue & mark it.
3. Pop vertex from queue.
4. Print the vertex.
5. Put all non-visited neighbours of the vertex on the queue and mark them.
6. Repeat 3-5 until queue is empty.



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Graph : Algorithms

• Graph Traversal Algorithms:

- Used to traverse all vertices in the graph.
- DFS Traversal (using Stack) and BFS Traversal (by using Queue)

• Shortest Path Algorithm:

- Single source SPT algorithm used to find minimum distance from the given vertex to all other vertices.
- Dijkstra's Algorithm (Doesn't work for -ve weight edges) $\Rightarrow O(V \log V)$.
- Bellman Ford Algorithm $\Rightarrow O(VE)$.

• All pair Shortest Path Algorithm:

- To find minimum distance from each vertex to all other vertices.
- Floyd Warshall Algorithm $\Rightarrow O(V^3)$
- Johnson's Algorithm $\Rightarrow O(V^2 \log V + VE)$

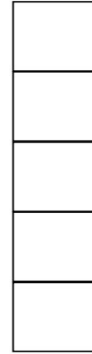
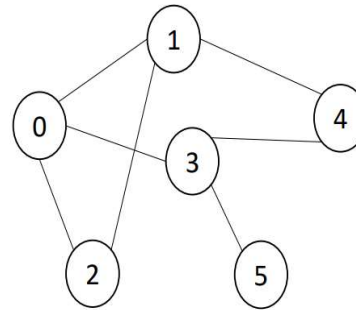


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Check Connected-ness

1. push start on stack & mark it.
2. begin counting marked vertices from 1.
3. pop and print a vertex.
4. push all its non-marked neighbors on the stack, mark them and increment count.
5. if count is same as number of vertex, graph is connected (return).
6. repeat steps 3-5 until stack is empty.
7. graph is not connected (return)

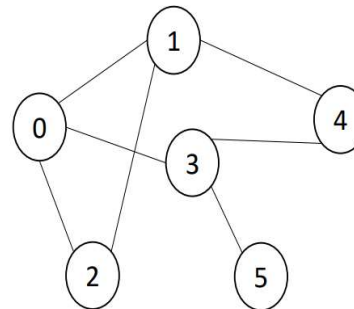


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Single Source Path Length

1. Create path length array to keep distance of vertex from start vertex.
2. push start on queue & mark it.
3. pop the vertex.
4. push all its non-marked neighbors on the queue, mark them.
5. For each such vertex calculate distance as $\text{dist}[\text{neighbor}] = \text{dist}[\text{current}] + 1$
6. print current vertex to that neighbor vertex edge.
7. repeat steps 3-6 until queue is empty.
8. Print path length array.



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Thank you!

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