

Data Structures and Algorithms

Data Structure

- **Data structure is organising data into memory for efficient access.**
- **on which we can perform various operations like add, remove, search, sort and update etc**
- **Abstract Data Types**
- **There are two types of Data structures**
 - 1. Linear Data structure (basic)**
 - **data is arranged linearly (sequentially)**
 - 1. Array**
 - 2. Structure / class**
 - 3. Queue**
 - 4. Stack**
 - 5. Linked List**
 - 2. Non Linear Data structure (Advanced)**
 - **data is arranged non-linearly (heirachical)**
 - 1. Tree**
 - 2. Graph**

Algorithm

- step by step solution to the given problem statement
- set of instructions to the human (programmer/developer)
- template ---> program

eg. Find sum of array elements

step 1 - create variable to store sum and initialize it with zero

step 2 - traverse the array from 0th index to size-1 index

step 3 - add every element of array into sum variable

step 4 - return /print the final sum

eg. searching, sorting

to achieve

- Abstraction
- Reusability
- Efficiency (time and space)

Traversing - visiting each and every element of collection/structure at least once

Analysis - time and space measurement of data structure and algorithm

Algorithm Analysis

Exact Analysis

- exact time and space required to execute
- space is dependent on no of elements and type of element
- time is dependent on type of machine, no of processes running at that time, size of data structure

Approximate Analysis

- approximate budget/measurement of time and space
- Asymptotic Analysis - mathematical approach to find time and space requirement of algorithm
- observing change in behavior of algorithm on change in input
- "Big O" / "O()" notation is used to indicate time and space

Time Measurement / Time Complexity

- time complexity is approximate time analysis
- to execute single statement will need 1 unit of time
- no of iterations of the loops are considered
- time is directly proportional to the no of iterations of the loop

1. Print 1D array

```
void printArray(int arr[], int n){  
    for(int i = 0 ; i < n ; i++)  
        sysout(arr[i]);  
}
```

No. of iterations
of loop = n

Time \propto No. of
Required Iterations

Time $\propto n$ unit

Time = n

$T(n) = O(n)$

2. Print 2D array

```
void print2DArray(int arr[][], int n, int m){  
    for(int i = 0 ; i < n ; i++)  
        for(int j = 0 ; j < m ; j++)  
            sysout(arr[i][j]);  
}
```

iterations
(outer loop) $r_1 = n$

iterations
(inner loop) $r_2 = m = n$

Total
iterations = $n * n = n^2$

Time $\propto n^2$

Time = n^2

$T(n) = O(n^2)$

3. Add two numbers

```
int addTwoNumbers(int num1, int num2){  
    int res = num1 + num2;  
    return res;  
}
```

irrespective of input values, this algorithm is taking some constant amount of time, it will be denoted as

$$T(n) = O(1)$$

4. print table of given number

```
void printTable(int num){  
    for(int i = 1 ; i <= 10 ; i++)  
        sysout(i * num);  
}
```

irrespective of input value, loop is going to iterate 10 time, so constant time will be needed.

$$T(n) = O(1)$$

5. Print binary of given decimal

```
void printBinary(int n){  
    int i = n;  
    while(i > 0){  
        int rem = i % 2;  
        sysout(rem);  
        i = i / 2;  
    }  
}
```

num = 9

9 > 0:

9 % 2 -> 1

9 / 2 -> 4

4 > 0:

4 % 2 -> 0

4 / 2 -> 2

2 > 0:

2 % 2 -> 0

2 / 2 -> 1

1 > 0:

1 % 2 -> 1

1 / 2 -> 0

i = 9, 4, 2, 1, 0

i = n, n/2, n/4, n/8, ...

i = $\frac{n}{2^0}, \frac{n}{2^1}, \frac{n}{2^2}, \frac{n}{2^3}, \dots, \frac{n}{2^{\text{itr}}}$

for i=1, last time loop condⁿ is true.

$$\frac{n}{2^{\text{itr}}} = 1$$

$$n = 2^{\text{itr}}$$

$$\log n = \log 2^{\text{itr}}$$

$$\text{itr} \log 2 = \log n$$

$$\text{itr} = \frac{\log n}{\log 2}$$

$$\text{Time} \propto \frac{1}{\log 2} \cdot \log n$$

$$\text{Time} = \log n$$

$$T(n) = O(\log n)$$

Time Complexity - $O(1)$, $O(\log n)$, $O(n)$, $O(n \log n)$, $O(n^2)$, $O(n^3)$, $O(2^n)$

modification - '+' / '-' \rightarrow time complexity will be in terms n

modification - '*' / '/' \rightarrow time complexity will be in terms $\log n$

$i = n = 9$ $i > 0$ $i = i / 2$ $\rightarrow i = 9, 4, 2, 1, \boxed{0}$ $\rightarrow \log n$
 $i = 1$ $i < 9$ $i = i * 2$ $\rightarrow i = 1, 2, 4, 8, \boxed{16}$ $\rightarrow \log n$

for($i=0$; $i < n$; $i++$)

for($i=n$; $i > 0$; $i--$)

for($i=0$; $i < n$; $i = i + 2$)

for($i=0$; $i < n$; $i = i + 100$)

} $T(n) = O(n)$

for($i=0$; $i < n$; $i++$) $\rightarrow n$

{ }

for($i=0$; $i < n$; $i++$) $\rightarrow n$

{ }

Time $\propto 2n$

$T(n) = O(n)$

for($i=0$; $i < n$; $i++$)
 for($j=0$; $j < n$; $j++$)
 { }

} $\rightarrow n^2$

for($i=0$; $i < n$; $i++$) $\rightarrow n$

Time $\propto n^2 + n$

$T(n) = O(n^2)$

Space Measurement / Space Complexity

- it is approximate measure of space required to execute the algorithm
- Total space is addition of input space and auxillary space

input space - actual space of memory to store input (data)

auxillary space - space of memory which is required to process actual input

- to store 1 value, will need 1 unit of space

```
int sumOfArray(int arr[], int n){  
    int sum = 0;  
    for(int i = 0 ; i < n ; i++)  
        sum += arr[i];  
    return sum;  
}
```

Input space - n units

Auxillary space - 3 units

Total space = Input space + Auxillary space
= n + 3
= n

$S(n) = O(n)$

Space Complexity - $O(1)$, $O(n)$, $O(n^2)$, $O(n^3)$

Searching Algorithms

- finding the key from collection of values
- There are two types of search
 1. linear search
 2. binary search

Linear search

- | | | |
|-----------------|----------------------------------|----------|
| 1. Best case | - key is found at initial places | - $O(1)$ |
| 2. Average case | - key is found at middle places | - $O(n)$ |
| 3. Worst case | - key is found at last places | |
| | - key is not found | - $O(n)$ |

Binary search

- | | | |
|-----------------|----------------------------------|---------------|
| 1. Best case | - key is found at initial levels | - $O(1)$ |
| 2. Average case | - key is found at middle levels | - $O(\log n)$ |
| 3. Worst case | - key is found at last levels | |
| | - key is not found | - $O(\log n)$ |

Approach

Iterative

implemented using loops

```
int factorial(int num){  
    int fact = 1;  
    while(num){  
        fact *= num;  
        num = num - 1;  
    }  
    return fact;  
}
```

Time complexity = no of iterations

$$T(n) = O(n)$$

Recursive

Recursive functions

$$n! = n * (n-1)!$$

if $n = 0$, stop

```
int recFactorial(int num){  
    if(num == 0)  
        return 1;  
    return num * recFactorial(num-1);  
}
```

Time complexity = no of recursive
function calls

$$T(n) = O(n)$$

Recursion

```
int rFact(int num){  
    if(num == 1)  
        return 1;  
    return num * rFact(num - 1);  
}
```

$5! = 5 * 4!$
 $= 5 * 4 * 3!$
 $= 5 * 4 * 3 * 2!$
 $= 5 * 4 * 3 * 2 * 1!$
 $= 5 * 4 * 3 * 2 * 1$
 $\text{if}(\text{num} == 1) \text{ return } 1$

