Practice Questions for Probability Distributions

1) Investment Advisors agree that near retirees, defined as people aged 55 to 65, should have balanced portfolios. Most advisors suggest that the near -retirees have no more 50% of their investments in stocks. However, during the huge decline in the stock market in 2008, 22% of near -retirees had 90% or more of their investments in stocks. Suppose you have a random sample of 10 people who have labeled as near -retirees in 2008. What is the probability that during 2008.

a) Zero had 90% or more of their investments in stocks?

b) Exactly one had 90% or more of his investments in stocks?

c) Two or fewer had 90% or more of their investment in stocks?

d) Three or more had 90% or more of their investments in stocks

Solution

Here, n = 10, p = 0.22

a) Probability that during 2008, Zero had 90% or more of their investments in stocks is given by P(X = 0)

It is given by pbinorm(0, 10, 0.22)

```
p1 = pbinom(0, 10, 0.22)

cat("\n a)  Probability that during 2008, Zero had 90% or more of their inve
stments in stocks is ", round(p1,4))
##
##  a)  Probability that during 2008, Zero had 90% or more of their investme
nts in stocks is  0.0834
```

b) Probability that during 2008, exactly one had 90% or more of his investments in stocks is given by P(X = 1)

It is given by pbinom(1, 10, 0.22) - pbinom(0, 10, 0.22)

```
p1 = pbinom(1, 10, 0.22) - pbinom(0, 10, 0.22)

cat("\nb) Probability that during 2008, exactly one had 90% or more of his i
nvestments in stocks is ", round(p1,4))
##
## b) Probability that during 2008, exactly one had 90% or more of his inves
tments in stocks is  0.2351
```

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c) Probability that during 2008, two or fewer had 90% or more of their investment in stocks is given by P(X ≤ 2)

It is given by pbinom(2, 10, 0.22)

```
p1 = pbinom(2, 10, 0.22)

cat("\nc)   Probability that during 2008, two or fewer had 90% or more of th
eir investment in stocks is  ", round(p1,4))
##
## c)   Probability that during 2008, two or fewer had 90% or more of their
investment in stocks is   0.6169
```

d) Probability that during 2008, three or more had 90% or more of their investments in stocks is given by P(X ≥ 3)

It is given by 1 - pbinom(2, 10, 0.22)

```
p1 = 1 - pbinom(2, 10, 0.22)

cat("\nd)   Probability that during 2008, three or more had 90% or more of t
heir investments in stocks is ", round(p1,4))
##
## d)   Probability that during 2008, three or more had 90% or more of their
investments in stocks is  1-0.6169=0.3831
```

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2) Assume that number of network errors experienced in a day on a local area network (LAN) is distributed as a Poisson random variable. The mean number of network errors experienced in a day is 2.4. What is the probability that in any given day

a) Zero network errors will occur?

b) Exactly one network error will occur?

c) Two or more network error will occur?

d) Fewer than three network errors will occur?

Solution

Here $\lambda = 2.4$

a. Zero network errors will occur is given by P(X = 0 | λ = 2,4) It is given by ppois(0, lambda = 2.4)

```
p1 = ppois(0, lambda = 2.4)
cat("\n a) Probability that Zero network errors will occur is ", round(p1,4)
)
```

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```
##
##  a) Probability that Zero network errors will occur is  0.0907
```

b) Exactly one network error will occur is given by P(X = 1 | λ = 2,4) It is given by ppois(1, lambda = 2.4) - ppois(0, lambda = 2.4)

```
p1 =  ppois(1, lambda = 2.4) - ppois(0, lambda = 2.4)
cat("\n b) Probability that Exactly one network error will occur is ", round
(p1,4))
##
##  b) Probability that Exactly one network error will occur is  0.2177
```

c) Two or more network error will occur is given by P(X > 1 | λ = 2,4) It is given by 1 - ppois(1, lambda = 2.4)

```
p1 = 1 - ppois(1, lambda = 2.4)
cat("\n c) Probability that Two or more network error will occur is ", round
(p1,4))

##
##  c) Probability that Two or more network error will occur is  0.6916
```

d) Fewer than three network errors will occur is given by P(X < 3 | λ = 2,4) It is given by ppois(2, lambda = 2.4)

```
p1 = ppois(2, lambda = 2.4)
cat("\n d) Probability that Fewer than three network errors will occur is  "
, round(p1,4))
##
##  d) Probability that Fewer than three network errors will occur is 0.5697
```

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3) The quality control manager of Marilyn's cookies is inspecting a batch of chocolate - chip cookies that has been just baked. If the production process is in control the mean number of chips parts per cookie is 6.0.

What is the probability that in any particular cookie being inspected:

a) Fewer than five chips part will be found?
b) Exactly five chips parts will be found?
c) Five or more chips part will be found?
d) Either four or five chips part will be found?

Solution

Here $\lambda = 6.0$

a) Probability that in any particular cookie being inspected fewer than five chips part will be found is given by $P(X < 5 \mid \lambda = 6.0)$

It is given by ppois(4, lambda = 6.0)

```
p1 = ppois(4, lambda = 6.0)
cat("\n a) Probability that Fewer than five chips part will be found is ", r
ound(p1,4))
##
##  a) Probability that Fewer than five chips part will be found is  0.2851
```

b) Probability that in any particular cookie being inspected exactly five chips parts will be found is given by $P(X = 5 \mid \lambda = 6.0)$

It is given by ppois(5, lambda = 6.0) - ppois(4, lambda = 6.0)

```
p1 = ppois(5, lambda = 6.0) - ppois(4, lambda = 6.0)

cat("\n b)  Probability that in any particular cookie being inspected exactl
y five chips parts will be found is ", round(p1,4))
##
##  b)  Probability that in any particular cookie being inspected exactly fi
ve chips parts will be found is  0.1606
```

c) Probability that in any particular cookie being inspected five or more chips part will be found is given by $P(X > 4 \mid \lambda = 6.0)$

It is given by 1 - ppois(4, lambda = 6.0)

```
p1 =  1 - ppois(4, lambda = 6.0)

cat("\n c)  Probability that in any particular cookie being inspected five o
r more chips part will be found is ", round(p1,4))
##
##  c)  Probability that in any particular cookie being inspected five or mo
re chips part will be found is  0.7149
```

d) Probability that in any particular cookie being inspected will have either four or five chips part is given by
$P(X = 4 \mid \lambda = 6.0) + P(X = 5 \mid \lambda = 6.0)$

It is given by ppois(5, lambda = 6.0) - ppois(3, lambda = 6.0)

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```
p1 =  ppois(5, lambda = 6.0) - ppois(3, lambda = 6.0)

cat("\n d)      Probability that in any particular cookie being inspected wi
ll have either four or five chips part is", round(p1,4))
##
##  d)      Probability that in any particular cookie being inspected will h
ave either four or five chips part is 0.2945
```

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4) In a recent year, about two thirds of US households purchased ground coffee. Consider the annual ground coffee expenditures for households purchasing ground coffee assuming that these expenditures are approximately distributed as a normal random variable with a mean of $65.16 and a Standard Deviation of $10.00

a) Find the probability that a household spent less than $35.00.

b) Find the probability that a household spent more than $60.00

c) What proportion of the households spent between $40.00 and $50.00?

d) 99% of households spent less than what amount?

Solution

Here μ = 65.16 and σ = 10.00

a) Probability that household spent less than USD 35.00 is given by P(X < 35 | μ = 65.16 and σ = 10)

It is given by 1 - pnorm(35, mean = 65.16, sd = 10.0, lower.tail=FALSE)

```
p1 =  1 - pnorm(35, mean = 65.16, sd = 10.0, lower.tail=FALSE)

cat("\n a)  Probability that household spent less than USD 35.00 is", round(
p1,4))
##
##  a)  Probability that household spent less than USD 35.00 is 0.0013
```

b) Probability that a household spent more than USD 60.00 is given by P(X > 60 | μ = 65.16 and σ = 10)

It is given by pnorm(60, mean = 65.16, sd = 10.0, lower.tail=FALSE)

```
p1 =  pnorm(60, mean = 65.16, sd = 10.0, lower.tail=FALSE)

cat("\n Probability that a household spent more than USD 60.00 is",
round(p1,4))
##
##  Probability that a household spent more than USD 60.00 is 0.6971
```

c) Probability that a households spent between USD 40.00 and USD 50.00 is given by pnorm(50, mean = 65.16, sd = 10.0, lower.tail= TRUE) - pnorm(40, mean = 65.16, sd = 10.0, lower.tail= TRUE)

```
p1 =  pnorm(50, mean = 65.16, sd = 10.0, lower.tail= TRUE) - pnorm(40, mean
= 65.16, sd = 10.0, lower.tail= TRUE)


cat("\n Probability that a households spent between USD 40.00 and USD 50.00
is", round(p1,4))
##
##  Probability that a households spent between USD 40.00 and USD 50.00 is 0
.0588
```

d) 99 percent of households spent less than what amount is given by qnorm(0.99, mean = 65.16, sd = 10.0)

```
value1 =  qnorm( 0.99, mean = 65.16, sd = 10.0)


cat("\n Amount that will be sepnt by    99 percent of households is USD ", r
ound(value1, 2))
##
##  Amount that will be sepnt by    99 percent of households is USD  88.42
```

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