Games and adversarial search

World Champion chess player Garry Kasparov is defeated by IBM's Deep Blue chess-playing computer in a six-game match in May, 1997

Why study games?

- Games are a traditional hallmark of intelligence
- Games are easy to formalize
- Games can be a good model of real-world competitive or cooperative activities
  - Military confrontations, negotiation, auctions, etc.

Types of game environments

	Deterministic	**Stochastic**
Perfect information (fully observable)	Chess, checkers, go	Backgammon, monopoly
Imperfect information (partially observable)	Battleships	Scrabble, poker, bridge

Alternating two-player zero-sum games

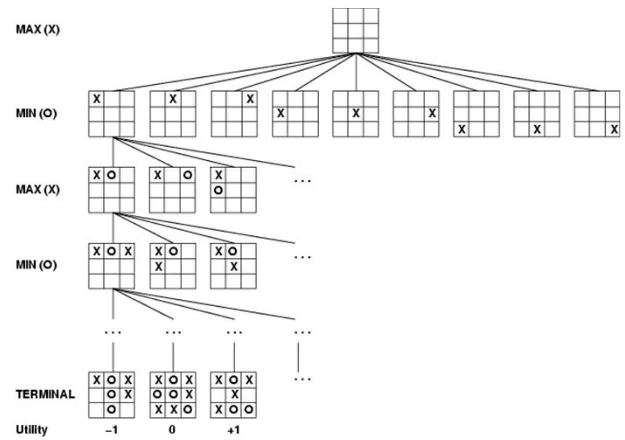
- Players take turns
- Each game outcome or **terminal state** has a **utility** for each player (e.g., 1 for win, 0 for loss)
- The sum of both players' utilities is a constant

Games vs. single-agent search

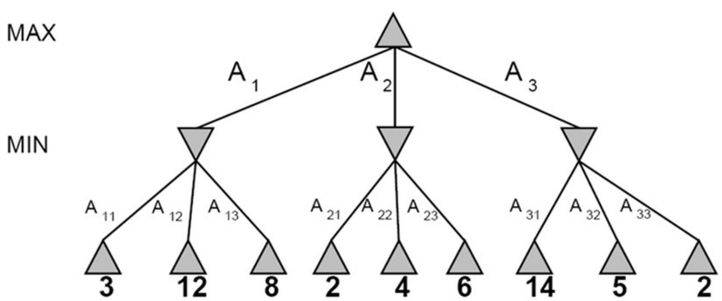
- We don't know how the opponent will act
  - The solution is not a fixed sequence of actions from start state to goal state, but a ***strategy*** or ***policy*** (a mapping from state to best move in that state)
- Efficiency is critical to playing well
  - The time to make a move is limited
  - The branching factor, search depth, and number of terminal configurations are huge
    - In chess, branching factor ≈ 35 and depth ≈ 100, giving a search tree of 10^{154} nodes
      - Number of atoms in the observable universe ≈ 10^{80}
  - This rules out searching all the way to the end of the game

Game tree

- A game of tic-tac-toe between two players, "max" and "min"



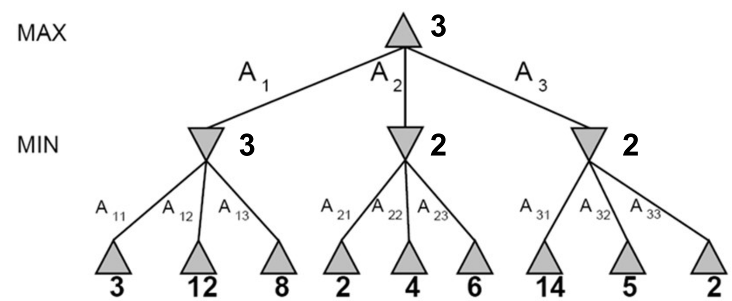
A more abstract game tree



Terminal utilities (for MAX)

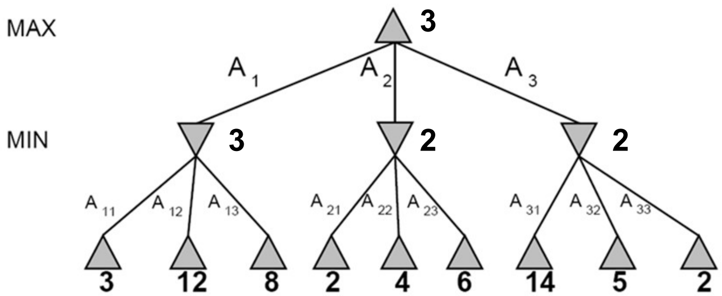
A *two-ply* game

Game tree search



- **Minimax value of a node**: the utility (for MAX) of being in the corresponding state, assuming perfect play on both sides
- **Minimax strategy:** Choose the move that gives the best worst-case payoff

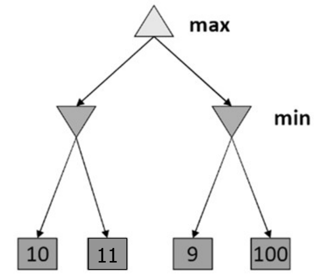
Computing the minimax value of a node



- **Minimax**(*node*) =
  - Utility(*node*) if *node* is terminal
  - $\max_{action}$ **Minimax**(Succ(*node, action*)) if *player* = MAX
  - $\min_{action}$ **Minimax**(Succ(*node, action*)) if *player* = MIN

Optimality of minimax

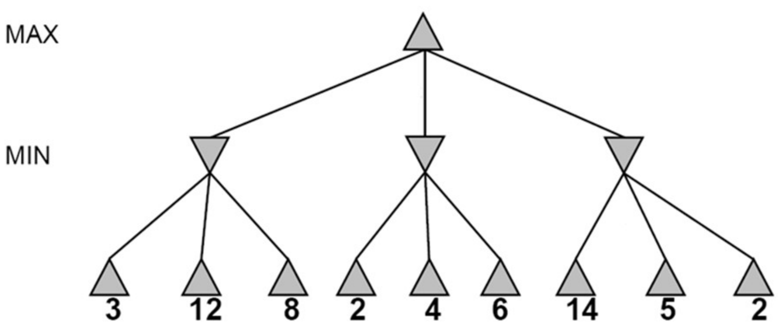
- The minimax strategy is optimal against an optimal opponent
- What if your opponent is suboptimal?
  - Your utility can only be higher than if you were playing an optimal opponent!
  - A different strategy may work better for a sub-optimal opponent, but it will necessarily be worse against an optimal opponent



Example from D. Klein and P. Abbeel

Alpha-beta pruning

- It is possible to compute the exact minimax decision without expanding every node in the game tree



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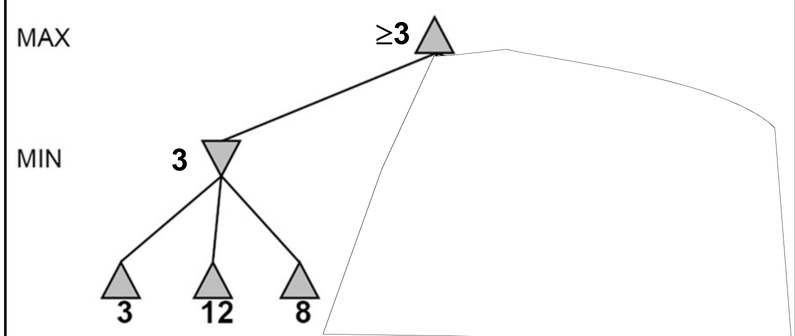

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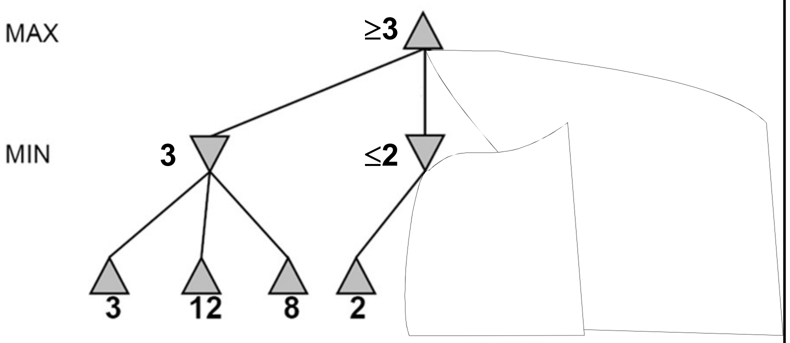

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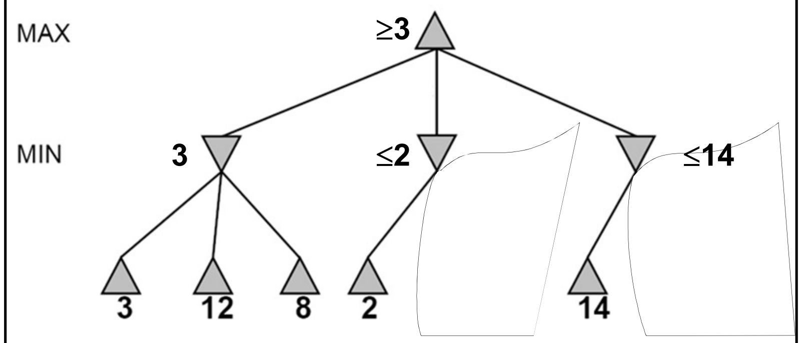

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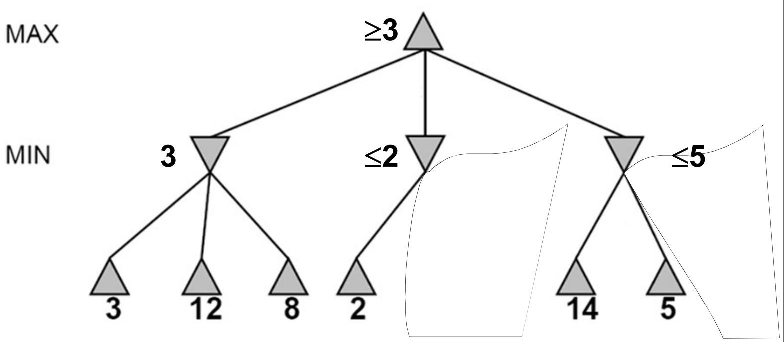

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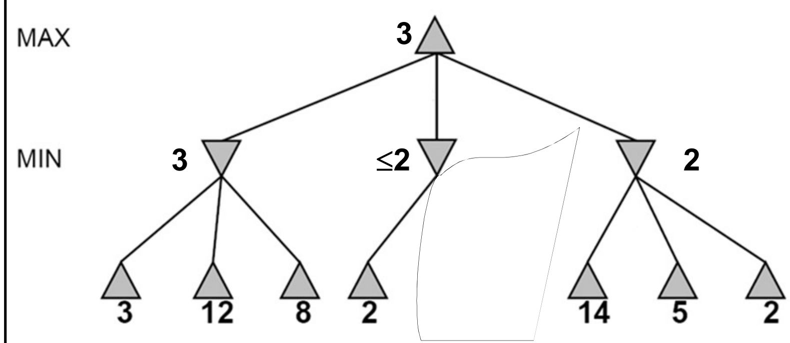
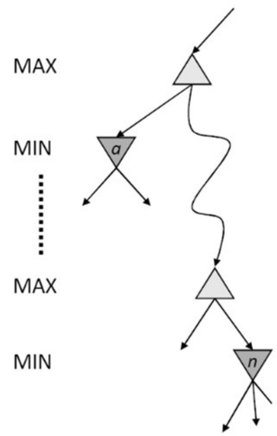


Alpha-beta pruning

- **α** is the value of the best choice for the MAX player found so far at any choice point above node *n*
- We want to compute the MIN-value at *n*
- As we loop over *n*'s children, the MIN-value decreases
- If it drops below **α**, MAX will never choose *n*, so we can ignore *n*'s remaining children
- Analogously, **β** is the value of the lowest-utility choice found so far for the MIN player

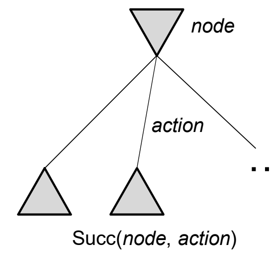


Alpha-beta pruning

Function *action* = **Alpha-Beta-Search**(*node*)
  v = **Min-Value**(*node*, −∞, ∞)
  return the *action* from *node* with value *v*

α: best alternative available to the Max player
β: best alternative available to the Min player

Function *v* = **Min-Value**(*node*, *α*, *β*)
  if Terminal(*node*) return Utility(*node*)
  v = +∞
  for each *action* from *node*
    v = Min(*v*, **Max-Value**(Succ(*node*, *action*), *α*, *β*))
    if *v* ≤ *α* return *v*
    β = Min(*β*, *v*)
  end for
  return *v*



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Function *action* = **Alpha-Beta-Search**(*node*)
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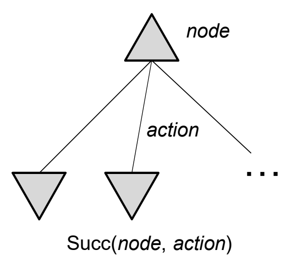
α: best alternative available to the Max player
β: best alternative available to the Min player

Function *v* = **Max-Value**(*node*, *α*, *β*)
  if Terminal(*node*) return Utility(*node*)
  v = −∞
  for each *action* from *node*
    v = Max(*v*, **Min-Value**(Succ(*node*, *action*), *α*, *β*))
    if *v* ≥ *β* return *v*
    α = Max(*α*, *v*)
  end for
  return *v*



Alpha-beta pruning

- Pruning does not affect final result
- Amount of pruning depends on move ordering
  - Should start with the "best" moves (highest-value for MAX or lowest-value for MIN)
  - For chess, can try captures first, then threats, then forward moves, then backward moves
  - Can also try to remember "killer moves" from other branches of the tree
- With perfect ordering, the time to find the best move is reduced to $O(b^{m/2})$ from $O(b^m)$
  - Depth of search is effectively doubled

Evaluation function

- Cut off search at a certain depth and compute the value of an **evaluation function** for a state instead of its minimax value
  - The evaluation function may be thought of as the probability of winning from a given state or the *expected value* of that state
- A common evaluation function is a weighted sum of *features*:

$$\boldsymbol{Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)}$$

  - For chess, $\boldsymbol{w_k}$ may be the **material value** of a piece (pawn = 1, knight = 3, rook = 5, queen = 9) and $\boldsymbol{f_k(s)}$ may be the advantage in terms of that piece
- Evaluation functions may be *learned* from game databases or by having the program play many games against itself

Cutting off search

- **Horizon effect:** you may incorrectly estimate the value of a state by overlooking an event that is just beyond the depth limit
  - For example, a damaging move by the opponent that can be delayed but not avoided
- Possible remedies
  - **Quiescence search:** do not cut off search at positions that are unstable – for example, are you about to lose an important piece?
  - **Singular extension:** a strong move that should be tried when the normal depth limit is reached

Advanced techniques

- **Transposition table** to store previously expanded states
- **Forward pruning** to avoid considering all possible moves
- **Lookup tables** for opening moves and endgames