

Prob - I

(a) $H_0: \mu = 25$

$H_1: \mu \neq 25$

This is correct because we can see that H_0 contains the no difference term and it is contradictory to H_0 .

(b) $H_0: \sigma > 10$

$H_1: \sigma = 10$

This is wrong because H_0 contains the type of expression $\sigma = 10$. Since it is the hypothesis of no difference.

(c) $H_0: \bar{x} = 50$

$H_1: \bar{x} \neq 50$

This is correct because we can see that H_0 contains the no difference term and it is contradictory to H_0 .

(d) $H_0: P = 0.1$

$H_1: P = 0.5$

This is wrong because H_1 contains the no diff claim and H_0 is not contradictory to H_1 with equal right hand side.

Problem Statement - 2

Soln

$$H_0: \mu \leq 52$$

$$\bar{x} = 52$$

$$H_1: \mu > 52$$

$$\sigma = 4.50$$

\therefore It is one tailed Right hand tail

$$n = 100$$

$$\bar{x} = 52.80$$

$$\alpha = 0.05$$

$$S.E = \frac{\sigma}{\sqrt{n}} = \frac{4.50}{\sqrt{100}}$$

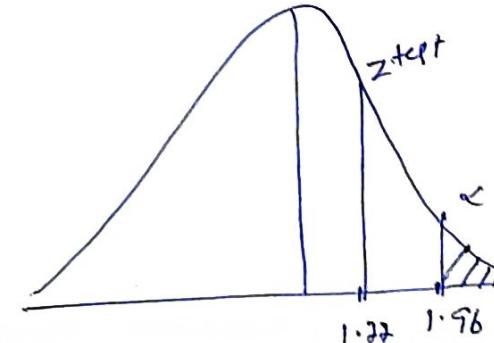
$$\boxed{S.E = 0.45}$$

$$\begin{aligned} Z(\text{test}) &= \frac{(\bar{x} - \mu)}{S.E} \\ &= \frac{52.80 - 52}{0.45} \end{aligned}$$

$$\boxed{Z(\text{test}) = 1.72}$$

$$\alpha = 0.05$$

$$\underline{Z(\alpha) = 1.96}$$



\therefore Here Z value doesn't fall under critical region.

Hence Accept the null hypothesis
Reject the alternate hypothesis

Prob - 3

$$\bar{x} = 34 \text{ ppm}$$

$$H_0: \mu = 34$$

$$\sigma = 8 \text{ ppm}$$

$$H_1: \mu \neq 34$$

$$\bar{x} = \cancel{32.5} \text{ ppm}$$

$$n = 50$$

$$\alpha = 1\%$$

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{8}{\sqrt{50}}$$

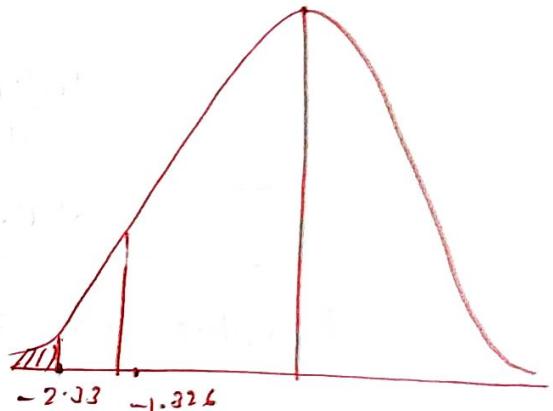
$$\boxed{SE = 1.131}$$

$$Z = \frac{32.5 - 34}{1.131}$$

$$\boxed{Z = -1.326}$$

$$\alpha = 1\% = 0.01$$

$$Z_{0.01} = -2.33$$



$\therefore Z$ value doesn't fall under the critical region Hence Accept the null hypothesis.

Prob - 4

$$H_0: \mu = 11.35$$

$$\mu = 11.35$$

$$H_1: \mu \neq 11.35$$

$$S = 240.37$$

$$\alpha = 0.05$$

$$n = 82$$

$$\bar{x} = 1031.22$$

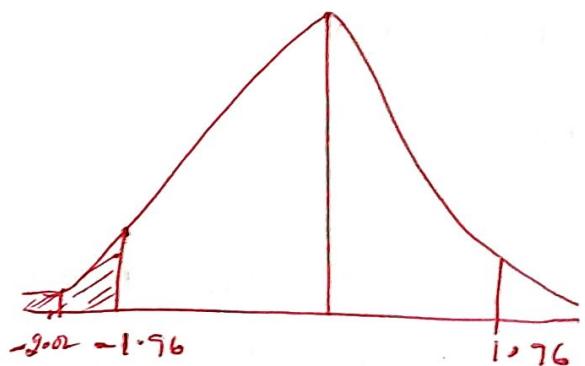
$$\begin{aligned}
 Z &= \frac{\bar{x} - \mu}{S/\sqrt{n}} \\
 &= \frac{1031.22 - 11.35}{240.37 / \sqrt{82}} \\
 &= -\frac{105.68}{51.25}
 \end{aligned}$$

$$\boxed{Z = -2.02}$$

The critical value is $z \in -1.96$ and $+1.96$

\therefore Z value falls under the critical region Hence reject the null hypothesis

Accept Alternate hypothesis



Problem - 5

$$\mu = 48,432$$

$$H_0: \mu = 48,432$$

$$\sigma = 2000$$

$$H_1: \mu \neq 48,432$$

$$n = 400$$

$$\bar{x} = 48,574$$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$\text{consider } \alpha = 10\%$$

$$= \frac{48,574 - 48,432}{\frac{2000}{\sqrt{400}}}$$

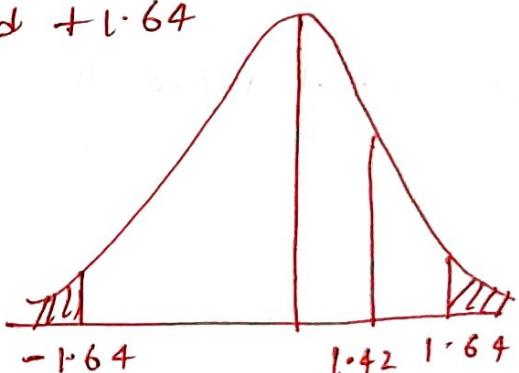
$$= \frac{142}{100} = 1.42$$

$$\boxed{Z = 1.42}$$

The critical value z is -1.64 and $+1.64$

$\therefore Z$ value doesn't fall under the critical region

Hence accept the null hypothesis



Prob - 6

$$\mu = 32.28$$

$$H_0: \mu = 32.28$$

$$S = 1.29$$

$$H_1: \mu \neq 32.28$$

$$n = 19$$

\therefore two tail test

$$\bar{x} = 31.67$$

$$\alpha = 0.05$$

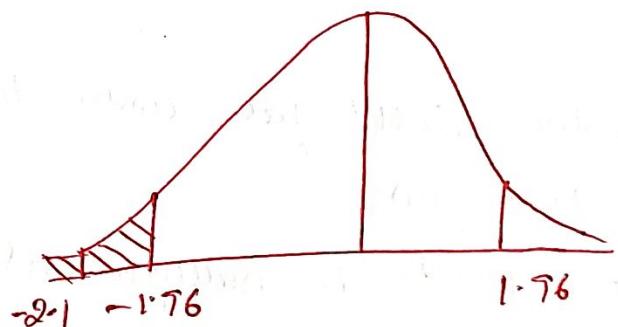
$$Z = \frac{\bar{x} - \mu}{S/\sqrt{n}}$$

$$= \frac{31.67 - 32.28}{1.29 / \sqrt{19}}$$

$$= \frac{-0.61}{0.29}$$

$$\boxed{Z = -2.1}$$

The critical value is -1.96 and $+1.96$



\therefore Z value fall under the critical region Hence
Accept the Alternate hypothesis

Prob - 8

$$n = 16$$

$$\bar{x} = 10$$

$$\bar{x} = 12$$

$$\sigma = 1.5$$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

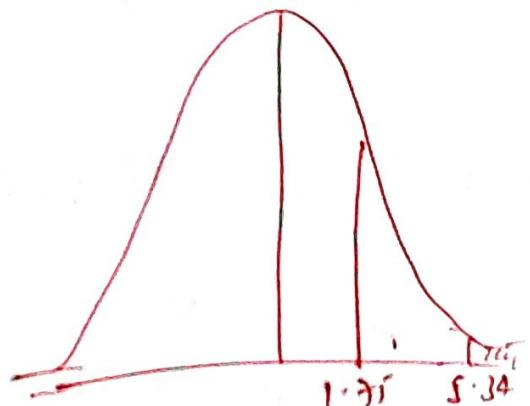
$$= \frac{12 - 10}{\frac{1.5}{\sqrt{16}}}$$

$$(t = 5.34)$$

$$df = 16 - 1 \\ = 15$$

$$(t \text{ score} \approx 1.75)$$

Assume $\alpha = 0.05$



Prob - 7

$$1 - \alpha = 0.99$$

$$\alpha = 0.01$$

$$t_{0.99} = -t_{0.01}$$

$$t_{0.99} = -t_{0.01}$$

$$= -2.602$$

$$df = n - 1$$

$$df = 16 - 1 \\ = 15$$

Prob - 10

$$n = 25$$

$$\sigma_x = \frac{\sigma}{\sqrt{n}}$$

$$\bar{x} =$$

$$= \frac{4}{\sqrt{25}}$$

$$\mu = 60$$

$$= 0.8$$

$$S = 4$$

$$-t_{0.05} < t < t_{0.10}$$

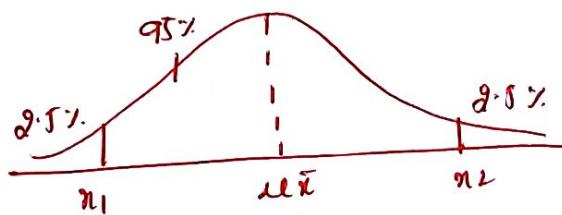
$$df = 25 - 1 \\ = 24$$

$$P(-t_{0.05} < t < t_{0.10})$$

exp

Let middle 95% of all sample means lies b/w

\bar{x}_1 and \bar{x}_2



t Score are

$$t_1 = t_{0.025}, 24 \\ = -2.064$$

$$t_2 = t_{0.025}, 24 = | -2.064 | = 2.064$$

$$\bar{x}_1 = 60 + 2.064 \times 4/5 \\ = 61.65$$

$$t_1 = \frac{\bar{x}_1 - \mu}{\sigma/\sqrt{n}}$$

$$\Rightarrow -2.064 = \frac{\bar{x}_1 - 60}{4/\sqrt{25}}$$

$$\bar{x}_1 - 60 = -2.064 \times 4/5$$

$$\bar{x}_1 = \frac{60 - 1.65}{4/5} \\ = 58.35$$

The 95% middle range is

$$[58.35 \text{ to } 61.65]$$

$$\begin{aligned} P(-0.05 < t < 0.10) \\ &= P(t < 0.10) - P(t < -0.05) \\ &= 0.5394 - 0.4803 \\ &= 0.0591 \end{aligned}$$

Prob - 11

$$n_1 = 1200$$

$$\bar{x}_1 = 452$$

$$\sigma_1 = 212$$

$$n_2 = 800$$

$$\bar{x}_2 = 523$$

$$\sigma_2 = 185$$

$$S.E = \sqrt{\left(\frac{\sigma_1^2}{n_1}\right) + \left(\frac{\sigma_2^2}{n_2}\right)}$$

$$S.E = \sqrt{\frac{(212)^2}{1200} + \frac{(185)^2}{800}}$$

$$S.E = 8.96$$

$$Z\text{test} = \frac{(\bar{x}_1 - \bar{x}_2)}{S.E}$$

$$Z\text{test} = \frac{(452 - 523)}{8.96}$$

$$= -7.926$$

$$\alpha = 0.05$$

$$\alpha/2 = 0.025$$

$$Z_{0.025} = 2.81$$

$$Z\text{test} < Z_{0.025}$$

$\therefore H_0$ will be rejected

PROB - 12

H_0 : Different people using different battery

H_1 : Same people using different battery

$$n_1 = 100$$

$$\bar{x}_1 = 308$$

$$\sigma = 84$$

$$n_2 = 100$$

$$\bar{x}_2 = 254$$

$$\sigma_2 = 67$$

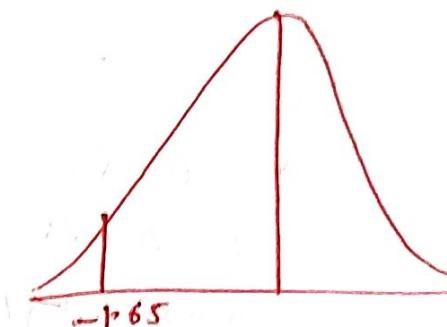
$$S.E = \sqrt{\left(\frac{\sigma_1^2}{n_1}\right) + \left(\frac{\sigma_2^2}{n_2}\right)}$$

$$\begin{aligned} S.E &= \sqrt{\frac{(84)^2}{100} + \frac{(67)^2}{100}} \\ &= 10.74 \end{aligned}$$

$$Z\text{ test: } \frac{(\bar{x}_1 - \bar{x}_2)}{S.E}$$

$$= \frac{308 - 254}{10.74} = 5.025$$

$$< 0.025 = -1.65$$



H₀ will be rejected because Z test
doesn't fall under Z_{α/2}

Prob - 13

$$X_1 = 0.317$$

$$H_0: \mu_1 = \mu_2$$

$$S_1 = 0.12$$

$$H_1: \mu_1 \neq \mu_2$$

$$n_1 = 14$$

$$n_2 = 9$$

$$X_2 = 0.21$$

$$S_2 = 0.11$$

$$\text{Pooled Std} = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}}$$

$$= \sqrt{\frac{13 \times 0.317^2 + 0.21^2}{14+9-2}}$$

$$\boxed{PS = 0.1162}$$

$$t = \frac{0.317 - 0.21}{0.1162 \times \sqrt{1/14 + 1/9}}$$

$$\boxed{t_{cal} = 1.552}$$

$$\begin{aligned} \text{Pr value} &= P(t_{n_1+n_2-2} > |t_{cal}|) \\ &= P(t_{21} > |2.1552|) \\ &= 0.031 \end{aligned}$$

As pr value < 0.05 we may reject H₀

Problem - 14

$$n_1 = 15$$

$$\bar{x}_1 = 6598$$

$$S_1 = 844$$

$$n_2 = 12$$

$$\bar{x}_2 = 6870$$

$$S_2 = 667$$

H_0 : Small price reduction is enough to increase sales

H_1 : Small price reduction is not enough to increase sales

$$S_{12} = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}}$$

$$= \sqrt{\frac{(15-1)(844)^2 + (12-1)(667)^2}{15+12-2}}$$

$$S_{12} = 771.7$$

$$S.E.$$

$$S.E. = S_{12} * \left(\sqrt{\frac{1}{n_1}} + \sqrt{\frac{1}{n_2}} \right)$$

$$S.E. = 289.96$$

$$t\text{-Stats} = \frac{|(\bar{x}_1 - \bar{x}_2)|}{S.E.}$$

$$t\text{-Stats} = \frac{|6598 - 6870|}{289.96}$$

$$t\text{-Stats} = 0.71$$

By making calculation every we will taking reference of P57.
to 0.05 and df = 295 will be = 1.708
t experimental < t0.05

H_0 will be accepted

Prob - 15

$N_1 = 1000$	population 1 : 1980
$X_1 = 53$	$P_1 = 0.53$
$P_1 = 0.53$	$n_1 = 100$ and not 1000
$N_2 = 100$	population 2 : 1985
$X_2 = 43$	$n_2 = 100$ So favourable are = 43
$P_2 = 0.53$	$\therefore P_2 = 0.43$ ie not 0.53

$$\alpha = 0.05$$

$$Z_{0.05} = 1.96$$

$$H_0: P_1 = P_2$$

$$H_1: P_1 \neq P_2$$

if R & tail Z test

Rejection region of 2 tail test if $\alpha = \{Z: |Z| > 1.96\}$

$$\hat{P}_1 = \frac{X_1}{N_1} = \frac{53}{100} = 0.53$$

$$\hat{P}_2 = \frac{X_2}{N_2} = 43/100 = 0.43$$

$$\bar{P} = \frac{X_1 + X_2}{N_1 + N_2}$$

$$\boxed{\bar{P} = \frac{53 + 43}{100 + 100} = 0.48}$$

$$Z = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\bar{P}(1-\bar{P})(1/n_1 + 1/n_2)}}$$

$$= \frac{0.53 - 0.43}{\sqrt{0.48(1-0.48)(1/100 + 1/100)}}$$

$$\boxed{Z = 1.415}$$

\therefore Since it is observed that $|Z| = 1.415 \leq Z_{0.05} = 1.96$ it is then
Accept the H_0

(6)

Population 1

$$n_1 = 300$$

$$x_1 = 120$$

$$\hat{p}_1 = 0.40$$

Population 2

$$n_2 = 700$$

$$x_2 = 140$$

$$\hat{p}_2 = 0.20$$

$$H_0: p_1 - p_2 \leq 0.10$$

$$H_1: p_1 - p_2 > 0.10$$

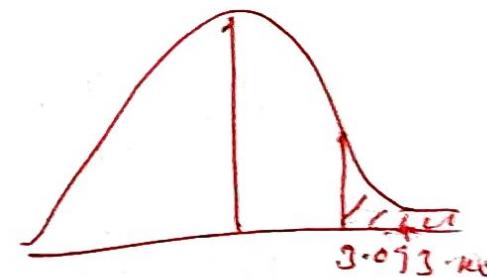
$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - D}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}} \\ = \frac{(0.40 - 0.20) - 0.10}{\sqrt{\left(\frac{0.40(0.60)}{300} + \frac{0.20(0.80)}{700}\right)}}$$

$$= \frac{0.10}{0.03202}$$

$$\boxed{Z = 3.118}$$

$$Z_{0.01} = 3.09$$

$\therefore H_0$ may be rejected



Prob - 17

Observed frequency (O)	Expected frequency (E)	$(O-E)^2$
15	22	49
20	22	4
25	22	9
18	22	49
29	22	49
28	22	$\frac{36}{196}$

B =>

Step 1 H_0 : die is unbiased

H_1 : The die is not unbiased

Step 2: on the hypothesis that the die is unbiased we should expect the frequency of each \approx to be

$$182/6 = 30.33$$

$$\begin{aligned} \therefore \chi_{\text{cal}}^2 &= \sum \frac{(O-E)^2}{E} \\ &= 198/22 \\ &= 8.91 \end{aligned}$$

Step 3: $2 \cdot 0.5(\alpha) = 0.08$

$$\begin{aligned} Df &= 6-1 \\ &= 5 \end{aligned}$$

Step 4 $(\chi_{\alpha})^2 = 11.0705$

Step 5 $\chi_{\text{cal}}^2 < (\chi_{\alpha})^2$

$\therefore H_0$ is accepted

\therefore Die is unbiased

Prob - 18

H₀: "Gender is independent of voting"
H_i: "Gender and voting are dependent"

	Men	Women	
voted	2791	3591	= 6386
Didnt vote	1486	2131	= 3617
	4278	5722	10,000

Expected value (men who voted) =

$$\frac{\text{Number (all who voted)} \times \text{Number (all men)}}{\text{Number (total no.)}}$$

$$= \frac{6386 \times 4278}{10,000}$$

$$= 2731$$

Expected value (women who voted) =

$$\frac{\text{Number (all who voted)} \times \text{Number (all women)}}{\text{Number (total number)}}$$

$$= \frac{6386 \times 5722}{10,000}$$

$$= 3652$$

Expected value (men who didn't vote) =

$$\frac{\text{Number (all who didn't vote)} \times \text{Number (all men)}}{\text{Number (total number)}}$$

$$= \frac{3617 \times 4278}{10,000}$$

$$= 1547$$

Prob - 18

Contn - -

Expected value (women who didn't vote) =

$$\frac{\text{Number (all who didn't vote)} \times \text{Number (all women)}}{\text{Number (total number)}}$$

$$= \frac{3617 \times 5722}{10,000}$$

$$= 2070$$

		men	women
voted	2731	3652	
didn't vote	1547	2070	

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

$$\begin{aligned} \chi^2 &= \frac{(2792 - 2731)^2}{2731} + \frac{(3591 - 3652)^2}{3652} + \frac{(1486 - 1547)^2}{1547} + \\ &\quad \frac{(2131 - 2070)^2}{2070}. \end{aligned}$$

$$\chi^2 = 1.4 + 1.0 + 2.4 + 1.8$$

$$\chi^2 = 6.6$$

$$\text{Degree of freedom} = 1 \times 1 = 1$$

Since the Chi-Square goodness of fit value (6.6)
Exceeds the critical $\chi^2 (3.84)$ we will
reject the null hypothesis.

19)

H_0 : All Candidates are equally popular

H_1 : All Candidates are not equally popular

$$\alpha = 0.05$$

$$\text{chi-square} = \sum \frac{(O-E)^2}{E}$$

$$N = 4$$

$$df = 4-1 \\ = 3$$

$$\lambda = 0.05$$

Critical value = 7.814

	O	E	$\frac{(O-E)^2}{E}$
Higgins	41	25	10.24
Pearson	19	25	1.44
White	24	25	0.04
Charlton	16	25	3.24
Total	100	100	14.96

$$\text{chi-square} = \sum \frac{(O-E)^2}{E} = 14.96$$

$$P\text{-value} = 0.0018$$

$$P\text{-value} < \alpha = 0.05$$

\therefore Reject the null hypothesis

Prob - 20

Observed	Data	A	B	C	Total
age of child	5-6 y	18	22	20	60
	7-8 y	9	28	10	47
	9-10 y	20	10	40	70
	Total	40	60	100	

Expected	Data	A	B	C
age of child	5-6 y	12	18	30
	7-8 y	14	21	35
	9-10 y	14	21	35

Expected value = $\frac{(\text{row total})(\text{column total})}{(\text{total value})}$

$$\chi^2 = \frac{(18-12)^2}{12} + \frac{(12-14)^2}{14} + \frac{(20-14)^2}{14} + \frac{(22-18)^2}{18} + \frac{(28-21)^2}{21} + \frac{(20-21)^2}{21} + \frac{(20-30)^2}{30} + \frac{(40-35)^2}{35} + \frac{(40-35)^2}{35}$$

$$= 29.60$$

$$\text{degree of freedom} = (3-1)(3-1) = 2 \times 2 = 4$$

at $P < 0.05 \approx 0.001$

$\chi^2_{0.001}$ at df = 4 will be 18.47

$$\chi^2_{\text{test}} > \chi^2_{0.01}$$

\therefore Reject the null hypothesis

Prob - 21

Observed frequency

	Support	No Support	Total
Conform	18	40	58
Not Conform	32	10	42
Total	50	50	100

Expected frequency

$$EF = \frac{\text{Row total} \times \text{column total}}{\text{grand total}}$$

	Support	No Support
conform	29	29
not conform	21	21

H₀: There is no relationship b/w the Support & no support condition in the frequency with which individuals are likely to conform

H₁: There is no relationship b/w Conform & no Conform condition in the frequency with which individuals are likely to conform.

$$\chi^2 = \sum \left(\frac{(O-E)^2}{E} \right)$$

$$\underline{\chi^2 = 19.87}$$

Also given that p-value < 0.05 ($\alpha = 0.05$ level of significance)

Contd... 2

If p-value > α -level of Significance then we
accept H_0 otherwise reject H_0

Here p-value ≤ 0.05

\therefore we reject H_0 at 5% level of Significance

Table calculation for Chi-Square test χ^2

Leadership	Height				Total	
	Short		Tall			
	Observed (O)	Expected (E)	Observed (O)	Expected (E)		
Leader	12	19.92	32	24.08	48 (P.)	
follower	22	16.29	14	19.71	36 (R.)	
Unclassified	9	6.79	6	8.21	15 (B.)	
Total	43 (e)	42	52 (o)	52	95 (T)	

$$\begin{aligned}
 \text{W.H.T} \quad \chi^2 &= \frac{\sum (O-E)^2}{E} \\
 &= \frac{(12-19.92)^2}{19.92} + \frac{(22-16.29)^2}{16.29} + \frac{(9-6.79)^2}{6.79} + \\
 &\quad \frac{(32-24.08)^2}{24.08} + \frac{(14-19.71)^2}{19.71} + \frac{(6-8.21)^2}{8.21} \\
 &= 3.15 + 2.00 + 0.32 + 2.60 + 1.65 + 0.59 \\
 &= 10.31
 \end{aligned}$$

$$\chi^2 = 10.31$$

$$df = (3-1)(2-1) = 2$$

\therefore H₀ is rejected

\therefore There is relationship b/w Height and leadership.

Continer - . . 2

If p value > α -level of Significance then we accept H_0 otherwise reject H_0

Here p-value ≤ 0.05

\therefore we reject H_0 as 5% level of Significance

Prob - 23

H₀: The distribution of labour force is same
for different marital status

H₁: The distribution of labour force is not same
for different marital status

(i)	married	widowed, divorced or separated	non married	total
Employed	679	103	114	896
unemployed	63	10	20	93
not in labour force	42	18	25	85
total	784	131	159	1074

Calculated Expected frequency by

$$\text{E} = \frac{\text{Row total} \times \text{Column total}}{\text{Grand total}}$$

E	Col 1	Col 2	Col 3
Row 1	654.06	109.28	132.6
Row 2	67.88	11.34	13.26
Row 3	62.04	10.36	12.58

$$\chi^2 = \frac{(O - E)^2}{E}$$

Observed Count (O)	Expected Count (E)	$\frac{(O - E)^2}{E}$
629	654.06	0.9509
191	189.28	0.3628
114	132.64	2.6154
63	67.88	0.3508
10	11.34	0.1583
20	13.28	2.8297
42	62.04	6.4712
18	10.36	5.6311
25	12.58	12.262

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

$$\boxed{\chi^2 = 31.6392}$$

$$Df = (r-1)(c-1)$$

$$= (3-1) \times (3-1)$$

$$\boxed{Df = 4}$$

$$\text{Critical value } \chi^2_{0.05} = 9.488$$

$\Rightarrow \chi^2_{0.05}$ has area of 0.05 to its right

\Rightarrow to find $\chi^2_{0.05}$ of any Chi-Square

$Df = 4$ & down from column 0.05
is 9.488

Contd prob. - 21

\Rightarrow to get p-value

chi-square. dist DS [31. 6592, 4]

The p-value is ≈ 0.0001

\Rightarrow The result is significant at $p < 0.05$

\therefore The test statistic is greater than
critical value we Reject null hypothesis

\Rightarrow At 5% level there is sufficient evidence

that the distribution are significantly different.