Coordinate Geometry

Using Matrix Analysis

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February 15, 2019

Question

One of the diameters of the circle, given by

$$x^2 + y^2 - 4x + 6y = 12$$

is a chord of the circle S whose centre is at (-3,2). Find the radius of S.

Question in MATRIX Form

One of the diameters of the circle, given by

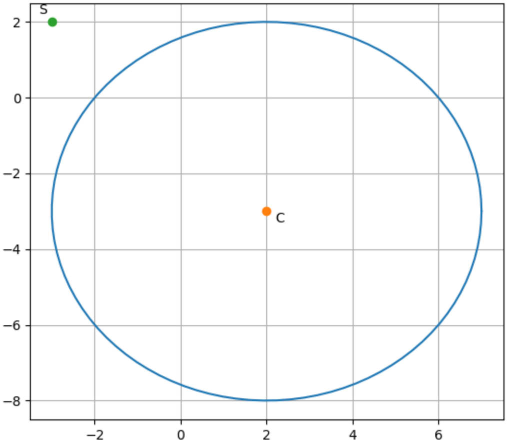
$$\mathbf{X^T X} + 2\begin{bmatrix}-2 & 3\end{bmatrix}\mathbf{X} = 12$$

is a chord of the circle S whose centre is at

$$\begin{bmatrix}-3\\2\end{bmatrix}$$

Find the radius of S.

Figure



Solution

Points on a circle with centre **C** and radius **R** would follow

$$(\mathbf{X}-\mathbf{C})^T(\mathbf{X}-\mathbf{C}) = R^2$$

Simplifying it :

$$\mathbf{X^TX}+\mathbf{C^TC}-\mathbf{C^TX}-\mathbf{X^TC} = R^2$$

In this case :

$$\mathbf{C^TX}=\mathbf{X^TC}$$

As **X** and **C** are one dimensional arrays.

Solution

So the equation of a circle centered at **C** and of radius **R** is

$$\mathbf{X}^{\mathbf{T}}\mathbf{X} - 2\mathbf{C}^{\mathbf{T}}\mathbf{X} = R^2 - \mathbf{C}^{\mathbf{T}}\mathbf{C}$$

Now comparing it with

$$\mathbf{X}^{\mathbf{T}}\mathbf{X} + 2\begin{bmatrix}-2 & 3\end{bmatrix}\mathbf{X} = 12$$

We get

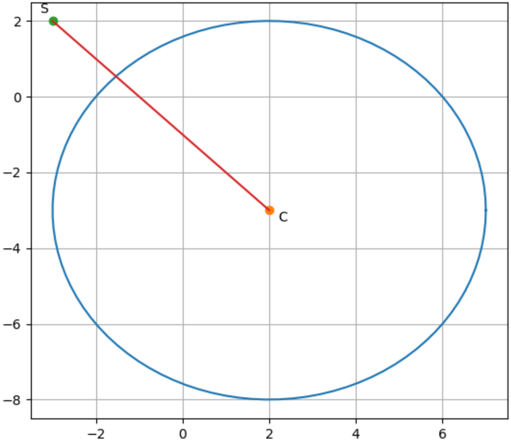
$$\mathbf{C} = \begin{pmatrix}2\\-3\end{pmatrix}$$

$$R^2 - \mathbf{C}^{\mathbf{T}}\mathbf{C} = 12$$

$$R = 5$$

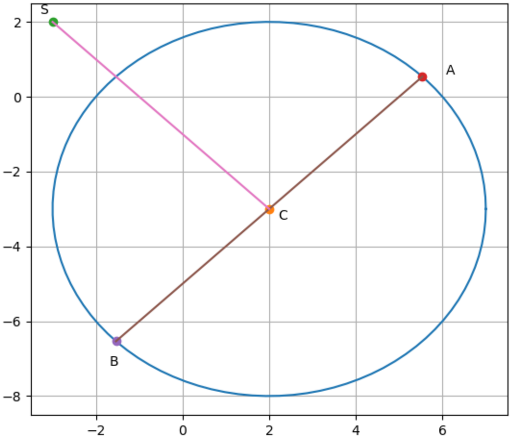
Solution

Given that chord of circle **S** is the diameter of the circle centered at **C**



Solution

CS would be perpendicular to the chord of **S** i.e, is perpendicular to a diameter of **C**



Solution

To find the points **A** and **B**, we find the points of intersection of the diameter perpendicular to **SC** and the circle.
If **D** is the direction vector of **SC**

$$D = S - C$$

$$D = \begin{pmatrix} -5 \\ 5 \end{pmatrix}$$

Solution

For the line **AB**, it's normal **N** is parallel to **SC**
So we can take

$$\mathbf{N} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Equation of **AB** is

$$\mathbf{N}^{\mathbf{T}}\mathbf{X} = p$$

As the centre of circle **C** passes through this line,

$$\mathbf{N}^{\mathbf{T}}\mathbf{C} = p$$

$$p = 5$$

Equation of **AB**

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{X} = 5$$

Solution

Equation of circle in parametric form is

$$\mathbf{X} = \mathbf{C} + R\mathbf{\Theta}$$

Where

$$\mathbf{\Theta} = \begin{pmatrix} cos\theta \\ sin\theta \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

Substituting this in the line equation would give us the value of θ at **A** and **B**

Solution

After substitution we get

$$\mathbf{N}^{\mathbf{T}}\left[\mathbf{C}+R\boldsymbol{\Theta}\right]=p$$

$$\mathbf{N}^{\mathbf{T}}\boldsymbol{\Theta}=\frac{p-\mathbf{N}^{\mathbf{T}}\mathbf{C}}{R}$$

We get

$$cos\theta=sin\theta$$

In the range $\theta \in [0, 2\pi]$

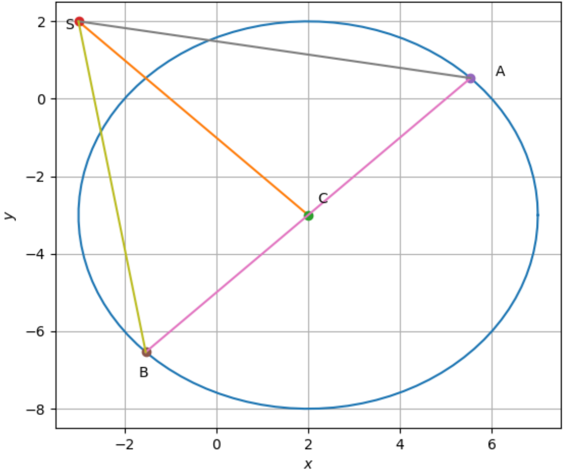
$$\theta=\frac{\pi}{4},\frac{5\pi}{4}$$

$$\mathbf{A}=\begin{pmatrix}2+\frac{5}{\sqrt{2}}\\-3+\frac{5}{\sqrt{2}}\end{pmatrix}$$

$$\mathbf{B}=\begin{pmatrix}2-\frac{5}{\sqrt{2}}\\-3-\frac{5}{\sqrt{2}}\end{pmatrix}$$

Solution

Δ**BCS** and Δ**ACS** are right angled triangles.



Solution

In Δ**ACS**

$$\vec{AS} = \mathbf{S} - \mathbf{A}$$

The radius of S

$$R_S = AS$$

$$R_S^2 = (\mathbf{S} - \mathbf{A})^T (\mathbf{S} - \mathbf{A})$$

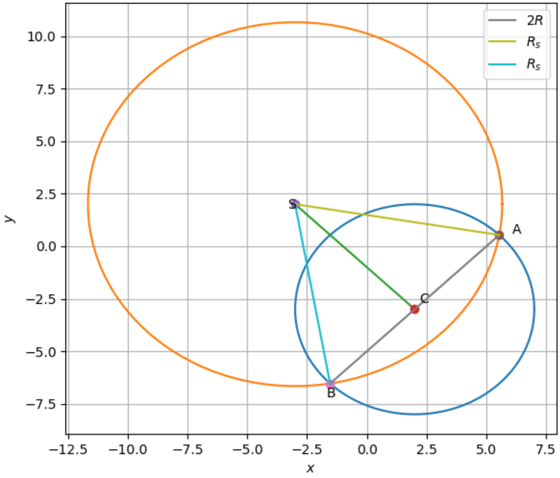
Where

$$\mathbf{A} = \begin{pmatrix} 2 + \frac{5}{\sqrt{2}} \\ -3 + \frac{5}{\sqrt{2}} \end{pmatrix}$$

$$\mathbf{S} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

$$\therefore R_S = 5\sqrt{3}$$

Figure



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