Empirical Finance Project: Volaitlity Forecasting, Factor Models & Portfolio Evaluation

April 3, 2025

WARWICK BUSINESS SCHOOL
THE UNIVERSITY OF WARWICK

Submitted in partial fulfilment of the requirements for Empirical Finance

Contents

1 Return Predictability

Examining whether conditional volatility (*cvol*), calculated by a GARCH(1,1) model, better predicts future market and excess returns when compared to historical realized volatility (*vol*), we use the dataset spanning from July 1963 to December 2022 and utilize daily excess market returns (Mkt-RF) from the Fama-French 5-factor database.

Volatility Characteristics. Calculating the realized volatility, defined as the annualized standard deviation of daily excess returns within each year. Realized volatility (Figure 1) shows notable spikes during crisis years—2008, 2020, and the early 2000s. These are clearly reflected in the distribution (Figure 2), which exhibits a right-skewed shape centered around 10–15%, with a long tail extending beyond 30%.The GARCH (1,1) conditional volatility is computed on the basis of daily returns and then aggregated into annual averages. Estimated GARCH coefficients (Table 1).

Table 1: **GARCH**

Parameter	Symbol	Value
mu	μ	0.050
omega	ω	0.011
alpha	α	0.100
beta	β	0.892

These values satisfy the GARCH stability condition:

$$\alpha + \beta < 1$$

and imply persistence in volatility shocks. The conditional volatility closely tracks realized volatility (Figure 1), but has a tendency to smooth out extreme shocks, aligning with theoretical expectations. The distribution (Figure 2) shows lower kurtosis and fewer outliers. Both volatilities remain within the expected range of 15%–25% over the long run. Realized vs Conditional Volatility, shows high co-movement between *vol* and *cvol* across time, with significant spikes during systemic crises.

Figure 1: **Realised vs Conditional Volatility**

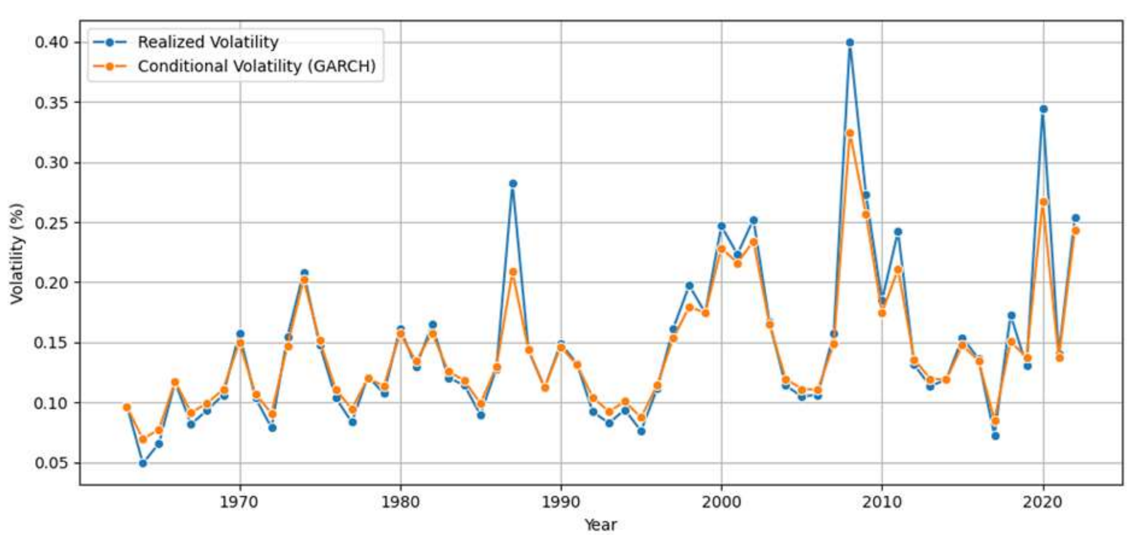


Figure 2: **Realised vs Conditional Volatility**

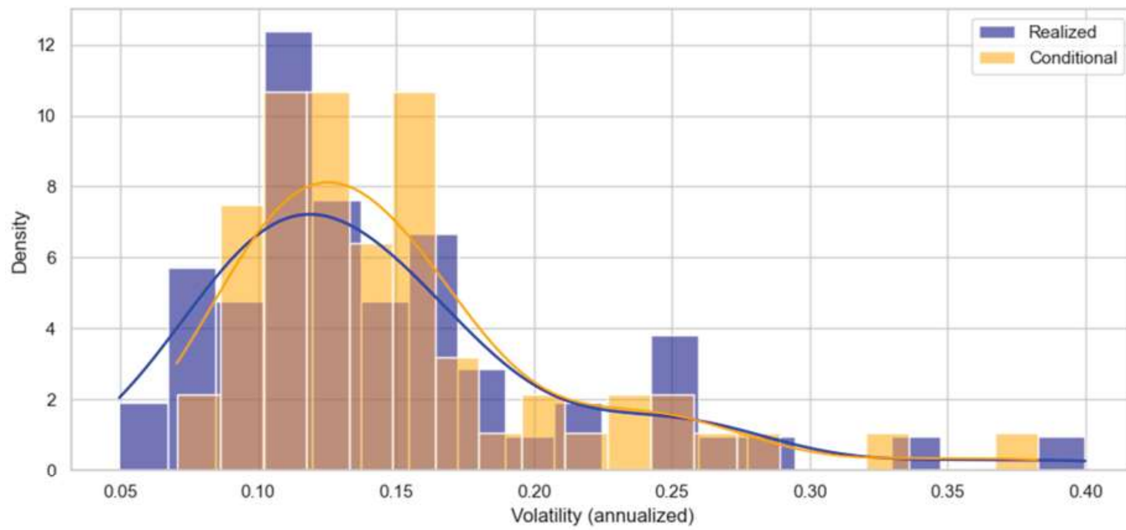


Table 2: **Predictive Regressions**

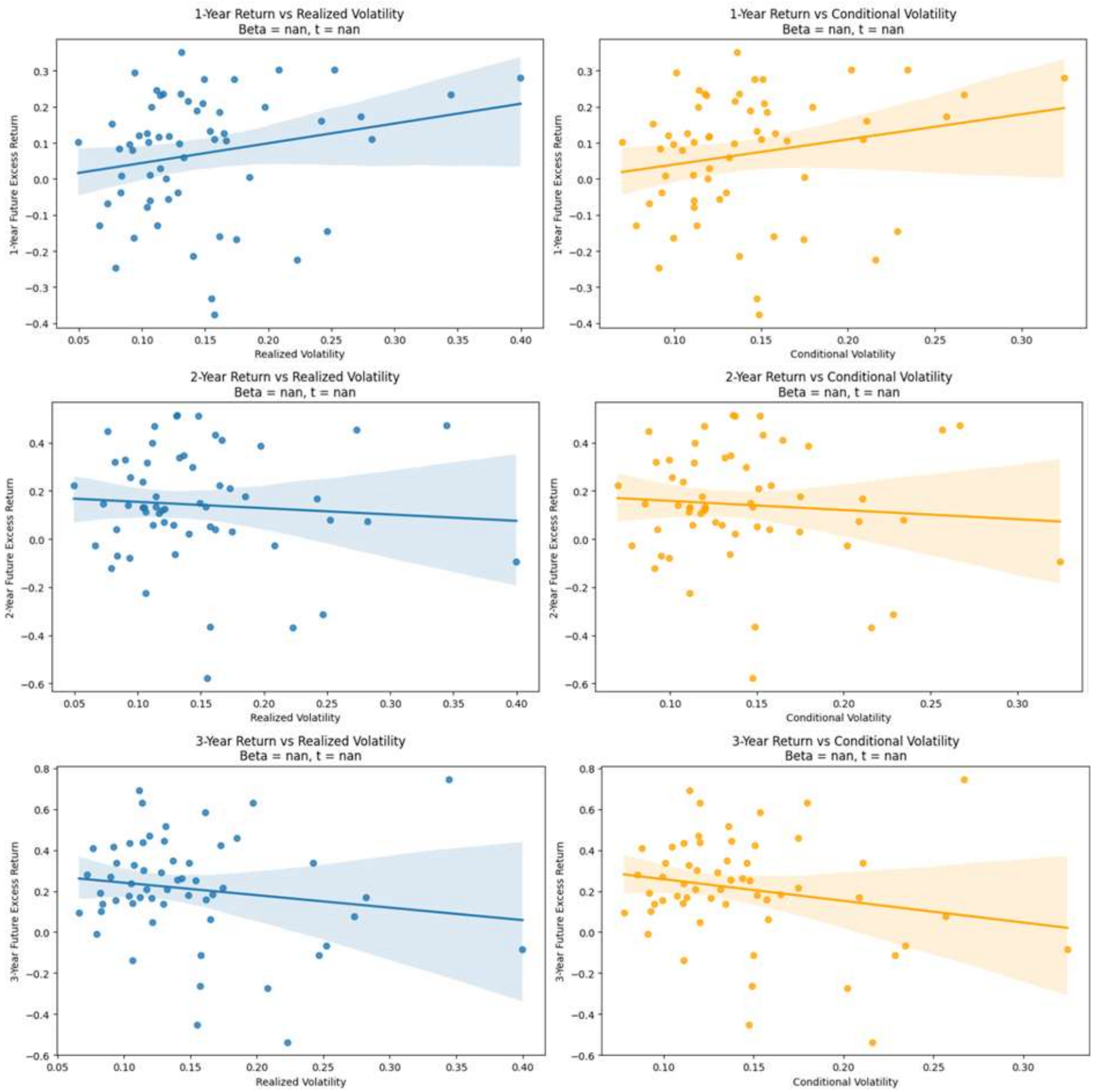
Horizon	Predictor	Beta	T-stat	R^2
1	Realized	0.548	2.262	0.046
1	GARCH	0.697	1.769	0.042
2	Realized	-0.262	-0.607	0.006
2	GARCH	-0.380	-0.624	0.007
3	Realized	-0.609	-0.913	0.025
3	GARCH	-1.058	-1.250	0.042
4	Realized	-0.506	-0.809	0.014
4	GARCH	-0.996	-1.215	0.030

At the 1-year horizon, both *vol* and *cvol* are positively and significantly related to future returns. However, realized volatility has a slightly stronger t-stat (2.26 vs. 1.77) and higher explanatory power (R^2 of 4.6% vs. 4.2%). This suggests that over short horizons, realized volatility contains more immediate predictive content, possibly due to more responsive incorporation of shocks.

At horizons of year 2 and 4, GARCH-based conditional volatility exhibits consistently higher betas and R^2 values than realized volatility. The 3-year mark GARCH R^2 (0.0415) is nearly double that of realized volatility (0.0247), and its beta magnitude is also greater. This suggests that while neither predictor is statistically significant in the long term, GARCH volatility better captures persistent, long-run risk dynamics.

The evidence here points to a short-run predictive power of realized volatility, but a more consistent signal from GARCH conditional volatility over medium-term horizons. This aligns with theoretical expectations (Andersen et al., 2003): realized volatility is more reactive, while GARCH smooths and embeds persistence, yielding more stable estimates for multiyear forecasting (Figure 3).

Figure 3: **Realised vs Conditional Volatility**



2 Non-stationary Time Series

Table 3: **Stationarity of Log Consumption**

Test Specification	**Test Statistic**	**p-value**	**5% Critical Value**	**Conclusion**
Intercept only (ARD model)	-4.0455	0.0012	-2.8739	Reject $H_0 \Rightarrow$ Stationary
Intercept + Trend (TS model)	-0.7174	0.9720	-3.4293	Fail to Reject $H_0 \Rightarrow$ Non-Stationary
Intercept + Trend + Lags (Lags=6)	-0.7174	0.9720	-3.4293	Fail to Reject $H_0 \Rightarrow$ Heavily Non-Stationary

This analysis explores if financial volatility and credit spreads can systematically forecast macroeconomic activity, specifically GDP growth, within the context of non-stationary economic time series. Using data spanning from 1963 Q1 to 2022 Q4, we test foundational assumptions of stationarity, investigate long-run equilibrium relations, and conduct prediction of quarterly volatility estimates.

Here we assess whether log consumption (log PCE) follows a stationary process. The *Augmented Dickey-Fuller (ADF)* test without a deterministic trend yields a test statistic of -4.0455 ($p = 0.0012$), allowing rejection of the unit root hypothesis at the 1% level. However, when we include a time trend, the test statistic becomes -0.7174 ($p = 0.9720$), indicating strong non-stationarity.

This divergence suggests that log(PCE) is not mean-stationary, but may instead be trend-stationary or integrated of order one, I(1) (Nelson and Plosser, 1982). This conclusion is visually confirmed in (Figure 4) and (Figure 5): the former shows a persistent upward trend, while the differenced series exhibits clear stationarity.

Figure 4: **Log Consumption (PCE) Time Series**

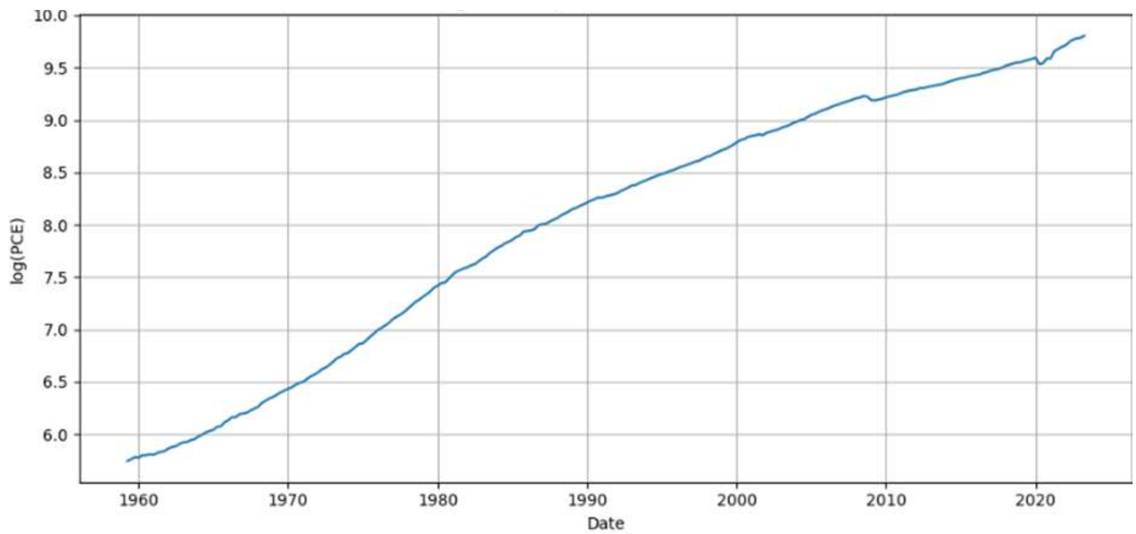
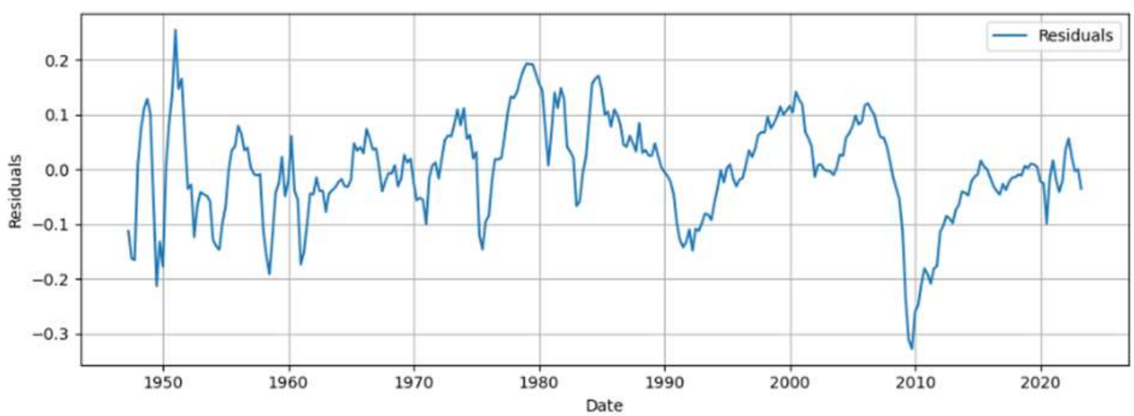
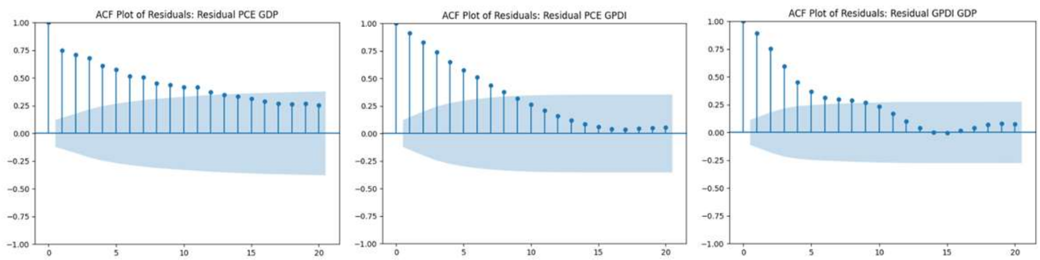


Figure 5: **Residuals from Regression**



Co-integration relationships among macroeconomic aggregates. Exploring a long run equilibrium relationship between macro variables via co-integration. All three variable pairs—*log(PCE) vs. log(GDP)* (ADF = –3.625, p = 0.005), *log(PCE) vs. log(Investment)* (ADF = –3.438, p = 0.0097), and *log(Investment) vs. log(GDP)* (ADF = –5.783, p ¡ 0.001)—reject the null of no co-integration at the 5% level. This hints towards strong evidence of stable long-run co-movement amongst key components of aggregate demand. Residual ACF plots confirm stationarity in the co-integrating errors (Figure 6).

Figure 6: **ACF Plots**



Volatility Aggregation and Its Macro Linkage. Daily excess market returns (Mkt – RF) were extracted from Kenneth French's database and aggregated into quarterly realized volatility, annualized using $\sqrt{252}$ scaling. The resulting series reflects well-known volatility events: Black Monday (1987), the Dot-Com bubble, the Global Financial Crisis (2008), and COVID-19. Volatility peaks at over 60% annualized. This volatility metric was merged with macroeconomic indicators from FRED for regression analysis.

Volatility and Credit Spread

OLS regressions were estimated of future GDP growth over horizons of 1q, 2q, 4q, 8q, and 16q against:

- Realized Volatility
- Credit Spread (BAA–AAA)
- Controls: $\Delta\log(\text{Investment})$, $\Delta\log(\text{Consumption})$

HAC standard errors have been employed to control for autocorrelation.

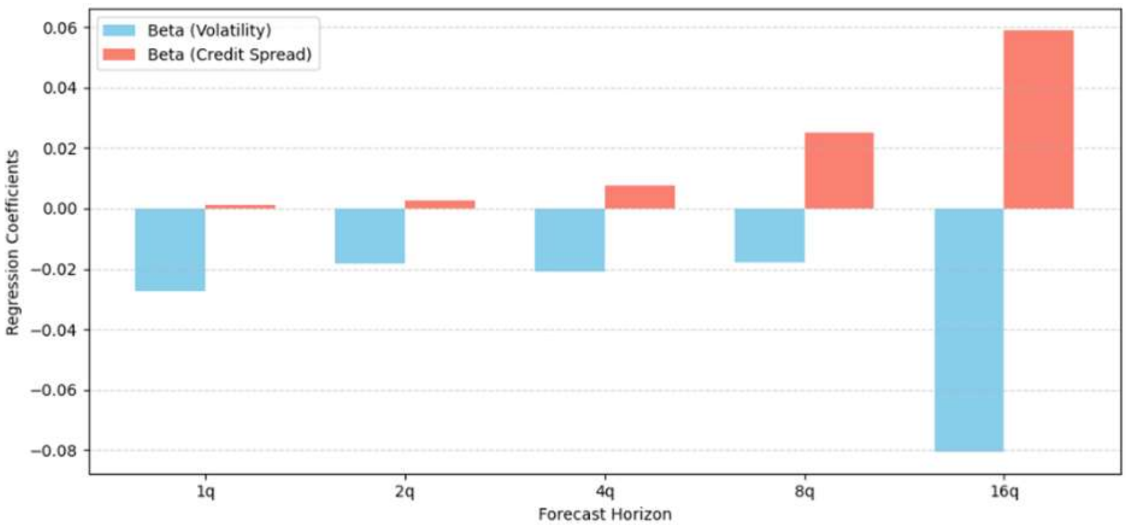
Table 4: **Volatility and Credit Spread**

Horizon	β_volatility	t-stat	β_credit	t-stat	R^2
1q	-0.028	-2.790	0.001	0.810	0.415
2q	-0.019	-0.820	0.003	0.860	0.269
4q	-0.021	-0.510	0.008	1.250	0.257
8q	-0.018	-0.240	0.025	2.230	0.249
16q	-0.081	-0.770	0.059	3.470	0.389

Several critical insights emerge. First, realized volatility is a strong short-run predictor, with a statistically significant negative coefficient at the 1-quarter horizon ($\beta = -0.02$, $t = -2.79$). This supports the idea that financial market uncertainty constrains near-term economic activity, possibly due to precautionary spending or investment delays (Bloom, 2009). However, volatility's predictive power diminishes at longer horizons, becoming statistically insignificant by 4q onward. This suggests volatility shocks are largely transitory and do not persistently drag long-term growth. In contrast, credit spreads rise in both economic magnitude and statistical significance over time, reaching $\beta = 0.059$ ($t = 3.47$) at 16 quarters. The spread likely captures systemic credit market stress and lending conditions that influence long-run investment decisions.

Figure 7: **Predictive Coefficients Across Horizons**

Notably, the opposite signs of volatility and credit spread imply contrasting dynamics: while volatility indicates short-term caution and macro fragility, rising spreads signal sustained financial distress that slows output growth. The results confirm that market-based indicators do offer predictive content for future GDP growth, but their effectiveness is horizon-dependent. Realized volatility negatively impacts short-run economic activity, while credit spreads dominate at medium-to-long-run horizons.



3 Factor Analysis

Table 5: **Performance Metrics**

The table displays the performance metrics of the four selected funds for the period 31 Jan 2000 to 31 Dec 2024. Moderate positive adjusted returns can be observed using the Sharpe Ratio,

$$S_a = \frac{E\left[R_a - R_b\right]}{\sigma_a}$$

The results demonstrate a trade-off between risk and return. VISGX's growth-oriented nature is reflected in the high return (0.7%) and risk (5.9%) reported. Conversely, FFIDX and PIODX offer more stability with lower volatility and correspondingly lower returns with Sharpe ratios of 0.424 and 0.418 respectively. However, FFIDX provides the best risk-adjusted return, making it ideal for risk-averse investors. Also, despite PRDSX's modest 0.6% return, it suffers from a poor Sharpe ratio of 0.372 due to its high volatility.

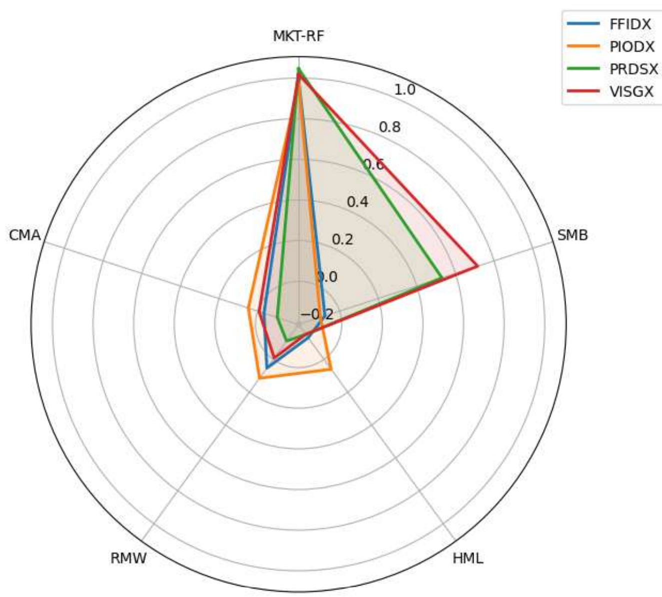
Fund	**Mean Excess Return**	**Std Dev**	**Sharpe Ratio**
FFIDX	0.006	0.046	0.424
PIODX	0.005	0.044	0.418
PRDSX	0.006	0.059	0.372
VISGX	0.007	0.059	0.427

Table 6: **Fama-French 5-Factor Model**

R^2 results (≥ 0.934) indicate that the Fama-French 5-factor model can explain most return variations of selected funds. Consequently, the Alphas are near zero, suggesting no significant outperformance after adjusting for the five factors. Figure 1 illustrates the explanatory power of each factor. Firstly, all funds have market betas (Mkt-RF) close to 1, moving in tandem with the market. VISGX's small cap (SMB) and growth (HML) bias align with its high mean return, and volatility. On the other hand, PIODX exhibits value (HML), profitability (RMW), and conservative (CMA) tilts, consistent with its low volatility. Meanwhile, FFIDX provides the most balanced returns due to its diversified exposure and uniform allocation across all factors. Lastly, PRDSX's underperformance may be attributed to a higher allocation in stocks with weak profitability (RMW) and aggressive investment strategies (CMA). This, along with a substantial small-cap exposure (SMB) can potentially leave it vulnerable to downturns.

Fund	Alpha	Mkt-RF	SMB	HML	RMW	CMA	R^2
VISGX	0.000	1.018	0.715	-0.153	-0.008	-0.008	0.934
FFIDX	-0.000	0.992	-0.076	-0.132	0.054	-0.033	0.958
PRDSX	0.000	1.046	0.530	-0.152	-0.111	-0.102	0.936
PIODX	-0.001	0.974	-0.100	0.060	0.115	0.048	0.939

Figure 8: Factor Analysis



4 Portfolio Creation and Evaluation

Table 7: **T-statistics**

T-statistics determine whether each factor's influence on a fund's return is statistically significant ($|t| > 2$ suggests significance). There are no significant alphas thus no true outperformance nor underperformance. However, all funds have extremely high Mkt-RF t-stats, implying that the market is the dominant driver for returns. The SMB factor statistically confirms VISGX and PRDSX as small-cap growth funds as opposed to FFIDX and PIODX, who both record a large-cap bias. Furthermore, PIODX t-stats show robust profitability (RMW) thus making it ideal for prudent investment management. Other than its large-cap bias of -3.632 SMB, FFIDX with an alpha of almost zero appears to be a passive fund.

Fund	Alpha	Mkt-RF	SMB	HML	RMW	CMA
VISGX	0.274	46.329	21.092	-4.243	-0.194	-0.150
FFIDX	-0.024	73.232	-3.632	-5.940	2.142	-0.998
PRDSX	0.203	48.449	15.915	-4.294	-2.803	-1.935
PIODX	-1.438	62.075	-4.115	2.349	3.978	1.251

Table 8: **Statistics**

Statistic	Value
Mean Daily Return	0.001
Standard Deviation	0.015
Skew	-0.172
Kurtosis	4.690

Table 9: **Sharpe Ratios**

Frequency	Portfolio	Nasdaq
Daily	0.079	0.040
Weekly	0.191	0.093
Monthly	0.423	0.212

Table 10: **Fama-French 5-Factor Regression**

Frequency	Alpha	Mkt-RF	R^2
Weekly	0.001	1.086	0.793
Monthly	0.000	1.157	0.823

Glossary

FFIDX Fidelity Freedom® Index 2010 Fund

PIODX PIMCO Income Fund Investor Class

PRDSX T. Rowe Price Spectrum Conservative Allocation Fund

VISGX Vanguard Small-Cap Growth Index Fund Admiral Shares

Mkt-RF Market Risk Premium (Market Return minus Risk-Free Rate)

SMB Small Minus Big (Size Premium)

HML High Minus Low (Value Premium)

RMW Robust Minus Weak (Profitability Premium)

CMA Conservative Minus Aggressive (Investment Premium)

References

Andersen, T.G. et al. (2003) ‘Modeling and Forecasting Realized Volatility’, Econometrica, 71(2), pp. 579–625.

Nelson, C.R. and Plosser, C.R. (1982) ‘Trends and random walks in macroeconmic time series’, Journal of monetary economics, 10(2), pp. 139–162.